HARMONY SEARCH ALGORITHM FOR OPTIMUM DESIGN OF SLAB FORMWORK

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Abstract– In this paper, harmony search algorithm is utilized for the optimum design of slab-formwork. This method is a numerical optimization technique developed recently that imitates the musical performance process when a musician searches for a better state of harmony.

The cost of the form components involved is considered as the objective function of the optimization problem. Constraints for the optimization problem are bending moment, shear, maximum deflection and imposed ACI code provisions. The use of the harmony search to formwork design problems provides optimum cross sections and optimum spacing of the form members (joist and stringer), while minimizing the total cost.

Keywords– Formwork design, harmony search algorithm, optimization, construction cost

1. INTRODUCTION

Slab formwork is normally designed to provide support for freshly placed concrete until it can support itself. A typical slab formwork consists of sheathing, joists, stringers and shores as illustrated in Fig. 1. Sheathing is in the form of sheets of plywood, joists and stringers behaving like beams, and shores that behave like columns. Joists, stringers and shores hold the sheathing in place and sheathing retains both the concrete and applied loads. Joists and stringers act horizontally and shores act vertically. Stringers are supported on shores and joists are supported on stringers.

Quality, safety and economy are three essential requirements for a good formwork [1]. Formwork should be rigid, watertight, braced and tied together so as to maintain position and shape during the construction activities. Formwork should be designed accurately, erected and kept rigid and tightly jointed with a proper finish. Formwork failure is a major cause for accidents during construction, and therefore a formwork has to be strong and safe. The form should be simple, easy to handle, standardized and reusable to fulfill the economic requirements.

The cost of formwork material and labor are about 30% of the concrete slab cost [2], as shown in Fig. 2. The cost of formwork material is approximately 12% of the total form cost [3]. Therefore, reduction of formwork material and labor cost decreases the overall construction cost. In this article, a harmony search-algorithm-based optimization technique is proposed for the design of the optimum sections of joist and stringer and their spacing.

The traditional design method uses the developed design tables and charts [4], taking into consideration the strength of forms to resist the applied loads and to provide sufficient stiffness to maintain an allowable deflection. Christian [5] used published data obtained from empirical guidelines and
recommendations given in [6], and Peurifoy [7] proposed an integrated microcomputer package for formwork design. Ringwald [8] proposed a set of design curves for specific types of wood, for which safe formwork design values, namely size and spacing of members, can be inferred. Hanna and Senousi [3] developed a computerized method for the optimum design of concrete-slab forms. Senousi and Ansari [2] developed a computer program for the design of slab formwork components. Some of these research works are based on empirical results and in all of them the design process evaluates only some of the possible solutions to find an optimum formwork design. It can be very unsatisfactory and uneconomical to select the sizes and spacing of form members (sheathing, joist, stringer and shore) empirically. Any under-designing and over-designing results in ultimate failure and excessive cost of the forms. Furthermore, one may not reach the complete cost advantage by selecting one of the near optimum solutions.

![Fig. 1. Components of a typical slab formwork](image1)

![Fig. 2. Typical cost breakdown for concrete slabs (Senousi and Ansari [2])](image2)

In the present method, the cost of the form components is minimized while maintaining the safety of the formwork. An optimal solution is evolved by the harmony search algorithm from a set of points limited by the given constraints. Formwork can be made of either wood or wood-metal composite. Metal alone can also be used for design, employing the proposed technique.
2. FORMWORK DESIGN

A formwork should be designed such that it safely supports all vertical and lateral loads that might be applied until such loads can be supported by the concrete structure. The design should be such that the formwork supports all the loads applied on it without excessive deflection or collapse leading to accidents. Vertical and lateral loads should be carried to the ground by the formwork system or by the in-place construction that has adequate strength for that purpose. The engineer can help overall economy in the structure by minimizing the formwork cost. Design of construction formwork involves a step-by-step analysis of sheathing and framing members. Formulas for shear, bending moment and deflection are traditionally used in designing and limiting the spacing of form members. Fig. 1 shows a typical slab form with its components. The procedure used in this paper for the formwork design is based on the standard equations available for the structural design contained in the ACI formwork standard, explained further by Hurd [4].

a) Loads

Loads applied on the forms are of two categories: (a) Vertical loads consisting of dead and live loads. The weight of the formwork plus the weight of the reinforcement and freshly placed concrete is the dead load. The live load includes the weight of the workers, equipment, material storage, and impact; and (b) lateral loads including the externally applied loads and the internally applied loads. The current formwork design specifications are based on allowable stresses developed due to these loads, as specified by Hurd [4].

Form members are designed to bear the effects of imposed loads. Major slab formwork loads include dead load, live load, wind load and equipment impact load. ACI Committee 347 recommends [9] that both vertical supports and horizontal framing components of the formwork should be designed for a minimum live load of 2.4 kPa (50 psf) of horizontal projection to provide for the weight of personnel, runways, screeds (equipment used for precise strike-off and consolidation of concrete surfaces) and other equipment. The formwork should be designed for a live load of not less than 2.4kPa (50psf) of the horizontal projection. When motorized carts are used, the live load should not be less than 3.6 kPa (75psf). The design load for the combined dead and live loads should not be less than 4.8 kPa (100psf) or 6.0 kPa (125psf) if motorized carts are used.

Density of the ordinary concrete including the reinforcement is taken as 24 kN/m$^3$ (150 lb/ft$^3$). The weight of the formwork usually becomes about 0.15-0.73kPa (3-15 psf). Live load is comprised of the weight of workmen, the construction equipment and storage materials together with the impact load of 2.4kPa (50psf). In this study loads are defined as follows:

\[
\text{Design load} = \text{Dead load} + \text{Live load}
\]

\[
\text{Dead load of concrete and steel} = t \times 7.2\text{kPa} = (t \times 150\text{psf})
\]

Where “t” is the thickness of the slab in meter or foot.

Taking the weight of the form as 0.348kPa (8psf) and the minimum recommended live load as 2.4kPa (50psf),

\[
\text{the total load} = t \times 7.2 + 2.4 + 0.384\text{kPa} = (t \times 150 + 50 + 8\text{psf})
\]

b) Stress

Bending members (sheathing, joists and stringers) are analyzed for bending moment, shear and deflection. These are considered as uniformly loaded and supported on three or more spans. Shoring is
analyzed for compressive loads and also for the bearing stresses developed at supports. The traditional stress equations as specified by Hurd [4] are used for the formulation of the constraints.

Partially seasoned wood which is normally used for the sheathing has a moisture content of more than 19% and hence, allowable stresses must be decreased by a factor of 0.86 for bending, 0.97 for horizontal shear and modulus of elasticity.

3. DESIGN STEPS

Step 1: Estimate design loads
Step 2: Sheathing thickness and spacing of its supports (joist spacing)
Step 3: Joist size and spacing of supports (stringer spacing)
Step 4: Stringer size and span (shore spacing)
Step 5: Shore design to support stringers
Step 6: Check bearing stresses

Based on the computed joist spacing, the joist itself is analyzed to determine its maximum allowable span. Joists are assumed to be continuous over three or more spans. Each joist must support the load from the sheathing halfway over to the adjacent joist on either side. Therefore, the width of the load carried by the joist is equal to the spacing of the joists. The selected joist span becomes the spacing of the stringers. Based on the obtained stringer spacing, the process is repeated to determine the maximum stringer span which is the distance between the vertical supports (shores). Joist loads will be acting on the stringer as a series of concentrated loads, but for the sake of simplicity the load is treated as a uniformly distributed load. Once the distance between the vertical supports (shores) is obtained, the load to be carried by each shore should be less than the safe working load available for the shores which can be calculated as the product of shore spacing, stringer spacing, and design load. A steel shore was assumed for this illustration.

Step 1: Estimate design loads

The total load = t×7.2+2.4+0.384kPa = (t×150+50+8psf) (1)

Step 2: Sheathing thickness and spacing of its supports (joist spacing)

Check bending:

For a beam uniformly loaded and continuous over three or more spans, the following expressions are used for the design:

\[
\text{Max bending moment (M)} = \frac{wl^2}{120} \text{ (in.lb)}
\]  
\[
\text{Max bending moment (M)} = \frac{wl^2}{10} \text{ (N.mm)}
\]

Resisting bending moment (M) = \( f \times S \) \hspace{1cm} (3)

where,

- \( f \) = calculated unit stress in bending (MPa or psi),
- \( w \) = uniform load (N/mm or lb/ft),
- \( S \) = section modulus (mm\(^3\) or in\(^3\)),
- \( M \) = maximum bending moment (N.mm or in.lb), and \( l \) = spacing (mm or in)
From (2) and (3) we obtain the followings:

\[
\text{Maximum spacing} \ (l) = 10.95 \sqrt{\frac{FS}{w}} \text{ (in.)} \quad (4)
\]

\[
\text{Maximum spacing} \ (l) = 3.16 \sqrt{\frac{FS}{w}} \text{ (mm)}
\]

Check deflection:

Forms must be designed so that the members will not deflect beyond certain maximum values to avoid the formation of bulges and cracks. Deflection is a governing factor where the depth of the member is relatively small compared to its span. Hence for sheathing, deflection will be the governing factor while for joists and stingers the bending and shear are often the governing factors.

\[
\text{Deflection} = \frac{wl^4}{12 \times 145EI} \text{ (in)} \quad (5)
\]

\[
\text{Deflection} = \frac{wl^4}{145EI} \text{ (mm)}
\]

For Allowable deflection = \( l/360 \) \quad (6)

\[
\text{Maximum allowable span} \ (l) = 1.69 \sqrt[3]{\frac{EI}{w}} \text{ (in)} \quad (7)
\]

\[
\text{Maximum allowable span} \ (l) = 0.738 \sqrt[4]{\frac{EI}{w}} \text{ (mm)}
\]

For Allowable deflection = 1/16 \quad (8)

\[
\text{Maximum allowable span} \ (l) = 3.23 \sqrt[4]{\frac{EI}{w}} \text{ (in)} \quad (9)
\]

\[
\text{Maximum allowable span} \ (l) = 0.79 \sqrt[4]{\frac{EI}{w}} \text{ (mm)}
\]

Check rolling shear:

\[
F_s = \frac{VQ}{Ib} \quad (10)
\]

Since \( V_{\text{max}} = 0.6wl \) \quad (11)

therefore

\[
l = \frac{F_s}{0.6w} \times \frac{Ib}{Q} \quad (12)
\]

Where, \( E \) = modulus of elasticity (Mpa or psi), \( I \) = moment of inertia (mm\(^4\) or in\(^4\))

**Step 3: Joist size and spacing of supports (stringer spacing)**

\[
w = \frac{\text{joist spacing (in)}}{12 \text{ (in/ft)}} \times \text{design load (psf)} \quad (13)
\]
Check bending:

Maximum spacing \( l = 10.95 \sqrt{\frac{F_b S}{w}} \) (in)  \hspace{1cm} (14)

Maximum spacing \( l = 3.16 \sqrt{\frac{F_S}{w}} \) (mm)

\( F_b \) = extreme fiber bending stress

Check deflection:

Maximum allowable span \( l = 1.69 \sqrt[3]{\frac{EI}{w}} \) (in)  \hspace{1cm} (15)

Maximum allowable span \( l = 0.738 \sqrt[3]{\frac{EI}{w}} \) (mm)

Check shear:

\begin{align*}
\frac{f_v}{bd} & = \sqrt{\frac{12}{290}} \\
I & = \frac{f_v b d}{0.9 w} + \frac{2d}{12}
\end{align*}

\hspace{1cm} (16) \hspace{1cm} (17)

**Step 4: Stringer size and span (shore spacing)**

\begin{align*}
w &= \frac{\text{stringer spacing (in)}}{12 \text{ (in/ft)}} \times \text{design load (psf)} \\
\frac{w}{120} &= \frac{\text{stringer spacing (cm)}}{100 \text{ (cm/m)}} \times \text{design load (kPa)}
\end{align*}

Check bending:

Maximum spacing \( l = 10.95 \sqrt{\frac{F_v S}{w}} \) (in)  \hspace{1cm} (19)

Maximum spacing \( l = 3.16 \sqrt{\frac{F_S}{w}} \) (mm)

Check deflection:

Maximum allowable span \( l = 1.69 \sqrt[3]{\frac{EI}{w}} \) (in)  \hspace{1cm} (20)

Maximum allowable span \( l = 0.738 \sqrt[3]{\frac{EI}{w}} \) (mm)

Check shear:

\begin{align*}
\frac{F_v}{bd} & = \frac{0.9w}{b d} (1 - \frac{2d}{12})
\end{align*}

\hspace{1cm} (21)
Harmony search algorithm for optimum design…

\[
I = \frac{F_v b d}{0.9 w} + \frac{2d}{12}
\]  

(22)

\(F_v\) = shear parallel to grain, and should be adjusted for horizontal shear by a factor of 2.

\[
F_v' = 2 \times F_v
\]

(23)

**Step 5: Shore design to support stringers**

Assume the head piece of the adjustable steel shore is 292×92mm (11½×3 5/8in). The load carrying capacity of steel shore is assumed as 22.2kN (98kips).

**Step 6: Check bearing stresses**

The bearing stresses are critical in the design of formwork and are produced when one member rests upon another. The bearing forces applied to the horizontal timber member cause compression perpendicular to grain. The allowable stresses for compression perpendicular to the grain are lower than those for compression parallel to the grain. These stresses need to be analyzed wherever joists rest on stringers, and where stringers rest on shores. Otherwise, the formwork will settle out of position or will have undesirable cracks and openings.

Stringers bearing on shore:

\[
\text{Bearing stress} = \frac{\text{total shore load}}{\text{bearing area}} \leq F_{c\perp}
\]

(24)

Joist bearing on Stringers:

Average load transmitted by joist to stringer is = joist spacing \(\times\) joist span \(\times\) form load

\[
\text{Bearing stress} = \frac{\text{total shore load}}{\text{bearing area}} \leq F_{c\perp}
\]

(25)

\(F_{c\perp}\) = allowable compression perpendicular to grain

**4. HARMONY SEARCH METHOD**

In the last decade meta-heuristic algorithms have found many applications in civil engineering optimization problems [10-12]. Harmony search (HS) algorithm was recently developed in an analogy with the music improvisation process where music players improvise the pitches of their instruments to obtain better harmony. HS is applied to some optimization problems in structural engineering, [13-16].

The steps in the procedure of harmony search are as follows:

1. **Step 1. Initialize the problem and algorithm parameters.**
2. **Step 2. Initialize the harmony memory.**
3. **Step 3. Improvise a new harmony.**
4. **Step 4. Update the harmony memory.**
5. **Step 5. Check the stopping criterion.**

These steps are described in the next five subsections.

**a) Initialize the problem and algorithm parameters**

In Step 1, the optimization problem is specified as follows:

Minimize \(f(x)\) subjected to \(x_i \in X\) \((i = 1, 2, \ldots, N)\)

(26)
Where \( f(x) \) is an objective function; \( x \) is the set of each decision variable \( x_i \); \( N \) is the number of decision variables, \( x_i \) is the set of the possible range of values for each decision variable, that is \( L x_i \leq x_i \leq U x_i \) and \( L x_i \) and \( U x_i \) are the lower and upper bounds for each decision variable. The HS algorithm parameters are also specified in this step. These are the harmony memory size (HMS), or the number of solution vectors in the harmony memory; harmony memory considering rate (HMCR); pitch adjusting rate (PAR); and the number of improvisations (NI), or stopping criterion.

The harmony memory (HM) is a memory location where all the solution vectors (sets of decision variables) are stored. This HM is similar to the genetic pool in the GA. Here, HMCR and PAR are parameters that are used to improve the solution vector. Both are defined in Step 3.

**b) Initialize the harmony memory**

Harmony memory matrix is initialized. Each row of harmony memory matrix contains the values of design variables which are randomly selected feasible solutions from the design pool for that particular design variable. Hence, this matrix has \( n \) columns where \( N \) is the total number of design variables and HMS rows which are selected in the first step. HMS is similar to the total number of individuals in the population matrix of the genetic algorithm. The harmony memory matrix has the following form:

\[
[H] = \begin{bmatrix}
    x_{1,1} & x_{2,1} & \cdots & x_{n-1,1} & x_{n,1} \\
    x_{1,2} & x_{2,2} & \cdots & x_{n-1,2} & x_{n,2} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    x_{1,hms-1} & x_{2,hms-1} & \cdots & x_{n-1,hms-1} & x_{n,hms-1} \\
    x_{1,hms} & x_{2,hms} & \cdots & x_{n-1,hms} & x_{n,hms} 
\end{bmatrix}
\]  

(27)

\( x_{ij} \) is the value of the \( i \)th design variable in the \( j \)th randomly selected feasible solution. These candidate designs are sorted such that the objective function value corresponding to the first solution vector is the minimum. In other words, the feasible solutions in the harmony memory matrix are sorted in descending order according to their objective function value. It is worthwhile to mention that not only the feasible designs which satisfy the constraints inserted into the harmony memory matrix, but those designs having a small infeasibility are also included in the harmony memory matrix with a penalty on their objective function.

**c) Improvise a new harmony**

In generating a new harmony matrix, the new value of the \( i \)th design variable can be chosen from any discrete value within the range of the \( i \)th column of the harmony memory matrix with the probability of \( HMCR \) which varies between 0 and 1. In other words, the new value of \( x_i \) can be one of the discrete values of the vector \( \{ x_{i,1}, x_{i,2}, \ldots, x_{i,hms} \}^T \) with the probability of \( HMCR \). The same is applied to all other design variables. In the random selection, the new value of the \( i \)th design variable can also be chosen randomly from the entire pool with the probability of \( 1 - HMCR \). That is

\[
x_i^{\text{new}} = \begin{cases} 
    x_i \in \{ x_{i,1}, x_{i,2}, \ldots, x_{i,hms} \}^T & \text{with probability } HMCR \\
    x_i \in \{ x_1, x_2, \ldots, x_{ns} \}^T & \text{with probability } (1 - HMCR)
\end{cases}
\]

(28)

Where \( ns \) is the total number of design variables in the pool. If the new value of the selected design variable is among those of the harmony memory matrix, this value is then checked whether it should be
pitch adjusted. This operation uses pitch adjustment parameter PAR that sets the rate of adjustment for the pitch chosen from the harmony memory matrix as follows:

\[
\text{Is } x_i^{\text{new}} \text{ to be pitch-adjusted?} \begin{cases} 
\text{Yes} & \text{with probability of PAR} \\
\text{No} & \text{with probability of (1 - PAR)} 
\end{cases}
\] (29)

Supposing that the new pitch-adjustment decision for \(x_i^{\text{new}}\) came out to be yes from the test and if the value selected for \(x_i^{\text{new}}\) from the harmony memory is the \(k\)th element in the general discrete set, then the neighboring value \(k+1\) or \(k-1\) is taken for new \(x_i^{\text{new}}\). This operation prevents stagnation and improves the harmony memory for diversity with a greater chance of reaching the global optimum.

d) Update harmony memory

After selecting the new values for each design variable the objective function value is calculated for the new harmony vector. If this value is better than the worst harmony vector in the harmony matrix, it is then included in the matrix while the worst one is taken out of the matrix. The harmony memory matrix is then sorted in descending order by the objective function value.

e) Check stopping criterion

Steps 3 and 4 are repeated until the termination criterion, which is the preselected maximum number of cycles, is reached. This number is selected large enough such that within this number of design cycles no further improvement is observed in the objective function.

5. IMPROVED HARMONY SEARCH ALGORITHM

The HMCR and PAR parameters introduced in Step 3 help the algorithm to find globally and locally improved solutions, respectively.

PAR in the HS algorithm is an important parameter in suitable tuning of the optimized solution vectors, and can be useful in adjusting the convergence rate of the algorithm to an optimal solution. Therefore, good adjustment of this parameter is of great importance. The traditional HS algorithm uses a fixed value for PAR. In the HS method the PAR value is adjusted in the initialization step (Step 1) and cannot be changed during new generations. The main drawback of this method corresponds to the number of iterations the algorithm needs to find an optimal solution. Small PAR values can cause poor performance of the algorithm and considerable increase in iterations will then be needed to find the optimum solution.

The main difference between the improved harmony search (IHS) algorithm developed by [17] and the classic HS method is in the way of adjusting the PAR. In order to improve the performance of the HS algorithm and to eliminate the drawbacks associated with the fixed values of PAR, IHS algorithm uses variable PAR in the improvisation step (Step 3). PAR change dynamically with the generation number by the following relationship:

\[
\text{PAR}(\text{gn}) = \text{PAR}_{\text{min}} + \frac{\text{PAR}_{\text{max}} - \text{PAR}_{\text{min}}}{\text{NI}} \times \text{gn}
\] (30)

Where

- PAR: the pitch adjusting rate for each generation;
- PAR_{\text{min}}: the minimum pitch adjusting rate;
- PAR_{\text{max}}: is the maximum pitch adjusting rate;
6. OPTIMUM DESIGN ALGORITHM

The input data consist of

- structural inputs (length of slab, thickness of slab, breadth of slab),
- sheathing inputs (sheathing thickness, sheathing width (breadth), sheathing length, bending stress, elastic modulus, shear stress),
- joist and stringer inputs (bending stress, elastic modulus, shear stress, compression perpendicular to the grain), and
- Shore inputs (bending stress, elastic modulus, shear stress, end plate dimensions, shore capacity).

The number of reuses can be judged based on the care exerted by the form handlers during the dismantling, erecting and storage of the formwork. The expected reuse was taken as 4 for sheathing, joist and stringer and 25 for the steel shores.

The objective function for the optimization problem can be formulated as:

\[ C = N_1 \times C_1 + N_2 \times C_2 + N_3 \times C_3 + N_4 \times C_4 + A \times t \times C_5 \]  

in which,

\[ N_1 = \text{Number of required sheathing}, \quad C_1 = \text{Unit cost of sheathing}, \]
\[ N_2 = \text{Number of required joists}, \quad C_2 = \text{Unit cost of joist}, \]
\[ N_3 = \text{Number of required stringers}, \quad C_3 = \text{Unit cost of stringers}, \]
\[ N_4 = \text{Number of required shores}, \quad C_4 = \text{Unit cost of shores}, \]
\[ A = \text{Area of slab}, \quad C_5 = \text{Unit labor cost for unit volume of concrete} \]
\[ t = \text{thickness of slab} \]

Harmony search algorithm initiates the design process by first selecting values randomly for joist and stringer sizes in each iteration. HS attempts to evolve the optimum dimensions for these members that will eventually result in minimum total cost.

The design algorithm consists of the following steps.

1. Select the values of harmony parameters. The harmony memory size HMS, the harmony memory considering rate HMCR and the pitch adjustment rate PAR are selected. These values are decided after carrying out several trials in the design example.
2. Generate a harmony memory matrix. Select values randomly for joist and stringer sizes.
3. The evolved shore and stringer sizes are used to compute the allowable spacing using the equations described in the design steps. The minimum of this value is then selected as the spacing for members. The spacing of shore, stringer, and joist are limited by considering the stringer bearing on shore, joist bearing on stringer, and load carrying capacity of each shore.
4. Check whether the new design vector selected should be pitch-adjusted as explained in step 3 of the harmony search method.
5. Calculate the objective function value for the newly selected design vector. If this value is better than the worst harmony vector in the harmony matrix it is then included in the matrix, while the worst one is taken out of the matrix. The harmony memory matrix is then sorted in descending order by the objective function value.
6. Repeat steps 2 and 6 until the pre-selected maximum number of iterations is reached.
7. OPTIMAL DESIGN EXAMPLE AND COMPARISON WITH OTHER WORKS

Hanna and Senouci [3] worked on design optimization of concrete-slab forms, however, they used a wooden shore. Results of the traditional design and design optimization of concrete-slab forms by Hanna and Senouci [3] are shown in Table 1 and Table 2, respectively.

### Table 1. Traditional design of each slab form

<table>
<thead>
<tr>
<th>Slab thickness in(cm)</th>
<th>Sheathing thickness in(cm)</th>
<th>Joist dimension in×in (cm×cm)</th>
<th>Stringer dimension in×in (cm×cm)</th>
<th>Shore dimension in×in (cm×cm)</th>
<th>Joist spacing in(cm)</th>
<th>Stringer spacing in(cm)</th>
<th>Shore spacing in(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(15.24)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>64(162.56)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>7(17.78)</td>
<td>0.625(1.59)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>24(60.96)</td>
<td>48(121.92)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>8(20.32)</td>
<td>0.625(1.59)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>48(121.92)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>9(22.86)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>48(121.92)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>10(25.4)</td>
<td>0.75(1.9)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>24(60.96)</td>
<td>32(81.28)</td>
<td>64(162.56)</td>
</tr>
<tr>
<td>11(27.94)</td>
<td>0.625(1.59)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>39(99.06)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>12(30.48)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>39(99.06)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>13(33.02)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>16(40.64)</td>
<td>48(121.92)</td>
<td>32(81.28)</td>
</tr>
<tr>
<td>14(35.56)</td>
<td>0.75(1.9)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>32(81.28)</td>
<td>48(121.92)</td>
</tr>
</tbody>
</table>

### Table 2. Design optimization of concrete-slab forms by Hanna and Senouci [3]*

<table>
<thead>
<tr>
<th>Slab thickness in(cm)</th>
<th>Sheathing thickness in(cm)</th>
<th>Joist dimension in×in (cm×cm)</th>
<th>Stringer dimension in×in (cm×cm)</th>
<th>Shore dimension in×in (cm×cm)</th>
<th>Joist spacing in(cm)</th>
<th>Stringer spacing in(cm)</th>
<th>Shore spacing in(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(15.24)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>64(162.56)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>7(17.78)</td>
<td>0.625(1.59)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>24(60.96)</td>
<td>48(121.92)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>8(20.32)</td>
<td>0.625(1.59)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>48(121.92)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>9(22.86)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>48(121.92)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>10(25.4)</td>
<td>0.75(1.9)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>24(60.96)</td>
<td>32(81.28)</td>
<td>64(162.56)</td>
</tr>
<tr>
<td>11(27.94)</td>
<td>0.625(1.59)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>39(99.06)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>12(30.48)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>16(40.64)</td>
<td>48(121.92)</td>
<td>32(81.28)</td>
</tr>
<tr>
<td>13(33.02)</td>
<td>0.625(1.59)</td>
<td>2×8 (5.08×20.32)</td>
<td>2×10 (5.08×25.4)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>39(99.06)</td>
<td>48(121.92)</td>
</tr>
<tr>
<td>14(35.56)</td>
<td>0.75(1.9)</td>
<td>2×6 (5.08×15.24)</td>
<td>2×12 (5.08×30.48)</td>
<td>4×4 (10.16×10.16)</td>
<td>20(50.8)</td>
<td>32(81.28)</td>
<td>48(121.92)</td>
</tr>
</tbody>
</table>

*The horizontal dimension of slab are 60×90 ft (18.288×27.432 m) and height of the slab forms was taken 10ft (3.048 m)
AL-Tabtabai et al. [18] used Genetic algorithm (GA) for optimum design of slab forms with steel shore. They explained the results of optimum design as:

"For a standard slab carrying standard load with thickness ranging from 10 cm to 15 cm, based on the constrains set, GA yields an optimum section of 5 cm by 5 cm joist and 5 cm by 20 cm stringer. For a slab thickness of 10 cm, 45 cm spacing is found to be adequate for the joist, while stringer and the shore need to be spaced at 58 cm and 220 cm respectively. With the increase in the thickness of slab to 15 cm, the dead load of slab increases and the spacing becomes 43 cm, 55 cm, and 210 cm, for the joist, stringer and shore respectively. When the slab size increases, the joist size remains the same but the stringer size varied to 5 cm by 20 cm. The spacing for the 10 cm thick slabs are 45 cm for joist, 60 cm for stringer, and 200 cm for shores. For the slab thickness of 15 cm the joist and stringer spacing remains the same but the shore spacing decreases to 190 cm."

**Example 1:** For comparison with the existing results, a 60×90 ft (18.288×27.432 m) concrete slab was selected for optimization of its formworks using harmony search algorithm. The lower and upper bound values of the variables are used in the present study and input data are given in Tables 3, 4 and 5. In this example, the following parameters are utilized:

HMS = 30; HMCR = 0.85; PAR = 0.45.

<table>
<thead>
<tr>
<th>No</th>
<th>Limiting values</th>
<th>Joist</th>
<th>Stringer</th>
<th>Joist spacing</th>
<th>Stringer spacing</th>
<th>Shore spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Limiting values</td>
<td>Height in(cm)</td>
<td>Width in(cm)</td>
<td>Height in(cm)</td>
<td>Width in(cm)</td>
<td>Height in(cm)</td>
</tr>
<tr>
<td>1</td>
<td>Lower bound values</td>
<td>2(5.08)</td>
<td>2(5.08)</td>
<td>2(5.08)</td>
<td>2(5.08)</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Upper bound values</td>
<td>10(25.4)</td>
<td>4(10.16)</td>
<td>14(35.56)</td>
<td>6(15.24)</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thickness in(cm)</th>
<th>Moment of inertia in^4/ft (cm^4/m)</th>
<th>Elastic modulus psi (Mpa)</th>
<th>Allowable bending stress psi (Mpa)</th>
<th>Allowable shear stress psi (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.625(1.59)</td>
<td>0.131(17.8)</td>
<td>1650000(11601)</td>
<td>1930(13.57)</td>
<td>72(0.51)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elastic modulus Psi (Mpa)</th>
<th>Allowable bending stress Psi (Mpa)</th>
<th>Allowable shear stress Psi (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500000(10546.2)</td>
<td>1250(8.79)</td>
<td>87.5(0.615)</td>
</tr>
</tbody>
</table>

The results of the HS are given in Table 6 and are illustrated in Figs. 3 to 5. The design histories using the HS and the IHS algorithms are illustrated in Fig. 6. It can be observed that IHS converges faster and the result is also better.

As can be seen from Fig. 3, the thickness of the slab is related to the joist spacing, and the increase of the thickness of the slab decreases the joist spacing.

In traditional design methods, the increase in the thickness of the slab reduces the distance between the main beams. However, in optimal design, as it can be seen from Fig. 4, there is no definite relationship between the thickness and the distance between the main beams. Here, it is possible to increase the slab thickness and yet have an increase in the distance between the beams.
### Table 6. Results of design optimization of concrete-slab forms by HS*

<table>
<thead>
<tr>
<th>Slab thickness in (cm)</th>
<th>Sheathing thickness in (cm)</th>
<th>Joist dimension in(\times)in ((\text{cm}\times\text{cm}))</th>
<th>Stringer dimension in(\times)in ((\text{cm}\times\text{cm}))</th>
<th>Joist spacing in (cm)</th>
<th>Stringer spacing in (cm)</th>
<th>Shore spacing in (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(10.16)</td>
<td>0.625(1.59)</td>
<td>(2\times6) (5.08\times15.24)</td>
<td>(2\times13) (5.08\times33.02)</td>
<td>21(53.34)</td>
<td>90(228.60)</td>
<td>90(228.60)</td>
</tr>
<tr>
<td>5(12.70)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>20(50.80)</td>
<td>72(182.88)</td>
<td>102(259.08)</td>
</tr>
<tr>
<td>6(15.24)</td>
<td>0.625(1.59)</td>
<td>(2\times6) (5.08\times15.24)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>19(48.26)</td>
<td>83(210.82)</td>
<td>90(228.60)</td>
</tr>
<tr>
<td>7(17.78)</td>
<td>0.625(1.59)</td>
<td>(2\times6) (5.08\times15.24)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>18(45.72)</td>
<td>83(210.82)</td>
<td>90(228.60)</td>
</tr>
<tr>
<td>8(20.32)</td>
<td>0.625(1.59)</td>
<td>(2\times6) (5.08\times15.24)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>18(45.72)</td>
<td>77(195.58)</td>
<td>90(228.60)</td>
</tr>
<tr>
<td>9(22.86)</td>
<td>0.625(1.59)</td>
<td>(2\times6) (5.08\times15.24)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>18(45.72)</td>
<td>77(195.58)</td>
<td>80(203.20)</td>
</tr>
<tr>
<td>10(25.4)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>17(43.18)</td>
<td>63(160.02)</td>
<td>90(228.60)</td>
</tr>
<tr>
<td>11(27.94)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>17(43.18)</td>
<td>60(152.40)</td>
<td>90(228.60)</td>
</tr>
<tr>
<td>12(30.48)</td>
<td>0.625(1.59)</td>
<td>(2\times6) (5.08\times17.78)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>16(40.64)</td>
<td>83(210.82)</td>
<td>72(182.88)</td>
</tr>
<tr>
<td>13(33.02)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>16(40.64)</td>
<td>60(152.40)</td>
<td>80(203.20)</td>
</tr>
<tr>
<td>14(35.56)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times13) (5.08\times33.02)</td>
<td>16(40.64)</td>
<td>56(142.24)</td>
<td>80(203.20)</td>
</tr>
<tr>
<td>15(38.10)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>16(40.64)</td>
<td>56(142.24)</td>
<td>80(203.20)</td>
</tr>
<tr>
<td>16(40.64)</td>
<td>0.625(1.59)</td>
<td>(2\times5) (5.08\times12.70)</td>
<td>(2\times14) (5.08\times35.56)</td>
<td>15(38.10)</td>
<td>56(142.24)</td>
<td>80(203.20)</td>
</tr>
</tbody>
</table>

*the horizontal dimension of the slabs are 60\(\times\)90 ft \((18.288\times27.432\ m)\) and height of the slab forms was taken \(10\text{ft}(3.048\ m)\)*

---

![Fig. 3. Optimum joist spacing for 60\(\times\)90 ft \((18.288\times27.432\ m)\) slab](image-url)
Fig. 4. Optimum stringer spacing for 60×90 ft (18.288×27.432 m) slab

Fig. 5. Optimum steel shore spacing for 60×90 ft (18.288×27.432 m) slab
As observed from Fig. 5, in general, increasing the thickness of the slab results in reduction between the shore spacing. However, in some cases this rule is violated and in increasing the slab thickness the distance between the shores is also increased. In fact, this is one of the differences between the traditional design methods and the design by optimization methods.

![Design convergence history for 6 inch thick slab of Example 1](image)

Fig. 6. Design convergence history for 6 inch thick slab of Example 1

From Fig. 6 it can be seen that the IHS algorithm converges after 3420 iterations, while the classic HS converges after 5310 iterations. The convergence rate of the IHS is better than that of HS.

**Sensitivity analysis for Example 1:**

Five cases are considered as indicated in Table 7, and the corresponding histories of the convergence are illustrated in Fig. 7.

![Design convergence histories for sensitivity analysis of Example 1](image)

Fig. 7. Design convergence histories for sensitivity analysis of Example 1

<table>
<thead>
<tr>
<th>Case</th>
<th>HMS</th>
<th>HMCR</th>
<th>PAR(_{\text{min}})</th>
<th>PAR(_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>0.85</td>
<td>0.4</td>
<td>0.85</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0.9</td>
<td>0.45</td>
<td>0.9</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>0.95</td>
<td>0.3</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.95</td>
<td>0.25</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>0.85</td>
<td>0.35</td>
<td>0.99</td>
</tr>
</tbody>
</table>
The sensitivity analysis performed for 5 cases presented in Table 7 shows that the parameters for Case 4 are more suitable for this example and increase the convergence rate, leading to an optimal answer with a smaller number of iterations.

**Example 2:** In this example, the harmony search algorithm is used for the optimum formwork design of a trapezoidal shape slab, as shown in Fig. 8. The parameters used for the classic HS are the same as those of Example 1. The results of the HS are shown in Table 8, and the design convergence histories for HS and the IHS are illustrated in Fig. 9. It can be observed that IHS converges faster and the result is also better.

![Fig. 8. A trapezoidal shape slab](image)

<table>
<thead>
<tr>
<th>Slab height</th>
<th>Stringer spacing in(cm)</th>
<th>Shore spacing in(cm)</th>
<th>Slab thick in(cm)</th>
<th>Sheathing thickness in(cm)</th>
<th>Joist spacing in(cm)</th>
<th>Joist thickness in(cm)</th>
<th>Stringer thickness in(cm)</th>
<th>Stringer spacing in(cm)</th>
<th>Joist thickness in(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10(3.048)</td>
<td>70(177.8)</td>
<td>68(172.72)</td>
<td>5(12.70)</td>
<td>2(5.08)</td>
<td>5(12.70)</td>
<td>10(25.40)</td>
<td>2(5.08)</td>
<td>68(172.72)</td>
<td>10(25.40)</td>
</tr>
<tr>
<td>6(15.24)</td>
<td></td>
<td></td>
<td>5(12.70)</td>
<td>2(5.08)</td>
<td>5(12.70)</td>
<td>10(25.40)</td>
<td>2(5.08)</td>
<td>68(172.72)</td>
<td>10(25.40)</td>
</tr>
<tr>
<td>0.625(1.58)</td>
<td></td>
<td></td>
<td>2(5.08)</td>
<td>5(12.70)</td>
<td>5(12.70)</td>
<td>10(25.40)</td>
<td>2(5.08)</td>
<td>68(172.72)</td>
<td>10(25.40)</td>
</tr>
<tr>
<td>2(5.08)</td>
<td></td>
<td></td>
<td>2(5.08)</td>
<td>5(12.70)</td>
<td>5(12.70)</td>
<td>10(25.40)</td>
<td>2(5.08)</td>
<td>68(172.72)</td>
<td>10(25.40)</td>
</tr>
</tbody>
</table>

![Fig. 9. Design convergence histories of the formwork of Example 2](image)

7. CONCLUDING REMARKS

In the traditional formwork design procedure, the section is kept constant by the designer and the spacing is calculated based on bending, shear and deflection limits without specific consideration of the involved cost. The spacing that satisfy the constraints are considered acceptable, but might not necessarily correspond to the optimum minimum cost. The harmony search method is used to develop an optimum design algorithm for slab forms. This method is a new stochastic random search approach that simulates the musical process of searching for a perfect state of harmony. It is shown that this technique is mathematically quite simple but effective in finding the solutions of combinatorial optimization problems. Application of the improved harmony search method of [17] accelerates the convergence and often leads...
to better results. This can easily be observed from the convergence histories provided for the examples. The results obtained show that the improved harmony search method is an efficient and robust technique which can successfully be utilized in the optimum design of slab formwork.

In this paper metal shores are used instead of the wooden shores in [3] and also the domain of the variables is considered broader. Here, we use HS and IHS instead of Genetic algorithm employed in [18]. The present method can accommodate more general shapes for the slabs, as shown in Example 2.

Acknowledgement- The first author is grateful to the Iran National Science Foundation for support.

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9. ACI, committee 347, (1988). Guide to formwork for concrete, American Concrete Institute, P.O. Box 1950, Redford Station, Detroit, MI 48219.