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EXPERIMENTAL EVALUATION OF THE H/V SPECTRAL RATIO CAPABILITIES IN ESTIMATING THE SUBSURFACE LAYER CHARACTERISTICS*

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Abstract– Microtremor data have been carried out for microzonation studies and disaster mitigation in urban areas. Over a period of two decades, the Nakamura's H/V spectral ratio method was recognized as a simple and cost effective method in seismological geotechnics. In order to identify the soil layers characteristics, microtremor measurements were performed in 6 different regions in Iran. These sites are located in different parts of the south, center and east of Iran. Regarding the study, 471 recorded microtremors on an area about 700 km^2 were processed. In addition, the results of 227 refraction tests, 386 electrical resistivity tests and 197 boreholes were evaluated. Based on the analysis results, the resonance frequency of each station was estimated from the peak of the H/V spectral ratio components. Conducting the boreholes or geophysical investigations enabled the thickness of the sedimentary cover to be determined. Different places were selected as study areas namely, Bam, Bushehr, Qeshm Island, Mashhad, South Pars, and Qom. In order to develop the sedimentary thickness, an attempt has been made to derive a formula to correlate the frequency of the horizontal-to-vertical, (H/V) spectral ratio peaks (f_0) to the sedimentary cover thickness (h). The obtained equations in the different sites indicate that the relationship between these two parameters has a power form and it is significantly affected by the subsurface topography and material properties. It is also observed that both shape of basin and sedimentary thickness have a significant influence on the relation formula parameters.

Keywords– Microtremor, H/V spectral ratio, microzonation, site effect

1. INTRODUCTION

Over a period of three decades, several huge earthquakes occurred. The terrible events in Mexico 1985, Manjil 1990, Northridge 1994, Kobe 1995, Izmit 1999, Bam 2003, Sichuan, China 2008, and Honshu, Japan 2011 killed or injured thousands of citizens, destroyed many buildings and a considerable amount of budget was spent on compensation for damages caused by these earthquakes. Consequently, the dynamic characteristic identification and subsurface layers recognition of the soils are regarded as significant factors in seismic hazard estimation at a certain site. Geotechnical engineers always look for the simplest and most cost effective methods to investigate the soil properties and subsurface layer condition. In this regard, the horizontal-to-vertical spectral ratio (H/V) of using the microtremors data has attracted the interest of many researchers and engineers.

The H/V method was first introduced by Nogoshi and Igarshi [1] and improved by Nakamura [2, 3]. Due to its simplicity and cost efficiency it was rapidly accepted and developed. It must be mentioned that three main categories are developed on the H/V technique. In the first category, researchers attempted to evaluate the validity and reliability of the H/V results via the result calculated from earthquake records.

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Their applications consisted of either the standard spectral ratio method proposed by Borcherdt [4], or the generalized inversion technique [5]. Many researches have been conducted in this category [6-11] which have successfully provided the various formulas necessary to estimate the fundamental resonance frequency of the site using seismic ambient noise. The second group of studies focused on the horizontal-to-vertical spectral ratio method to evaluate the resonance frequency as a mapping tool in the microzonation studies. In this group, the results were evaluated based on analyzing the earthquake fore-shock records [12-16] or after-shock records to evaluate the building damage by the identification of a local site [17, 18]. The last category of research, which was recently performed, is concentrated on estimating a relationship between the resonance frequency, shear-wave velocity and thickness of the soil layers. In order to find this relationship, the researchers have used the H/V spectral ratio method as a geophysical exploration tool [19-21]. Practically, based on the soil layer depth obtained from boreholes or geophysical methods, and the horizontal-to-vertical resonance frequency value, the average shear-wave velocity of the soil column can be estimated.

In this paper, an effort has been made to evaluate the H/V spectral ratio collected from different study areas located in Iran. This model study was initiated with the intention of developing a series of unique formulas to estimate the depth of seismological bed from fundamental frequencies. Furthermore, it attempts to recognize the parameters that have a more significant effect on these relationship formulas.

a) The microtremor usage in identification of the resonant frequency of site

The resonance frequency of the sites can be acquired using microtremor analysis. This is clear evidence of the shear wave velocity contrast between bed-rock and sedimentary cover: the higher the amplitudes, the larger the velocity contrast will be. In addition, the amplitude of the H/V ratio depends not only on the velocity contrast, but also on the source-depth and source-distance distributions. Therefore, amplitudes of the H/V ratio qualitatively introduce the possible resonance effects. In cases where the shear wave velocity of the soft sediments is known from geotechnical or geophysical measurements, H/V spectral ratios can be used to estimate the thickness of the soft sedimentary layer. Since the site response during the earthquake and microtremor excitation is the same, by knowing the resonance frequency of the sites it is also possible to qualitatively know the expected amplification of the soil layer as a function of frequency.

In order to estimate the soil layers characterization, the use of resonance frequency or the shear wave velocity has been taken into consideration. Some researchers have suggested the depth of the bedrock as a power function, as shown in relation (1). [19-21]

$$h = af_0^{-b} \tag{1}$$

where h is the depth, f_0 is the resonance frequency, and a and b are the constants. The coefficients a and b are related to the geometric and geotechnical properties of the site. It must be pointed out that all of the previous researches were performed around the Rhine River using the soil features of that region. Table 1 shows the amounts of a and b for these studies.

2. MATERIALS AND METHODS

a) Geological and geotechnical setting of the studied sites

In this study the microzonation data from the 6 sites was utilized. Bushehr, South Pars and the Qeshm Island sites are situated in the south of Iran, on the northern part of the Persian Gulf and the remained sites consisting of Mashhad City, the city of Qom and Bam City are located in the north-east, center and east of Iran respectively. Fig. 1 illustrates the studied sites located in different locations in Iran. The southern sites are situated at the end of the folded region of the main south-eastern zone of the Zagros, which has approximately similar subsurface characteristics. The subsurface investigation indicates that the layer specifications vary with regions, and generally consist of coastal sand surface layers, sedimentary clay in the middle and the clay-marl layers at the bed. In addition, a coral compacted layer appears in some low depth sub-surfaces, for instance, in the Bushehr site, which does not appear in the other sites. Geological investigation shows that most parts of Bam are covered with deposits including gravel, sand and silt. The thickness of dense layers varies from a few meters on the hill to more than 50 meters in the center of Bam City. The Mashhad alluvium consists of three layers. The first layer is made up of surface alluvium which is generally less than 13 meters; below this surface layer lie deep sediments which vary from 13 to 200 meters and, finally, the investigated region of Qom consists of expanded alluvium outcrops i.e., the compacted macro grain materials in the south as well as the young and small grains in the Qom plain. Generally, the first layer of the Qom site from zero to 5 meters is composed of sand and macro grain silt, the second layer is made up of silt and sand or silt stone from 5 to 25 meters, and the third layer, located below the second layer, consists of sand and gravel. The sites specifications are shown separately in Table 2.

Table 1. The suggested a and b for the equation 1 in the previous researches

Site	Researchers	Soil type	Suggested parameters	
			a	*b*
Aachen	Ibs-Von Seht and Wohlenberg,1999	Gravel, sand, clay	96	1.388
Cologne	Parolai, 2002	Gravel, sand, clay	108	1.588
Cologne	Hinzen,2004	Gravel, sand, clay	107	1.119

Fig. 1. Location of investigated sites and important cities in Iran

b) Data and data analysis

The data used in this research were taken from some microzonation studies that have been manipulated by different research teams mainly from IIEES (Tehran), except the data of Mashhad,
prepared by Mashhad University. In order to recognize the sites specifications, in total, 471 recorded microtremors on an area about 700 km^2 were processed. In addition, the results of 227 refraction tests, 386 electrical resistivity tests and 197 boreholes were evaluated. The data from each site have been processed using GEOPSY software, following the same procedure and using the same processing parameters recommended in SESAME manual [22]:

- The mean of the entire recorded signal is deducted from each sample value.

Table 2. The specification of soil layers and suggested equation for studied sites

Site	No. of Geophysics and Geotechnical Tests				No. of Microtremor Stations	Sedimentary Thickness(m)	Soil Types
	Down-hole	Refraction	Resistivity	Boreholes			
Bushehr and Aalishahr	6	66	37	35	46	0-2	Silt, firm sand
						2-6	Corallites cap-rock
						6-35	Sand Stone
						>35	Clay- Marl
South Pars, Phase 3	12	95	95	270	123	0-5	Mixture Gravel, Sand, Silt and Clay – low to medium density
						5-40	Dense Sand and Silt, or Dense Gravel and Sand
Qeshm Island	_	62	_	46	46	0-12	Sand, Silt and Clay
						>12	Sandstone
City of Bam	18	36	_	_	18	0-5	Silt and Clay
						5-50	Dense Gravel and Sand
City of Qom	5	100	43	51	60	0-5	Course Sand and Silt With medium density
						5-25	Dense Sand and silt
City of Mashhad	18	_	_	64	200	10-13	Poorly graded Sand
						13-200	Gravel, Well or poorly graded

- Determination of stable windows, in the 30–40 second range, using an anti-trigger, with a 1-s STA, a 30-s LTA, and STA/LTA ratio thresholds of 0.3 and 2.0;
- For each window, a butter-worth's filter with order 4 is applied on both sides of the window signal of the vertical (V), North-South (NS), and East-West (EW) components;
- For each window, a FFT is applied to the signal of the three components to obtain the three spectral amplitudes, to which a Konno and Ohmachi smoothing [1998] is applied, with a bandwidth of 40 and an arithmetical average;

- Spectral ratios (NS/V, EW/V, and average (NS, EW)/ V) are computed.

For the sake of simplicity, only the velocity spectral amplitudes of the vertical component and the average (NS, EW)/ V ratio are presented in the figures. For illustrating the used data, the results of microtremor spectral analysis for seismic stations of the Bushehr Site are presented in Fig.2.

3. RESULTS AND DISCUSSION

a) The h-f_0 relationship from microtremor data analysis

Previous studies on microtremors allow a quantitative interpretation of the relation between the resonance frequency and sediment thickness cover; a depth estimate has been derived from the measured H/V peak frequencies. With reference to the Nakamura method, which approved the origin of the main peak observed in the H/V spectral ratio that is related to the average shear wave velocity in a sediment layer, the sedimentary thickness *h* can be related to the H/V resonance peak frequency. By selecting the depth of the seismic contrast *h*, as recommended by technical committee of the international society of soil mechanics and geotechnical engineering (ISSMGE) obtained from the geotechnical and geophysical procedure [23], it is possible to derive a modified relation between the H/V peak frequency *f* and depth *h*. The statistical results indicate that the best relation is fitted by the power equation trend line as shown in the equation (1). To develop the required formula for each site, the following discussed trend was used to derive the equations. For instance, the Bushehr site including 33 various stations were selected to describe the research procedure. Using the geophysical and geotechnical investigated data, resonance frequencies and the shear wave of site were evaluated at 33 stations. Evaluating the H/V spectral ratio showed that 23 curves have a clear peak and the others have flat curves or include more than one peak. The resonance frequencies vary from 0.5 to 30 Hz and the shear wave velocities vary from 200 to 950 m/s. Fig. 3 illustrates the correlation formula on the Bushehr site which recognizes the depth of bed in the function of resonance frequencies. The relationship between the mean thickness and frequency of the main peak in the H/V spectra is given in the Eq. (2).

$$h = 29.86(f_0)^{-0.62} \tag{2}$$

The results show that the correlation relationship (1) is reliable for all the studied sites including Bam, Bushehr and Alishahr, Qeshm, Mashhad, South Pars and Qom. The independent information about the sediment data can be used to obtain the parameters *a* and *b* empirically by fitting the data sets of the H/V frequencies *f* on different sites where the thickness *h* is available. Fig. 2 illustrates the calculated H/V spectral ratio at the selected stations in the Bushehr Site with a strong velocity contrast between sediment and bedrock.

The other sites have been evaluated by the same trends. Figs. 3 to 6 and Table 3 show the soil layer specifications used in this research. Estimating the resonance frequency (f_0) from the peak in the H/V ratio and the thickness of the sedimentary cover obtained from the borehole data, a set of nonlinear regression fits is obtained for 6 investigated sites. The derived formula has been summarized in equations (2) and (4) to (9) in Table 3.

b) Evaluation of the h-f_0 relationship with the geophysical data analysis

To evaluate the resonance frequency of the sites, another approach was adopted. For this purpose, the resonance spectral ratio frequency was compared with the frequencies obtained from the average of the shear wave velocity of the sedimentary cover. This frequency was determined in the conventional Eq. (3) introduced by Dobry et al. [24].

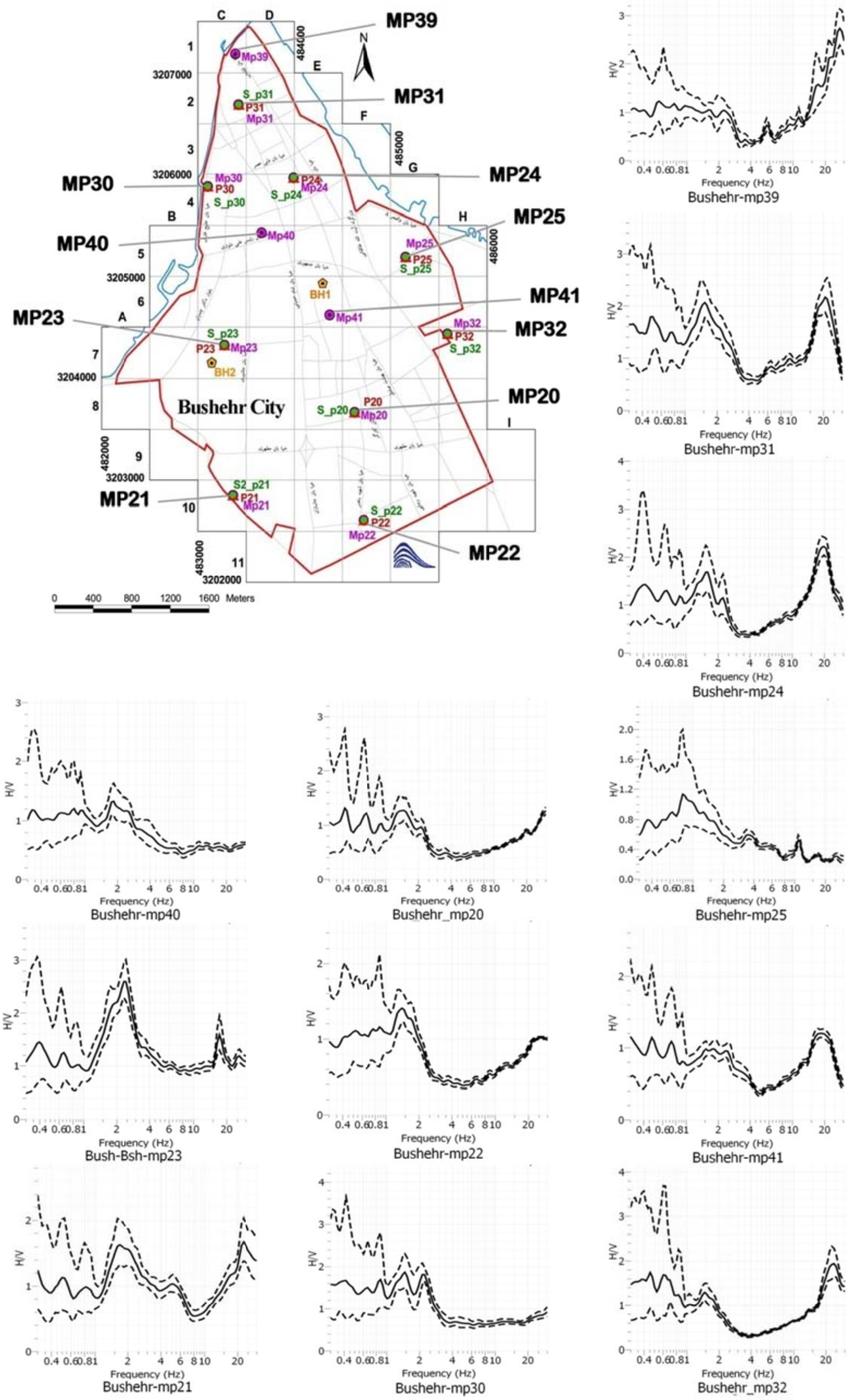

Fig. 2a. The analyzed spectral ratio of stations in the Bushehr site (city of Bushehr); these pictures have been presented as an example to show the microtremor analysis procedure of the sites.

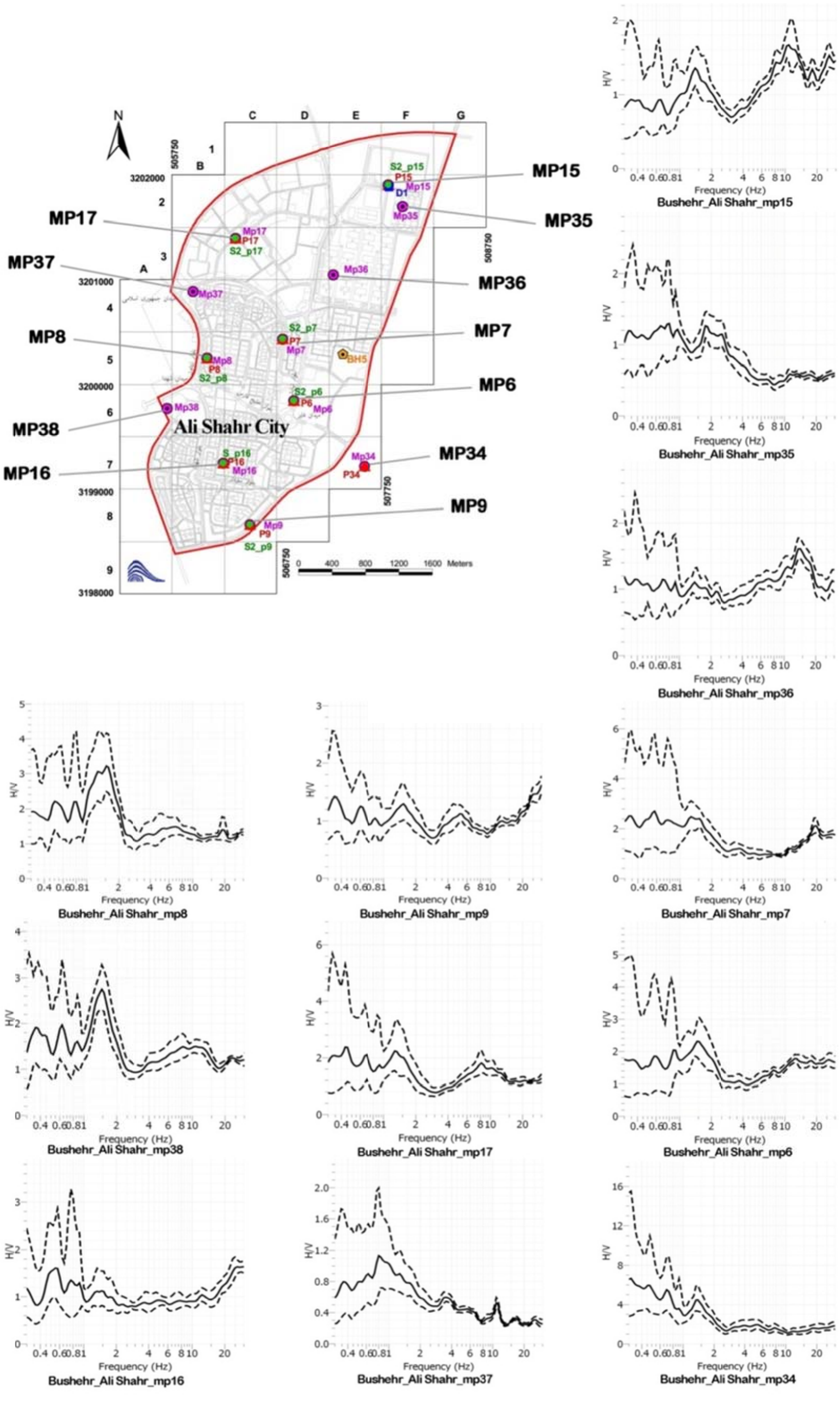

Fig. 2b. The analyzed spectral ratio of stations in the the Bushehr site (city of Ali-Shahr)

$$f_0 = \frac{v_s}{4h} \tag{3}$$

This procedure was used to evaluate the trend discussed in the suggested equations obtained previously by the Eq. (1).

The relationship (2) could be compared with the f_0 which was used to obtain the Eq. (3). Fig. 4 shows the equation introduced in the Eq. (2) for the Bushehr site. By comparing the frequencies obtained from the two methods as discussed before, Fig. 5 confirms a good agreement between the mean frequencies obtained from the H/V spectral ratio curve and those achieved by the Eq. (3). The other sites imitate the same trends. For all sites the parameters are acquired from the Eqs. (1) and (3). The results and relationship equations have been summarized in Fig. 6.

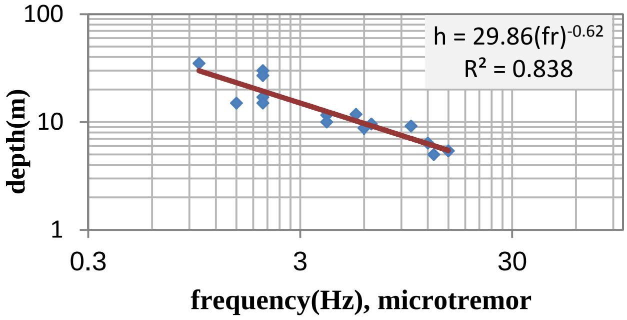

Fig. 3. Suggested relationship between sedimentary depth and resonant frequencies obtained from H/V spectral ratio curves in the Bushehr site

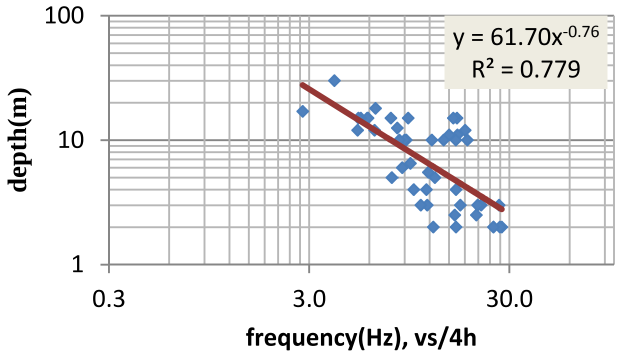

Fig. 4. The relationship between sedimentary depth and frequencies calculated from relation f_0=Vs/4h in the Bushehr site

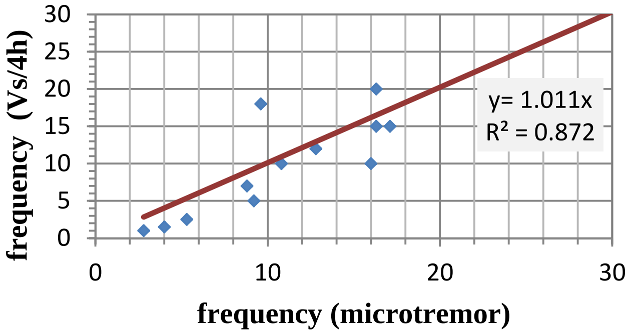

Fig. 5. Comparision of the frequencies obtained from H/V method with the frequencies calculated with relation f_0=Vs/4h in the Bushehr site

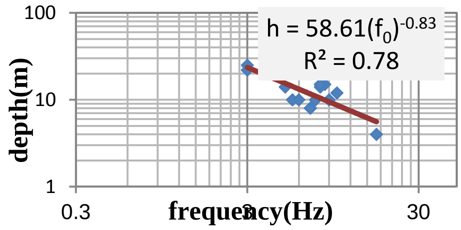

Fig. 6a1. The relation formula for the Bam site

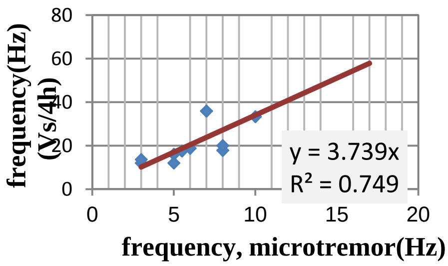

Fig. 6b1. Comparision of the frequencies obtained from two different methods in the Bam site

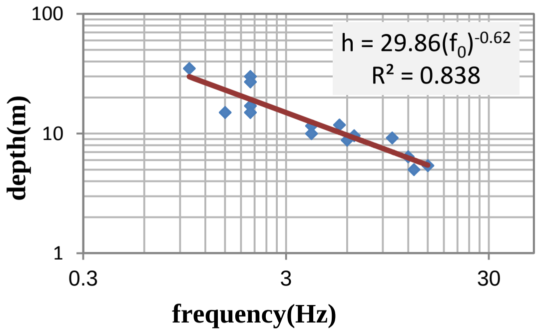

Fig. 6a2. The relation formula for the Bushehr site

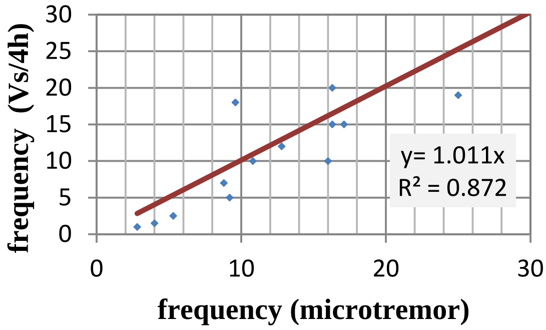

Fig. 6b2. Comparison of the frequencies obtained from two different methods in the Bushehr site

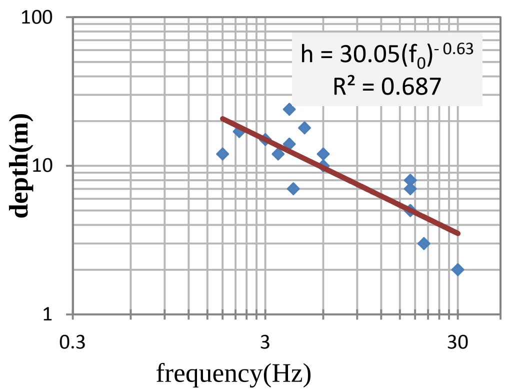

Fig. 6a3. The relation formula for the Qeshm site

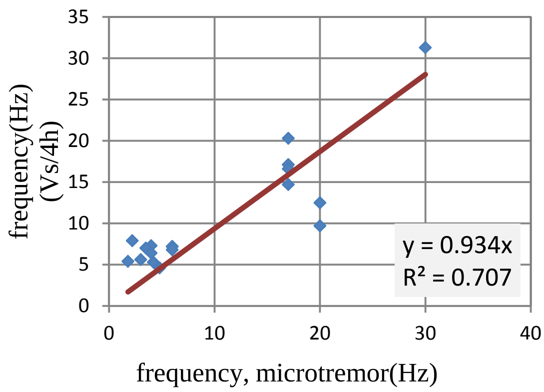

Fig. 6b3. Comparison of the frequencies obtained from two different methods in the Qeshm site

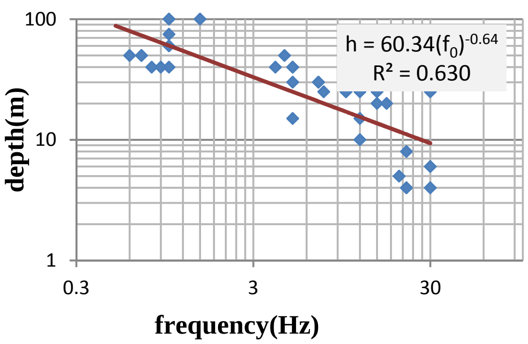

Fig. 6a4. The relation formula for Qom site

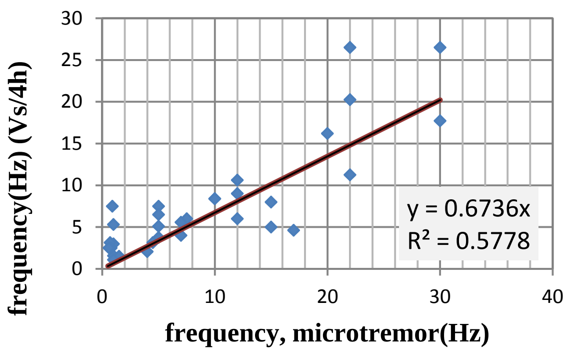

Fig. 6b4. Comparision of the frequencies obtained from two different methods in the Qom site

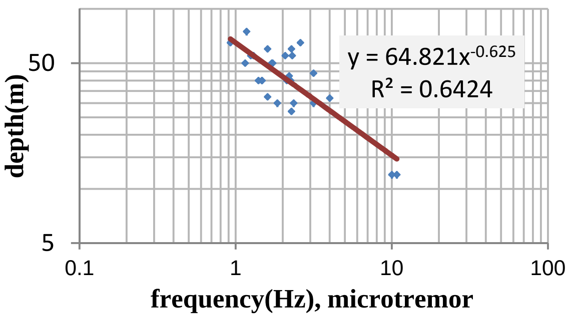

Fig. 6a5. The relation formula for the city of Mashhad

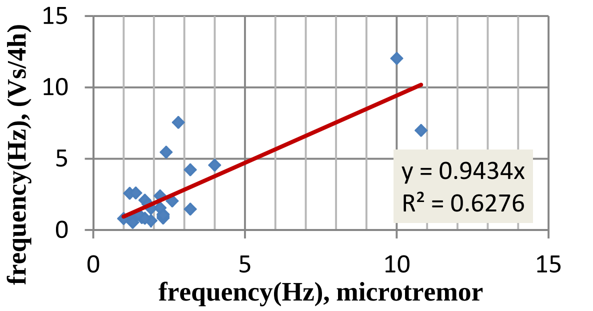

Fig. 6b5. Comparison of the frequencies obtained from two different methods in the city of Mashhad

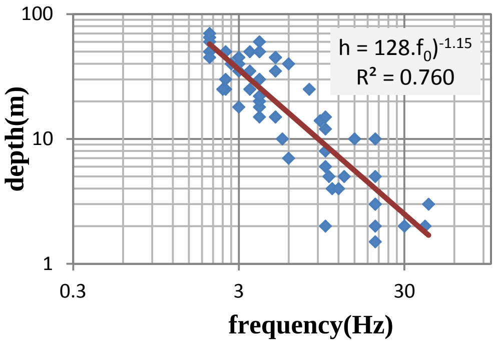

Fig. 6a6. The relation formula for the South Pars site

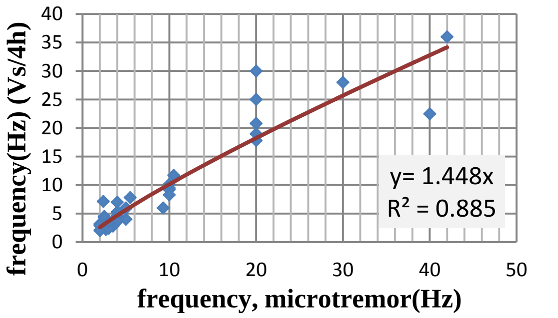

Fig. 6b6 Comparison of the frequencies obtained from two different methods in the South Pars site

Fig. 6. a) Suggested relationship equations between sedimentary depth and resonant frequencies, and b) Comparision of the frequencies obtained from H/V method with frequencies calculated with relation f_0=Vs/4h for the studied sites

c) Discussion

It seems that the sediments thickness, basin shape, soil properties and sea climate are the parameters having the most significant effects on *a* and *b*. As is shown in the relations (4) to (9). On the Bushehr Port and Qeshm Island the amount of *a* is equivalent to 30 and *b* is close to 0.63; on the Qom and Mashhad sites the soil was coarse and the sedimentary cover thick (over 150 meters), the factor *a* is about 65 and the value of the coefficient *b* is 0.63. On the Bam City site, which markedly resembles these specifications, the amount of a and b are close to those related to other sites, except the amount of a in the Bam site, which is about 58 due to its U-shape topography. Since the shape of the basin and the soil specification are two important parameters in site amplification, Eq. (9) has different amounts of *a* and *b*; the correlation equation for Site 3, South Pars, is close to the equations presented by Parolai (2002) and Hinzen (2004) for the Cologne basin near the Rhine River, due to the similarity of subsurface structures i.e., the nearby flat structure. Furthermore, on the studied sites the amplification factors are between 3 and 4 with an average 3.6, except the Qom site where the amplification factor is about 5. A comparison of the suggested equations sites located near the Persian Gulf with a drier environment area, for instance Bam or Mashhad, suggests that the water table level has no essential effect on the resonance frequencies. In addition, it was observed that the thickness of soil layers is a effective factor in the relation between the depth and frequencies. From a practical perspective, the equations presented in this research have appropriate utility for estimating the bed thickness in the absence of the borehole data.

Table 3. The suggested relationship equations for the studied sites

Site	Relation Formula: $h=a(f_0)^{-b}$	
City of Bam	$h=59(f_0)^{-0.83}$	(4)
Bushehr and Aalishahr	$h=29.86(f_0)^{-0.63}$	(5)
Qeshm Island	$h=30(f_0)^{-0.63}$	(6)
City of Qom	$h=60.34(f_0)^{-0.64}$	(7)
City of Mashhad	$h=65(f_0)^{-0.63}$	(8)
South Pars, Phase 3	$h=128(f_0)^{-1.15}$	(9)

4. CONCLUSION

The H/V spectral ratio of the microtremor was calculated for 6 stations namely, the Bushehr region, South Pars (phase 3), Qeshm Island, the city of Qom, the city of Bam and Mashhad City. The soil profile characteristics from the inversion of H/V Spectral ratio technique were estimated. In order to determine the geometric specifications of sedimentary layers, 471 recorded microtremors as well as 227 refraction tests, 386 electrical resistivity and 197 boreholes data were collected. A set of correlation formulas between the sediment depth and the frequency of the main peak in the H/V spectral ratio were estimated. The following conclusions could be drawn from this research.

1- The results confirm that the one-station H/V spectral ratio method could be a suitable approach to estimate the geometric specifications of the soil layers, especially for the soil layers with a clear contrast between the sedimentary cover and bedrock.
2- For all sites, a power function confirms the relation between seismic bed depth and resonance frequencies.
3- The sediments thickness, basin shape, soil properties and marine weather are the parameters which have greater effects on parameters a and b in Eq. (1).
4- The subsurface topography has a greater effect on the parameters *a* and *b*.
5- Due to the similarity in the subsurface topography and thickness of sedimentary cover, the obtained correlations in Mashhad, Qom and Bam are similar. Furthermore, the relationship formulas for Qeshm and Bushehr have a similar trend.
6- The reliability of the equations obtained from the H/V techniques is comparable with the other techniques such as down-hole and resistivity method.
7- The similarities between the equations in the studied sites are good evidence for the utilization of this method in the site investigation.

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REFERENCES

1. Nogoshi, M. & Igarashi, T. (1971). On the amplitude characteristics of microtremor (part 2). *Journal of Seismological Society of Japan*, Vol. 24, pp. 26-40.
2. Nakamura, Y. (1989). A method for dynamic characteristics estimation of subsurface using microtremor on the ground surface. *Quarterly Report Railway Tech. Res. Inst*., Vol. 30, pp. 25-30.
3. Nakamura, Y. (2000). Clear identification of fundamental idea of Nakamura's technique and its applications. *Proceedings of the 12th World Conference on Earthquake Engineering*. Auckland, New Zealand

4. Borcherdt, R. D. (1970). Effects of local geology on ground motion near San Francisco Bay. *Bulletin of the Seismological Society of America*, Vol. 60, pp. 29-61.
5. Castro, R. R., Anderson, J. G. & Singh, S. K. (1990). Site response attenuation and source spectra of S waves along the Guerrero. Mexico, Ubduction zone. *Bulletin of the Seismological Society of America*, Vol. 80, pp. 1481–1503.
6. Field, E. & Jacob, K. (1995). A comparison of various site-response estimation techniques, including three that are not reference-site dependent. *Bulletin of the Seismological Society of America,* Vol. 85, pp. 1127–1143
7. Gue´guen, P., Chatelain, J. L., Guillier, B. & Yepes, H. (2000). An indication of the soil topmost layer response in Quito (Ecuador) using H/V spectral ratio. *Soil Dynamics and Earthquake Engineering,* Vol. 19, pp. 127–133.
8. Haghshenas, E., Bard, P. Y., Theodulidis, N. & SESAME WP04 Team (2008). Empirical evaluation of microtremor H/V spectral ratio. *Bulletin of the Seismological Society of America,* Vol. 6, pp. 74-108.
9. Lachet, C. & Bard, P. Y. (1994). Numerical and theoretical investigations on the possibilities and limitations of Nakamura's technique. *Journal of Physics of the Earth,* Vol. 42, pp. 377–397.
10. Lebrun, B., Hatzfeld, D. & Bard, P. Y. (2001). Site effect study in urban area: experimental results in Grenoble (France). *Pure and Applied Geophysics*, Vol. 158, pp. 2543–2557.
11. Lermo, J. & Chavez-Garcia, F. J. (1993). Site effects evaluation using spectral ratios with only one station. *Bulletin of the Seismological Society of America.* Vol. 83, pp. 1574–1594.
12. Alfaro, A., Pujades, L., Goula, X., Susagna, T., Navarro, B. M., Sanchez, F. J. & Canas, J. A. (2001). Preliminary map of soil's predominant periods in Barcelona using microtremor. *Pure and Applied Geophysics,* Vol. 158, pp. 2499–2511.
13. Fäh, D. (1997). Microzonation of the city of Basel. *Journal of Seismology*, Vol. 1, pp. 87-102.
14. Gue´guen, P., Chatelain, J. L., Guillier, B. & Yepes, H. (2000). An indication of the soil topmost layer response in Quito (Ecuador) using H/V spectral ratio. *Soil Dynamics and Earthquake Engineering,* Vol. 19, pp. 127–133.
15. Lebrun, B., Duval, A. M., Bard, P. Y., Monge, O., Bour, M., Vidal, S. & Fabriol, H. (2004). Seismic microzonation: a comparison between geotechnical and seismological approaches in Pointe-à-Pitre (French West Indies). *Bull of Earthquake Engineering,* Vol. 2, pp. 27–50.
16. Haeri, S. M. & Hajialilue Bonab, M. (2001). Vertical and horizontal seismic microzonation by microtremor measurement. *Iranian Journal of Science and Technology,* Vol. 25, pp 709-715.
17. Duval, A. M., Méneroud, J. P., Vidal, S. & Singer, A. (1998). Relation between curves obtained from microtremor and site effects observed after Caracas 1967 earthquake. *Proceedings of the 11th European Conference on Earthquake Engineering,* Paris, France.
18. Nakamura Y. (1996). Real-time information systems for hazards mitigation. *Proceedings of the 11th World Conference on Earthquake Engineering,* Acapulco, Mexico.
19. Hinzen, K. G., Scherbaum, F. & Weber, B. (2003). On the resolution of H/V measurements to determine sediment thickness, a case study across a normal fault in the Lower Rhine Embayment. Germany. *Submitted to JEE*.
20. Lermo, J. & Chavez-Garcia F. J. (1994). Are microtremors useful in site response evaluation. *Bulletin of the Seismological Society of America,* Vol., pp.1350–1364.
21. Parolai, S., Bormann, P. & Milkereit, C. (2002). New relationships between Vs, thickness of sediments, and resonance frequency calculated by the H/V ratio of seismic noise for the Cologne area (Germany). *Bulletin of the Seismological Society of America,* Vol. 92, pp. 2521–2527.
22. Bard and SESAME manual, (2004). Guidelines for the implementation of the H/V spectral ratio technique on ambient vibration. *European Commission. Research General Directorate Project*, No. EVG1-CT-2000-00026 SESAME
23. TC4, ISSMGE. (1999). *Manual for zonation on seismic geotechnical hazards (revised version),* the Japanese Geotechnical Society.
24. Dobry, R, Oweis, I. & Urzua, A. (1976). Simplified procedures for estimating the fundamental period of a soil profile. *Bulletin of the Seismological Society of America,* Vol. 66, pp. 1293–1321.