IJST, Transactions of Civil Engineering, Vol. 37, No. C+, pp 409-421
Printed in The Islamic Republic of Iran, 2013


# TEACHING-LEARNING-BASED OPTIMIZATION ALGORITHM FOR SHAPE AND SIZE OPTIMIZATION OF TRUSS STRUCTURES WITH DYNAMIC FREQUENCY CONSTRAINTS*

A. BAGHLANI[1**] AND M. H. MAKIABADI[2]

[1]Faculty of Civil and Environmental Engineering, Shiraz University of Technology, Shiraz, I. R. of Iran
Email: baghlani@sutech.ac.ir
[2]Faculty of Civil Engineering, Shiraz University of Technology, I. R. of Shiraz, Iran

**Abstract–** The complicated problem of truss shape and size optimization with multiple frequency constraints is investigated in this paper. A recently developed metaheuristics called teaching-learning-based optimization (TLBO) algorithm is used for the first time to solve this kind of problem. Contrary to other metaheuristics, the procedure of TLBO is simple to implement since no tuning parameters need to be adjusted. Analyses of structures are performed by a finite element code in MATLAB which is used in conjunction with an optimization code based on TLBO. Various benchmark problems are solved with this technique and the results are compared with those found by other methods including metaheuristics such as PSO, HS and FA. In all test cases, the results show that TLBO leads to very satisfactory results i.e. lighter structures which satisfy all frequency constraints. The results of this study indicate excellent inherent capacity of the approach in dealing with complicated dynamic non-linear optimization problems.

**Keywords–** Truss structures, non-linear dynamic optimization, frequency constraints, teaching-learning-based optimization (TLBO)

## 1. INTRODUCTION

Truss optimization with frequency constraints is a highly non-linear dynamic optimization problem. In spite of difficulties in addressing this type of problem, considerable progress has been achieved in solution methods where the geometry of the structure is prescribed and cross-sectional areas have to be optimized, i.e. size optimization of trusses [1-4]. However, it is well known that the structural shape has a great influence on the size of elements. Hence, taking both shape and sizing optimization of trusses with frequency constraints into account increases the complexity of the problem extensively. Most of the difficulties in shape and sizing optimization of trusses simultaneously with multiple frequency constraints, are attributable to high non-linearity of the problem with respect to design variables and entirely different physical representation of shape and sizing variables, since their values are different orders of magnitude. Thus, coupling these variables sometimes leads to ill-conditioning problems and divergence [5-7]. In spite of its difficulty, from an engineering application point of view, this type of problem is very useful. As has been mentioned by Grandhi [8], the problem is advantageous for structural designers in manipulating the selected frequency to improve the performance of the structure under dynamic excitation. In other words, the designer can control the selected frequencies in a desired manner in order to improve the dynamic characteristics of the structure. Because of this important application in correct design of structures under dynamic loads, various efforts have been made in the literature to deal with this kind of problem [1, 3, 5, 7, 9, 10].

---

*Received by the editors November 11, 2012; Accepted February 5, 2013.
**Corresponding author

Due to the aforementioned obstacles in optimization of shape and sizing of structures with multiple frequency constraints, the choice of the solution method is of great importance and local search algorithms are not appropriate. As indicated by Gomes [11], traditional optimization methods based on gradients also have difficulties in problems with repeated eigenvalues and may trap into local optima. As powerful alternative methods, modern metaheuristic algorithms, which are not based on gradients, can be successfully employed to solve the problem. They are also capable of solving highly non-linear problems with complex objective functions. Due to their effectiveness in dealing with real-life complicated problems, new metaheuristics are frequently proposed. For example, among the most recently developed metaheuristics, Water Cycle Algorithm (WCA) [12], Mine Blast Algorithm (MBA) [13], Cuckoo Optimization Algorithm (COA) [14] can be mentioned. Some hybrid optimization algorithms have also been presented which make use of two or several metaheuristics and local search methods in order to improve their performance in dealing with trusses and other structures [15, 16]. However, use of metaheuristic optimization methods in optimization of shape and size of trusses with multiple frequency constraints have received little attention in the literature. Recently, Gomes [11] implemented Particle Swarm Optimization (PSO) algorithm to optimize the shape and size of truss structures with multiple frequency constraints. Later, Miguel and Miguel [17] utilized two other metaheuristics, i.e. Harmony Search (HS) method and Firefly Algorithm (FA) to solve this kind of problem. A combination of the Charged System Search (CSS) and the Big Bang-Big Crunch (BBC) algorithms has been recently employed by Kaveh and Zolghadr [18] for truss optimization with natural frequency constraints.

In this paper, a recently developed metaheuristic, called teaching-learning-based optimization (TLBO) algorithm is used for the first time to solve the problem of truss shape and size optimization with multiple frequency constraints. TLBO has some inherent capabilities and advantages compared to other metaheuristic approaches. It is reported that it outperforms most metaheuristics regarding constrained benchmark functions, constrained mechanical design, and continuous non-linear numerical optimization problems [19]. However, the effectiveness of TLBO in shape and size optimization of truss structures with multiple frequency constraints has not been investigated up to now. In this paper, the effectiveness of TLBO in solving the complex problem of shape and size optimization of truss structures with multiple frequency constraints is investigated through optimizing shape and size of some benchmark trusses. The results are compared with those found by other methods in the literature, including other metaheuristics such as harmony search (HS), particle swarm optimization (PSO) and firefly algorithm (FA). The results show that TLBO results in lighter structures which satisfy all frequency constraints, indicating outstanding capabilities of this modern approach. The results of applying TLBO in shape and size optimization of truss structures with frequency constraints show the excellent capacity of the method in dealing with complicated structural engineering problems.

## 2. PROBLEM FORMULATION

In dealing with truss shape and size optimization problems, the truss topology is prescribed and it is assumed to be unchanged during the optimization procedure. However, the cross-sectional areas of elements and nodal coordinates are considered as design variables which should be optimized. The natural frequencies are considered as design constraints to avoid resonance with the external excitations. The weight of the structure should be minimized subject to some prescribed constraints. The problem can be mathematically represented as follows:

Minimize:

$$W = \sum_{e=1}^{n} L_e \, \rho_e \, A_e \tag{1}$$

Subject to

$$\omega_i \geq \omega_i^* , \quad i = 1,...,q_1 \tag{2}$$

$$\omega_i \leq \omega_i^* , \quad i = q_1 + 1,...,q \tag{3}$$

$$\nu_{low} \leq \nu_j \leq \nu_{up} , \quad j = 1,...,k \tag{4}$$

where *W* is the total weight of structure, $L_e$ , $\rho_e$ and $A_e$ are length, material density, and cross sectional area of the *e*th element, respectively. Total number of elements is denoted by *n* and the number of independent design variables is denoted by *k*. Frequency constraint (2) represents that some natural frequencies $\omega_i$ , numbering $q_1$, should exceed the prescribed lower limits. Frequency constraint (3) represents that other natural frequencies, numbering $q - q_1$, should be less than the prescribed upper limits. Inequality (4) indicates that the design variables $\nu_j$ , including either a shape or sizing variable must take a value between its lower bound $\nu_{low}$ and upper bound $\nu_{up}$ , respectively.
It is worthy to note that the natural frequency is highly non-linear and implicit with respect to design variables.

## 3. TEACHING-LEARNING-BASED- OPTIMIZATION (TLBO) ALGORITHM

One of the most recently developed metaheuristics is teaching-learning-based- optimization (TLBO) algorithm [20]. TLBO has many similarities to evolutionary algorithms (EAs): an initial population is randomly selected, moving on the way to the teacher and classmates is comparable to mutation operator in EA, and selection is based on comparing two solutions in which the better one always survives [19].

Similar to most other evolutionary optimization methods, TLBO is a population-based algorithm inspired by learning process in a classroom. The searching process consists of two phases, i.e. Teacher Phase and Learner Phase. In teacher phase, learners first get knowledge from a teacher and then from classmates in learner phase. In the entire population, the best solution is considered as the teacher ($X_{teacher}$). On the other hand, learners learn from the teacher in the teacher phase. In this phase, the teacher tries to enhance the results of other individuals ($X_i$) by increasing the mean result of the classroom ($X_{mean}$) towards his/her position $X_{teacher}$. In order to maintain stochastic features of the search, two randomly-generated parameters *r* and $T_F$ are applied in update formula for the solution $X_i$ as:

$$X_{new} = X_i + r.( X_{teacher} - T_F . X_{mean} ) \tag{5}$$

where *r* is a randomly selected number in the range of 0 and 1 and $T_F$ is a teaching factor which can be either 1 or 2:

$$T_F^{\,i} = round\left[1 + rand(0,1)\{2-1\}\right] \tag{6}$$

Moreover, $X_{new}$ and $X_i$ are the new and existing solution of *i*, [20-21].

In the second phase, i.e. the learner phase, the learners attempt to increase their information by interacting with others. Therefore, an individual learns new knowledge if the other individuals have more knowledge than him/her. Throughout this phase, the student $X_i$ interacts randomly with another student $X_j$ ($i \neq j$) in order to improve his/her knowledge. In the case that $X_j$ is better than $X_i$ (i.e. $f(X_j) < f(X_i)$ for minimization problems), $X_i$ is moved toward $X_j$. Otherwise it is moved away from $X_j$:

$$X_{new} = X_i + r.( X_j - X_i ) \quad if \quad f(X_i) > f(X_j) \tag{7}$$

$$X_{new} = X_i + r.(X_i - X_j) \quad if \quad f(X_i) < f(X_j) \tag{8}$$

If the new solution $X_{new}$ is better, it is accepted in the population. The algorithm will continue until the termination condition is met. The pseudo code shown in Table 1 demonstrates the TLBO algorithm step-by-step.

Table 1. The pseudo code for TLBO

```
Set k = 1;
Objective function f(X),    X = (x_1, x_2, ....., x_d)^T        d=no. of design variables
Generate initial students of the classroom randomly   X^i , i = 1,2,...,n      n=no. of students
Calculate objective function f(X) for whole students of the classroom
  WHILE (the termination conditions are not met)
     {Teacher Phase}
     Calculate the mean of each design variable  X_Mean
     Identify the best solution (teacher)
     FOR i = 1 → n
        Calculate teaching factor   T_F^i = round[1 + rand(0,1){2 − 1}]
        Modify solution based on best solution(teacher)   X_new^i = X^i + rand(0,1)[X_teacher − (T_F^i · X_mean)]
        Calculate objective function for new mapped student    f(X_new^i)
        IF  X_new^i is better than X^i , i.e f(X_new^i) < f(X^i)
             X^i = X_new^i
        END IF  {End of Teacher Phase}
        {Student Phase}
        Randomly select another learner  X^j , such that  j ≠ i
        IF  X^i is better than X^j , i.e f(X^i) < f(X^j)
             X_new^i = X^i + rand(0,1)(X^i − X^j)
        Else
             X_new^i = X^i + rand(0,1)(X^j − X^i)
        END IF
        IF  X_new^i is better than X^i , i.e f(X_new^i) < f(X^i)
             X^i = X_new^i
        END IF  {End of Student Phase}
     END FOR
     Set k = k + 1
  END WHILE
Postprocess results and visualization
```

## 4. BENCHMARK DESIGN EXAMPLES

To investigate the effectiveness of TLBO algorithm in shape and size optimization of truss structures with frequency constraints, four benchmark problems have been solved. A finite element code in MATLAB is developed for analysis of structures which is used with an optimization code based on TLBO. The results are compared with the results found by other researchers and those found by other metaheuristic approaches. Standard penalty function method has been used to handle the frequency constraints.

### *a) 10 bar plane truss*

For the first example, the 10 bar planar truss shown in Fig. 1 is considered. This truss has been already investigated by Wang et al. [7], Grandhi [8] using evolutionary node shift methods, Lingyun et al. [5] using Niche Hybrid Genetic Algorithm (NHGA), Lingyun et al. [22] based on parallel genetic algorithm, Gomes [11] using PSO, Zuo et al. [23] using adaptive eigenvalue re-analysis methods, and

Miguel and Miguel [17] using both harmony search (HS) and firefly algorithm (FA) methods. Therefore, the results of this study can be compared with various approaches.

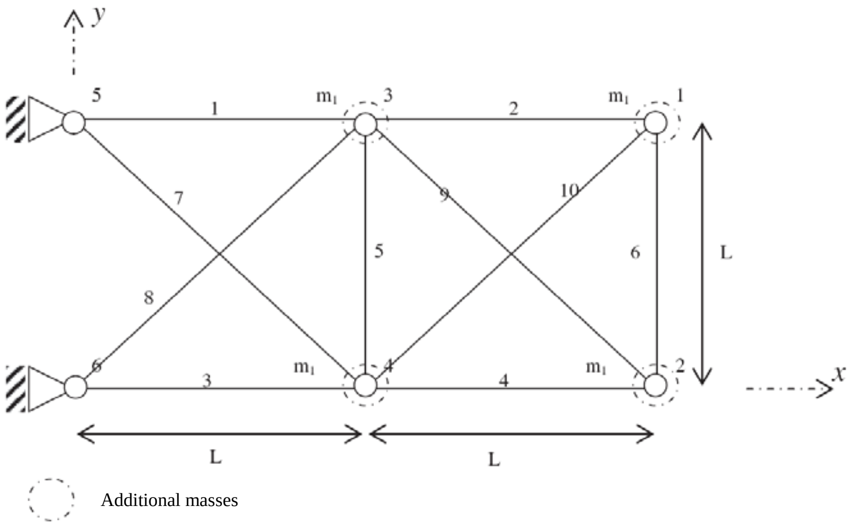


Fig. 1. 10-bar truss structure with added masses

The structure is made of aluminum with modulus of elasticity $E = 68.95\ Gpa$ $(10^7\ psi)$ and material density $\rho = 2767.99\ kg/m^3$ $(0.1\ lb/in^3)$. The lower bound of cross sectional area is $6.45\times10^{-5}\ m^2$ $(0.1\ in^2)$ for all elements. A nonstructural mass of $453.6\ kg$ $(1000\ lb)$ is attached at each of the four free nodes. The frequency constraints are as follows: $\omega_1 \geq 7\ Hz$ , $\omega_2 \geq 15\ Hz$ and $\omega_3 \geq 20\ Hz$ .

The optimal solutions of the cross-sectional areas, structural weight and frequencies obtained from various methods are tabulated in Tables 2 and 3, respectively. As is clear from Table 2, TLBO gives excellent results compared to all other methods. The optimal weight of 530.76 kg which is found by TLBO is the best result among the aforementioned methods in Table 2. Table 3 shows that with design variables obtained by TLBO, all frequency constraints are met. It is noteworthy that the first and the third frequencies obtained by TLBO are exactly the lower bounds of frequency constraints, similar to the results found by PSO [11]. However, the optimal weight of 537.98 kg has been found by PSO which is much heavier than the optimal solution found by TLBO. Compared to two other metaheuristic methods, i.e. HS and FA, TLBO shows a better performance as well. Table 4 gives statistical results for five independent runs of TLBO for this example. A little standard deviation from the mean value of the independent runs (2.23 kg) shows that TLBO is very effective in shape and size optimization of truss structures with frequency constraints. A value of standard deviation of 2.49 kg for HS and 3.64 kg for FA for five independent runs has been reported by Miguel and Miguel [17] which are greater than the value found by TLBO, indicating better performance of TLBO, compared to other metaheuristic methods as well.

***b) 37 bar planar truss***

The simply supported 37 bar planar truss shown in Fig. 2 is investigated in this example. The results of employing TLBO in mass minimization of this truss are compared with other methods [5, 7, 17, 22]. The truss is made of steel with modulus of elasticity of $E = 210\,Gpa$ and material density of $\rho = 7800\ kg/m^3$ . The truss is optimized on shape and size for its mass minimization with multiple frequency constraints. Nodal coordinates in the upper chord and cross-sectional areas of members are considered as design variables. All members on the lower chord have fixed cross sectional areas of $4\times10^{-3}\ m^2$ and the others have initial cross sectional areas of $1\times10^{-4}\ m^2$ . A nonstructural mass of $m=10$ *kg* is attached at each of the nodes on the lower chord. In the optimization process, nodes on the upper chord can be shifted vertically. In addition, nodal coordinates and member areas are linked to maintain the structural symmetry about *y-z* plane. Therefore, only five shape variables and fourteen sizing variables will be redesigned for optimization. Moreover, the lower bounds on cross sectional areas are $1\times10^{-4}\,m^2$ for all bar elements. The natural frequency constraints are $\omega_1 \geq 20\ Hz$ , $\omega_2 \geq 40\ Hz$ and $\omega_3 \geq 60\ Hz$ .

Table 2. Optimum design of cross sections (cm$^2$) for the 10 bar truss from various methods

| Member | Areas(cm$^2$) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Grandhi [8] | Sedaghati et al. [3] | Wang et al. [7] | Lingyun et al. [5] | Lingyun et al. [22] | Gomes [11] | Zuo et al. [23] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| 1 | 36.584 | 38.245 | 32.456 | 42.234 | 36.63 | 37.712 | 37.810 | 34.282 | 36.198 | 35.494 |
| 2 | 24.658 | 9.9160 | 16.577 | 18.555 | 13.043 | 9.959 | 9.518 | 15.653 | 14.030 | 14.777 |
| 3 | 36.584 | 38.619 | 32.456 | 38.851 | 34.229 | 40.265 | 36.463 | 37.641 | 34.754 | 36.203 |
| 4 | 24.658 | 18.232 | 16.577 | 11.222 | 15.289 | 16.788 | 19.095 | 16.058 | 14.900 | 15.387 |
| 5 | 4.1670 | 4.4190 | 2.115 | 4.783 | 0.645 | 11.576 | 2.851 | 1.069 | 0.654 | 0.6451 |
| 6 | 2.0700 | 4.1940 | 4.467 | 4.451 | 4.8472 | 3.955 | 5.526 | 4.740 | 4.672 | 4.5896 |
| 7 | 27.032 | 20.097 | 22.810 | 21.049 | 22.14 | 25.308 | 19.463 | 22.505 | 23.467 | 23.211 |
| 8 | 27.032 | 24.097 | 22.810 | 20.949 | 27.983 | 21.613 | 26.400 | 24.603 | 25.508 | 24.561 |
| 9 | 10.346 | 13.890 | 17.490 | 10.257 | 15.034 | 11.576 | 14.346 | 12.867 | 12.707 | 12.482 |
| 10 | 10.346 | 11.4516 | 17.490 | 14.342 | 10.216 | 11.186 | 10.643 | 12.099 | 12.351 | 12.324 |
| Mass (kg) | 594.0 | 537.01 | 553.8 | 542.75 | 535.14 | 537.98 | 535.61 | 534.99 | 531.28 | 530.76 |

Table 3. Optimum design of natural frequencies (HZ) for the 10 bar truss from various methods

| Frequency No. | Natural frequencies (HZ) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Grandhi [8] | Sedaghati et al. [3] | Wang et al. [7] | Lingyun et al. [5] | Lingyun et al. [22] | Gomes [11] | Zuo et al. [23] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| 1 | 7.059 | 6.992 | 7.011 | 7.008 | 7.0003 | 7.000 | 7.008 | 7.0028 | 7.0002 | 7.000 |
| 2 | 15.895 | 17.599 | 17.302 | 18.148 | 16.08 | 17.786 | 17.146 | 16.7429 | 16.164 | 16.201 |
| 3 | 20.425 | 19.973 | 20.001 | 20.000 | 20.002 | 20.000 | 20.084 | 20.0548 | 20.0029 | 20.000 |
| 4 | 21.528 | 19.977 | 20.100 | 20.508 | 20.172 | 20.063 | 21.438 | 20.3351 | 20.0221 | 20.001 |
| 5 | 28.976 | 28.173 | 30.869 | 27.797 | 27.12 | 27.776 | 27.655 | 28.5232 | 28.5428 | 28.425 |
| 6 | 30.189 | 31.029 | 32.666 | 31.281 | 30.336 | 30.939 | 31.047 | 29.2911 | 28.922 | 28.907 |
| 7 | 54.286 | 47.628 | 48.282 | 48.304 | 48.199 | 47.297 | 47.873 | 49.0342 | 48.3538 | 48.708 |
| 8 | 56.546 | 52.292 | 52.306 | 53.306 | 50.706 | 52.286 | 52.565 | 51.7451 | 50.8004 | 51.217 |

Table 4. Statistical results for five independent runs of TLBO for the 10 bar truss structure

| Mean mass using (kg) | Standard deviation (kg) | Coefficient of variation (%) | Mean no. of searches |
|---|---|---|---|
| 533.35 | 2.23 | 0.42 | 6500 |

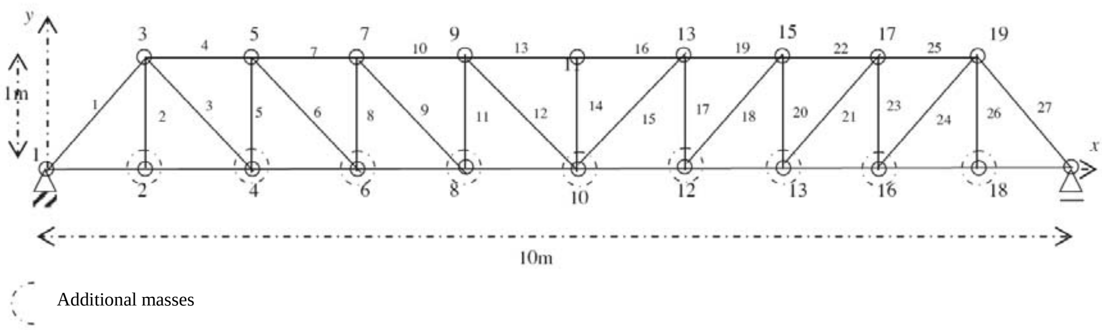


Fig. 2. Initial configuration for the 37-bar truss structure with added masses

TLBO is used to optimize this truss for its shape and size with multiple frequency constraints. The results are shown in Table 5 and the results obtained by other researches with other methods have been included for the sake of comparison as well. Similar to the previous example, Table 5 shows that excellent results are obtained by using TLBO for this example as well. The optimal weight of 359.98 kg found by TLBO is the best result among others. Again, the results show that TLBO outperforms other metaheuristics such as PSO, HS and FA. Table 6 gives the natural frequencies obtained by the present work and other methods. As the table indicates, all frequency constraints are satisfied with design variables shown in Table 5. In Table 7, statistical results for five independent runs are reported. A slight value of standard deviation from the mean value of the independent runs shows that TLBO is very efficient in shape and size optimization of truss structures with frequency constraints. Final configuration of the truss after designing with TLBO has been depicted in Fig. 3.

Table 5. Optimum design for the simply supported 37 bar truss from various methods

| Design variable | Y coordinates (m) and areas ($cm^2$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Initial | Wang et al. [7] | Lingyun et al. [5] | Lingyun et al. [22] | Gomes [11] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| Y3, Y19 | 1.0 | 1.2086 | 1.1998 | 1.09693 | 0.9637 | 0.8415 | 0.9392 | 0.9357 |
| Y5,Y17 | 1.0 | 1.5788 | 1.6553 | 1.45558 | 1.3978 | 1.2409 | 1.3270 | 1.2941 |
| Y7,Y15 | 1.0 | 1.6719 | 1.9652 | 1.59539 | 1.5929 | 1.4464 | 1.5063 | 1.4758 |
| Y9,Y13 | 1.0 | 1.7703 | 2.0737 | 1.76551 | 1.8812 | 1.5334 | 1.6086 | 1.6173 |
| Y11 | 1.0 | 1.8502 | 2.3050 | 1.67981 | 2.0856 | 1.5971 | 1.6679 | 1.7001 |
| A1,A27 | 1.0 | 3.2508 | 2.8932 | 2.62463 | 2.6797 | 3.2031 | 2.9838 | 2.8471 |
| A2,A26 | 1.0 | 1.2364 | 1.1201 | 1.00000 | 1.1568 | 1.1107 | 1.1098 | 1.0023 |
| A3,A24 | 1.0 | 1.0000 | 1.0000 | 1.00176 | 2.3476 | 1.1871 | 1.0091 | 1.0000 |
| A4,A25 | 1.0 | 2.5386 | 1.8655 | 2.07586 | 1.7182 | 3.3281 | 2.5955 | 2.5114 |
| A5,A23 | 1.0 | 1.3714 | 1.5962 | 1.22071 | 1.2751 | 1.4057 | 1.2610 | 1.0684 |
| A6,A21 | 1.0 | 1.3681 | 1.2642 | 1.48922 | 1.4819 | 1.0883 | 1.1975 | 1.2712 |
| A7,A22 | 1.0 | 2.4290 | 1.8254 | 2.30847 | 4.6850 | 2.1881 | 2.4264 | 2.9509 |
| A8,A20 | 1.0 | 1.6522 | 2.0009 | 1.43236 | 1.1246 | 1.2223 | 1.3588 | 1.3501 |
| A9,A18 | 1.0 | 1.8257 | 1.9526 | 1.64678 | 2.1214 | 1.7033 | 1.4771 | 1.5152 |
| A10,A19 | 1.0 | 2.3022 | 1.9705 | 2.87072 | 3.8600 | 3.1885 | 2.5648 | 2.8262 |
| A11,A17 | 1.0 | 1.3103 | 1.8294 | 1.50405 | 2.9817 | 1.0100 | 1.1295 | 1.2135 |
| A12,A15 | 1.0 | 1.4067 | 1.2358 | 1.31328 | 1.2021 | 1.4074 | 1.3199 | 1.3549 |
| A13,A16 | 1.0 | 2.1896 | 1.4049 | 2.32277 | 1.2563 | 2.8499 | 2.9217 | 2.4864 |
| A14 | 1.0 | 1.0000 | 1.0000 | 1.04258 | 3.3276 | 1.0269 | 1.0004 | 1.0001 |
| Mass (kg) | 336.29 | 366.5 | 368.84 | 363.032 | 377.2 | 361.5 | 360.05 | 359.997 |

Table 6. Optimum design of natural frequencies (HZ) for the 37 bar truss from various methods

| Frequency No. | Natural frequencies (HZ) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Initial | Wang et al. [7] | Lingyun et al. [5] | Lingyun et al. [22] | Gomes [11] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| 1 | 8.8778 | 20.0850 | 20.0013 | 20.0819 | 20.0001 | 20.0037 | 20.0024 | 20.0000 |
| 2 | 29.2135 | 42.0743 | 40.0305 | 40.0961 | 40.0003 | 40.0050 | 40.0019 | 40.0020 |
| 3 | 48.5539 | 62.9383 | 60.0000 | 60.0321 | 60.0001 | 60.0082 | 60.0043 | 60.0003 |
| 4 | 67.7487 | 74.4539 | 73.0444 | 73.4648 | 73.0440 | 77.9753 | 77.2153 | 76.5735 |
| 5 | 84.2484 | 90.0576 | 89.8244 | 88.7942 | 89.8240 | 99.2564 | 96.9900 | 96.6969 |

Table 7. Statistical results for five independent runs of TLBO for the simply supported 37 bar truss

| Mean mass using (kg) | Standard deviation (kg) | Coefficient of variation (%) | Mean no. of searches |
|---|---|---|---|
| 561.08 | 1.06 | 0.29 | 11400 |

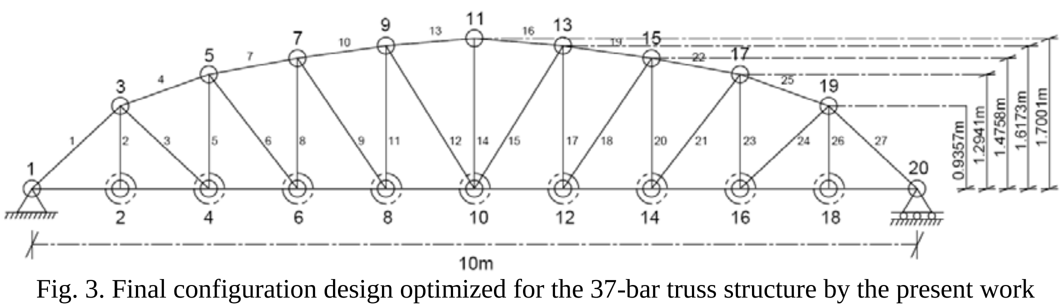


Fig. 3. Final configuration design optimized for the 37-bar truss structure by the present work

### *c) 52 bar dome structure*

The next benchmark example is 52 bar spatial truss with the initial configuration shown in Fig. 4 (top view) and Fig. 5 (lateral view). This example is a highly non-linear dynamic optimization problem with multiple frequency prohibited band constraints. A nonstructural mass of 50 kg is attached to each free node (nodes 1-13) of the structure. The dome is made of steel with modulus of elasticity of $E = 21 \times 10^{10}\ pa$ and material density of $\rho = 7800\ kg\ /\ m^3$. Shape and size optimization of this truss to minimize its mass with multiple natural frequency constraints is considered. Nodal coordinates and cross sectional areas of the members are considered as design variables. In order to maintain the structural symmetry in the design, the 52 bars are linked into eight groups as shown in Table 8. Thus, there are 13 independent design variables including five shape and eight sizing variables. The natural frequency constraints are $\omega_1 \le 15.9155\ Hz$ and $\omega_2 \le 28.6479\ Hz$ . The cross sectional area of each bar is initially equal to $2\times10^{-4}\,m^2$, and is permitted to vary between $1\times10^{-4}\,m^2$ and $1\times10^{-3}\,m^2$. The three coordinates (*x, y, z*) of each movable node are taken as independent variables and the movable range of each coordinate is $\pm 2\ m$ .

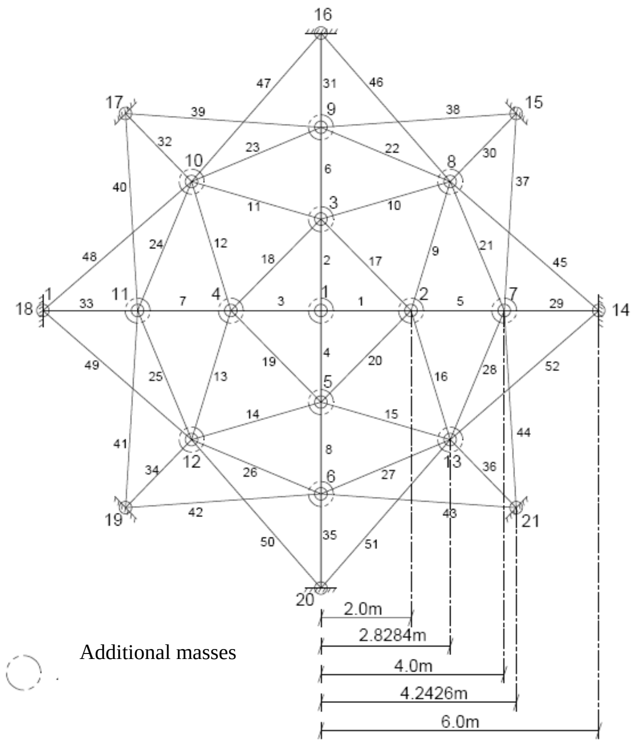


Fig. 4. Initial configuration for the 52-bar dome structure with added masses (top view)

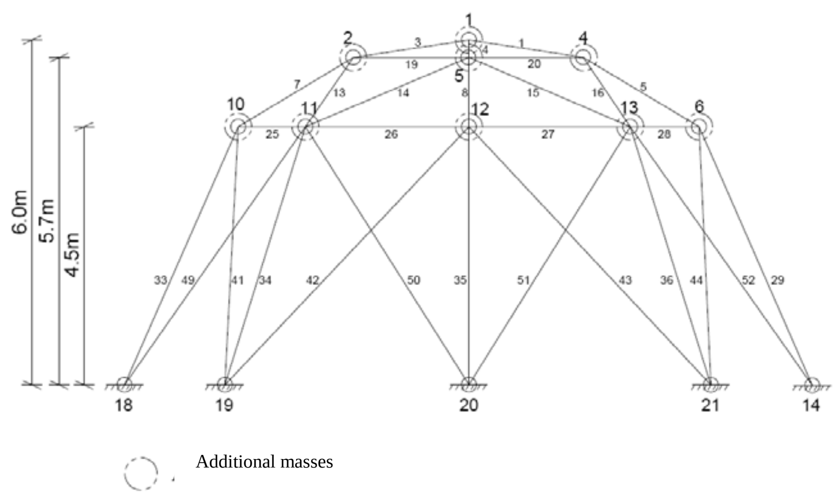


Fig. 5. Initial configuration for the 52-bar dome structure with added masses (lateral view)

Table 8. Member linking detail for the 52 bar space truss

| Group number | Members |
|---|---|
| 1 | 1-2, 1-3, 1-4, 1-5 |
| 2 | 2-6, 3-8, 4-10, 5-12 |
| 3 | 2-7, 3-7, 3-9, 4-9, 4-11, 5-11, 5-13, 2-13 |
| 4 | 2-3, 3-4, 4-5, 2-5 |
| 5 | 6-7, 7-8, 8-9, 9-10, 10-11, 11-12, 12-13, 6-13 |
| 6 | 6-14, 7-15, 8-16, 9-17, 10-18, 11-19, 12-20, 13-21 |
| 7 | 6-15, 8-15, 8-17, 10-17, 10-19, 12-19, 12-21, 6-21 |
| 8 | 7-14, 7-16, 9-16, 9-18, 11-18, 11-20, 13-20, 13-14 |

This problem has already been investigated by Lin et al. [10] using Optimal Criteria (OC), Lingyun et al. [5] using Niche Hybrid Genetic Algorithm (NHGA), Lingyun et al. [22] using parallel genetic algorithm, Gomes [11] using PSO, Zuo et al. [23] using adaptive eigenvalue re-analysis methods, and Miguel and Miguel [17] using both Harmony search (HS) and Firefly algorithm (FA) methods. Thus, it is a suitable example for comparing TLBO with other approaches.

The optimal design variables, natural frequencies and statistical results for five independent runs are shown in Tables 9-11, respectively.

Table 9. Optimum design for the 52 bar space truss from various methods

| Design variable | Coordinates (m) and areas ($cm^2$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Initial | Lin et al. [10] | Lingyun et al. [5] | Lingyun et al. [22] | Gomes [11] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| Z1 | 6.0 | 4.3201 | 5.8851 | 5.62509 | 5.5344 | 4.7374 | 6.4332 | 5.9749 |
| X2 | 2.0 | 1.3153 | 1.7623 | 2.17028 | 2.0885 | 1.5643 | 2.2208 | 2.2801 |
| Z2 | 5.7 | 4.1740 | 4.4091 | 3.89059 | 3.9283 | 3.7413 | 3.9202 | 3.7241 |
| X6 | 4.0 | 2.9169 | 3.4406 | 4.02326 | 4.0255 | 3.4882 | 4.0296 | 3.9734 |
| Z6 | 4.5 | 3.2676 | 3.1874 | 2.50583 | 2.4575 | 2.6274 | 2.5200 | 2.5000 |
| A1 | 2.0 | 1.00 | 1.0004 | 1.00001 | 0.3696 | 1.0085 | 1.0050 | 1.0000 |
| A2 | 2.0 | 1.33 | 2.1417 | 1.19040 | 4.1912 | 1.4999 | 1.3823 | 1.0982 |
| A3 | 2.0 | 1.58 | 1.4858 | 1.30267 | 1.5123 | 1.3948 | 1.2295 | 1.1993 |
| A4 | 2.0 | 1.00 | 1.4018 | 1.25795 | 1.5620 | 1.3462 | 1.2662 | 1.4621 |
| A5 | 2.0 | 1.71 | 1.9116 | 1.52709 | 1.9154 | 1.6776 | 1.4478 | 1.4041 |
| A6 | 2.0 | 1.54 | 1.0109 | 1.000001 | 1.1315 | 1.3704 | 1.0000 | 1.0000 |
| A7 | 2.0 | 2.65 | 1.4693 | 1.64556 | 1.8233 | 1.4137 | 1.5728 | 1.5958 |
| A8 | 2.0 | 2.87 | 2.1411 | 1.68189 | 1.0904 | 1.9378 | 1.4153 | 1.3701 |
| Mass (kg) | 338.69 | 298.0 | 236.05 | 207.2711 | 228.38 | 214.94 | 197.53 | 193.141 |

Table 10. Optimum design of natural frequencies (HZ) for the 52 bar truss from various methods

| Frequency No. | Natural frequencies (HZ) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Initial | Lin et al. [10] | Lingyun et al. [5] | Lingyun et al. [22] | Gomes [11] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| 1 | 22.7817 | 15.2196 | 12.8051 | 13.4193 | 12.751 | 12.2222 | 11.3119 | 11.5580 |
| 2 | 25.2693 | 29.2837 | 28.6489 | 28.6479 | 28.649 | 28.6577 | 28.6529 | 28.6479 |
| 3 | 25.2693 | 29.2837 | 28.6489 | 28.6479 | 28.649 | 28.6577 | 28.6529 | 28.6479 |
| 4 | 31.7347 | 31.6847 | 29.5398 | 28.6500 | 28.803 | 28.6618 | 28.803 | 28.6482 |
| 5 | 34.094 | 33.1547 | 30.2443 | 29.9412 | 29.230 | 30.0997 | 28.803 | 28.6500 |

Table 11. Statistical results for five independent runs of TLBO for the 52bar space truss

| Mean mass using (kg) | Standard deviation (kg) | Coefficient of variation (%) | Mean no. of searches |
|---|---|---|---|
| 196.43 | 2.38 | 1.21 | 12500 |

It is clear from Table 9 that the results obtained by the present work are better than all results reported in the literature. It is worth pointing out that none of the natural frequency constraints were violated using TLBO as can be seen in Table 10. The slight value of standard deviation from mean value for five independent runs given in Table 11 again shows the usefulness of the present study for solving this kind of complex problems. It should be mentioned that the results presented for comparison are the best results obtained among the runs.

Figure 6 depicts the final design of the structure optimized by TLBO.

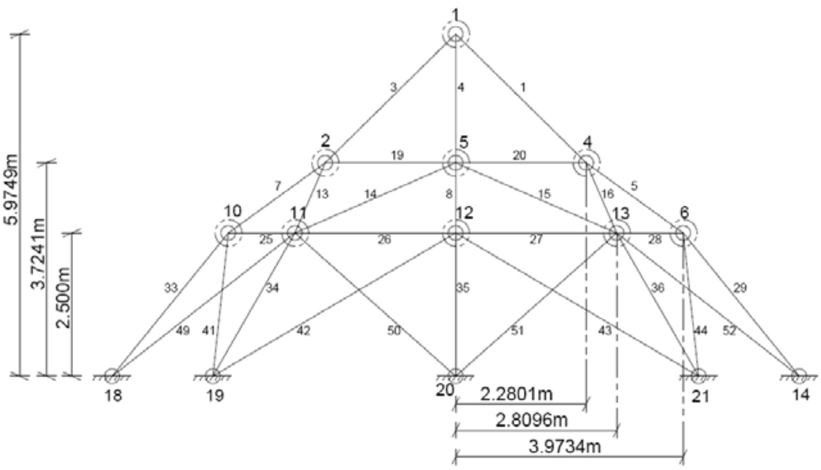


Fig. 6. Final configuration design optimized for the 52-bar dome structure by the present work

***d) 72 bar space truss***

The final benchmark problem is devoted to 72 bar space truss shown in Fig. 7. This problem has been studied by various researchers and it is appropriate for comparison [11, 17, 23, 24, 25].

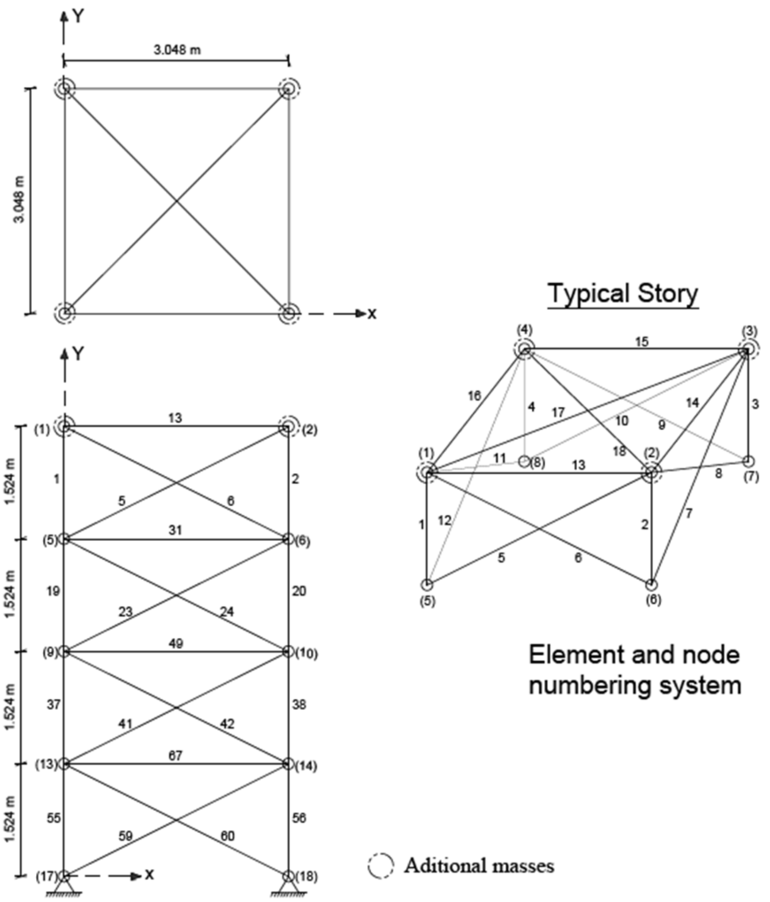


Fig. 7. 72-bar space truss structure with added masses (dimensions in m)

The design variables are the cross sectional areas of the members which are linked into 16 groups in order to maintain the structural symmetry, as shown in Table 12. A nonstructural mass of $2268\,kg$ $(5000\ lb)$ is attached to four nodes on the top of the structure (nodes 1-4). The structure is made of aluminum with modulus of elasticity $E = 68.95\ Gpa\ (10^7\ psi)$ and material density $\rho = 2767.99\, kg/m^3\ (0.1\ lb/in^3)$. The natural frequency constraints are $\omega_1 = 4\ Hz$ and $\omega_3 \geq 6\ Hz$. The allowable minimum area of the cross sectional area is $6.45\times10^{-5}\ m^2\ (0.1\ in^2)$.

The optimal design variables, natural frequencies and statistical results for five independent runs are shown in Tables 13-15, respectively.

Table 12. Member linking detail for the 72 bar space truss

| Group number | Members | Group number | Members |
|---|---|---|---|
| 1 | 1–4 | 9 | 37–40 |
| 2 | 5–12 | 10 | 41–48 |
| 3 | 13–16 | 11 | 49–52 |
| 4 | 17–18 | 12 | 53–54 |
| 5 | 19–22 | 13 | 55–58 |
| 6 | 23–30 | 14 | 59–66 |
| 7 | 31–34 | 15 | 67–70 |
| 8 | 35–36 | 16 | 71–72 |

Table 13. Optimum design for the 72 bar space truss from various methods

| Design variable | Areas($cm^2$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | Konzelman [24] | Sedaghati [25] | Gomes [11] | Zuo et al. [23] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| 1 | 3.499 | 3.499 | 2.987 | 3.169 | 3.6803 | 3.3411 | 3.7165 |
| 2 | 7.932 | 7.932 | 7.849 | 10.102 | 7.6808 | 7.7587 | 7.9297 |
| 3 | 0.645 | 0.645 | 0.645 | 0.687 | 0.6450 | 0.6450 | 0.6456 |
| 4 | 0.645 | 0.645 | 0.645 | 0.778 | 0.6450 | 0.6450 | 0.6450 |
| 5 | 8.056 | 8.056 | 8.765 | 14.563 | 9.4955 | 9.0202 | 8.0152 |
| 6 | 8.011 | 8.011 | 8.153 | 6.598 | 8.2870 | 8.2567 | 7.9660 |
| 7 | 0.645 | 0.645 | 0.645 | 0.751 | 0.6450 | 0.6450 | 0.6450 |
| 8 | 0.645 | 0.645 | 0.645 | 1.012 | 0.6461 | 0.6450 | 0.6451 |
| 9 | 12.812 | 12.812 | 13.450 | 12.033 | 11.451 | 12.0450 | 12.8138 |
| 10 | 8.061 | 8.061 | 8.073 | 7.689 | 7.8990 | 8.0401 | 8.1643 |
| 11 | 0.645 | 0.645 | 0.645 | 0.852 | 0.6473 | 0.6450 | 0.6459 |
| 12 | 0.645 | 0.645 | 0.645 | 0.718 | 0.6450 | 0.6450 | 0.6455 |
| 13 | 17.279 | 17.279 | 16.684 | 13.054 | 17.4060 | 17.3800 | 17.1437 |
| 14 | 8.088 | 8.088 | 8.159 | 6.844 | 8.2736 | 8.0561 | 8.0600 |
| 15 | 0.645 | 0.645 | 0.645 | 0.719 | 0.6450 | 0.6450 | 0.6467 |
| 16 | 0.645 | 0.645 | 0.645 | 0.983 | 0.6450 | 0.6450 | 0.6450 |
| Mass (kg) | 327.605 | 327.605 | 328.823 | 326.67 | 328.334 | 327.691 | 327.603 |

Table 14. Optimum design of natural frequencies (HZ) for the 72 bar truss from various methods

| Frequency no | Natural frequencies (HZ) | | | | | | |
|---|---|---|---|---|---|---|---|
| | Konzelman [24] | Sedaghati [25] | Gomes [11] | Zuo et al. [23] | Miguel and Miguel [17] HS | Miguel and Miguel [17] FA | Present study |
| 1 | 4.000 | 4.000 | 4.000 | 3.8899 | 4.0000 | 4.0000 | 4.0000 |
| 2 | 4.000 | 4.000 | 4.000 | 3.8899 | 4.0000 | 4.0000 | 4.0000 |
| 3 | 6.000 | 6.000 | 6.000 | 5.8629 | 6.0000 | 6.0000 | 6.0000 |
| 4 | 6.247 | 6.247 | 6.219 | 6.8512 | 6.2723 | 6.2468 | 6.2567 |
| 5 | 9.074 | 9.074 | 8.976 | 9.3833 | 9.0749 | 9.0380 | 9.0984 |

Table 15. Statistical results for five independent runs of TLBO for the 72 bar space truss

| Mean mass using (kg) | Standard deviation (kg) | Coefficient of variation (%) | Mean no. of searches |
|---|---|---|---|
| 329.72 | 1.57 | 0.48 | 35100 |

As Table 13 shows, Konzelman [24] using Dual Method (DM) and Sedaghati [25] using the Force Method (FM) found the same values for the design variables, resulting in optimal mass of 327.605 kg. Further, Gomes [11] using PSO found a heavier structure and Zuo et al. [23] using adaptive eigenvalue reanalysis methods found the optimal solution of 326.67 which violates the frequency constraints (see Table 14). Recently, Miguel and Miguel [17] by using two metaheuristics, i.e. HS and FA, found feasible solutions for this problem with optimal weights of 328.334 kg and 327.691 kg, using HS and FA, respectively. Table 13 shows that the optimal feasible solution found by TLBO with the weight of 327.603

kg is the best feasible solution among the feasible solutions obtained by other methods, and is very close to optimal solutions found by Konzelman [24] and Sedaghati [25] but with different cross sectional areas. It is important to note that from Table 14, the solution found by TLBO satisfies all frequency constraints, unlike the solution found by Zuo et al. [23].

## 5. CONCLUSION

Shape and size optimization of truss structures with multiple frequency constraints which is a highly non-linear problem was investigated in this paper. A recently developed approach, i.e. teaching-learning-based optimization (TLBO) was used for this purpose. The method is simple to implement since no tuning parameter should be calibrated in the algorithm. Some benchmark problems were solved via the proposed approach and the results were compared with other methods including other metaheuristic approaches such as PSO, HS and FA. In all examples TLBO gives very satisfactory results which satisfy all frequency constraints.

The results of this paper show that TLBO is an outstanding approach suitable for solving complicated optimization problems with highly non-linear behavior.

## REFERENCES

1. Grandhi, R. V. & Venkayya, V. B. (1988). Structural optimization with frequency constraints, *AIAA Journal*, Vol. 26, pp. 858–866.
2. Khot, N. S. (1985). Optimization of structures with multiple frequency constraints. *Computers and Structures*, Vol. 20, pp. 869–876.
3. Sedaghati, R., Suleman, A. & Tabarrok, B. (2002). Structural optimization with frequency constraints using finite element force method. *AIAA Journal*, Vol. 40, pp. 382–388.
4. Tong, W.H. & Liu, G.R. (2001). An optimization procedure for truss structure with discrete design variables and dynamic constraints. *Computers and Structures*. Vol. 79, pp. 155–162.
5. Lingyun, W., Mei, Z., Guangming, W. & Guang, M. (2005). Truss optimization on shape and sizing with frequency constraints based on genetic algorithm. *Computational Mechanics*, Vol. 25, pp. 361–368.
6. Rozvany, G. I. N., Bendsoe, M. P. & Kirsh, U. (1995). Layout optimization of structures. *Applied Mechanics Reviews*, Vol. 48, pp. 41–119.
7. Wang, D., Zhang, W. H. & Jiang, J. S. (2004). Truss optimization on shape and sizing with frequency constraints. *AIAA Journal*, Vol. 42, pp. 622–630.
8. Grandhi, R. V. (1993). Structural optimization with frequency constraints – A review. *AIAA Journal*, Vol. 31, pp. 2296–2303.
9. Bellagamba, L. & Yang, T. Y. (1981). Minimum-mass truss structures with constraints on fundamental natural frequency. *AIAA Journal*, Vol. 19, pp. 1452–58.
10. Lin, J. H., Chen, W. Y. &. Yu, Y. S. (1982). Structural optimization on geometrical configuration and element sizing with static and dynamic constraints. *Computers and Structures*, Vol. 15, pp. 507–515.
11. Gomes, H. M. (2011). Truss optimization with dynamic constraints using a particle swarm algorithm. *Expert Systems with Applications*, Vol. 38, pp. 957–968.
12. Eskandar, H., Sadolla, A., Bahreininejad, A. & Hamdi, M. (2012). Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems. *Computers and Structures*, Vol. 110–111, pp. 151–166.
13. Sadolla, A., Bahreininejad, A., Eskandar, H. & Hamdi, M. (2012). Mine blast algorithm: A new population based algorithm for solving constrained engineering optimization problems. *Applied Soft Computing*, http://dx.doi.org/10.1016/j.asoc.2012.11.026.

14. Rajabioun, R. (2011). Cuckoo optimization algorithm. *Applied Soft Computing*, Vol. 11, pp. 5508–5518.
15. Kaveh, A. & Laknejadi, K. (2011). A hybrid multi-objective optimization and decision making procedure for optimal design truss structure. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, Vol. 35, No. C2, pp. 137-154.
16. Kaveh, A. & Malakouti Rad, K. (2010). Hybrid genetic algorithm and particle swarm optimization for the force method–based simultaneous analysis and design. *Iranian Journal of Science & Technology, Transaction B: Engineering*, Vol. 34, No. B1, pp. 15-34.
17. Miguel, L.F.F. & Miguel, L.F.F. (2012). Shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms. *Expert Systems with Applications.* Vol. 39, pp. 9458-9467.
18. Kaveh, A. & Zolghadr, A. (2012). Truss optimization with natural frequency constraints using a hybridized CSS-BBBC algorithm with trap recognition capability. *Computers and Structures*, Vol. 103, pp. 14–27.
19. Crespinsek, M., Liu, S. H. & Mernik, L. (2012). A note on teaching-learning-based optimization algorithm. *Information Sciences*, Vol. 212, pp. 79-93.
20. Rao, R. V., Savsani, V. J. & Vakharia, D. P. (2012). Teaching-learning-based optimization: An optimization method for continuous non-linear large scale problems. *Information Sciences*, Vol. 183, pp. 1-15.
21. Rao, R. V., Savsani, V. J. & Vakharia, D. P. (2011). Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. *Computer-Aided Design*, Vol. 43, pp. 303–15.
22. Lingyun, W., Tianbing, T., Xianghong, X. & Wenjie, S. (2011). Truss optimization on shape and sizing with frequency constraints based on parallel genetic algorithm. *Struct Multidisc Opt*, Vol. 43, pp. 665-682.
23. Zuo, W., Xu, T., Zhang, H. & Xu, T. (2011). Fast structural optimization with frequency constraints by genetic algorithm using adaptive eigenvalue reanalysis methods. *Struct Multidisc Opt*, Vol. 43, pp. 799-810.
24. Konzelman, C. J. (1986). Dual methods and approximation concepts for structural optimization. M.A.Sc. thesis, Department of Mechanical Engineering, University of Toronto.
25. Sedaghati, R. (2005). Benchmark case studies in structural design optimization using the force method. *International Journal of Solids and Structures*, Vol. 42, pp. 5848–5871.