

TWO PHASE FLOW MODELING IN SHAFT-SPILLWAYS USING VOLUME OF FLUID (VOF) METHOD*

F. ASADSANGABI^{1**}, N. TALEBBEYDOKHTI² AND M. RAHNAVARD³

¹Dept. of Civil and Environmental Engineering, Shiraz University, Shiraz, I. R. of Iran
Email: f.sangabi@gmail.com

²Dept. of Civil and Environmental Engineering, Head of Center for Environmental Research and Sustainable Development, Shiraz University, Shiraz, I. R. of Iran

³Dept. of Civil and Environmental Engineering, Islamic Azad University, Sarvestan Branch, I. R. of Iran

Abstract– In this paper, the optimum inlet shape of morning-glory spillway is discussed. The effect of some parameters including maximum discharge coefficient and minimum possibility of cavitation (cavitation index) are investigated. For this purpose, using computational fluid dynamics (CFD) in the form of finite volume method, 3D Navier-Stokes equations of flow at different inlet shapes are solved. Free water surface is modeled by water-air two phase flow and equations are solved by volume of fluid (VOF) method. Flow turbulence is modeled by “K-ε model”. Based on experimental results, models are verified and discharge, velocity, pressure and cavitation index for different inlet shapes are computed and compared with each other. The morning-glory spillway inlet is modeled using different types of equations including WAGNER equation ($Y=aX^{3.88}$), CREAGER equation ($Y=aX^{1.87}$) and circular form for their funnel profile shapes. In each model changing the head (h) versus discharge (Q) diagram is drawn and results are compared.

Keywords– Morning glory spillway, cavitation, VOF method, CFD

1. INTRODUCTION

a) Morning-glory spillway

Morning-Glory spillway is one of the common spillways for discharging the overflow water behind dams, these kinds of spillways are constructed in dams with small reservoirs. The most important advantage of Morning-Glory spillway is its lower construction costs by using diversion tunnel constructed at the time of dam construction as a horizontal tunnel.

Morning-Glory spillway has three main parts: Cup-shape inlet funnel, vertical shaft, and horizontal diversion shaft (In some cases, a basin is constructed at the end of horizontal tunnel). The design discharge should develop the free flow in the horizontal shaft because the capacity of vertical shaft and diversion shaft should be more than the capacity of spillway inlet structure. An aerator is placed in vertical shaft in order to have the atmospheric pressure all along the spillway [1]. This spillway usage is recommended when:

1. The earthquake probability is low [2].
2. The horizontal spillway can be connected to the current diversion shaft [2].
3. The rate of trash and floating materials is not significant [2].

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**Corresponding author

4. There is a space limitation for constructing the direct spillway [3].
5. The geotectonic conditions with regard to structure subsidence is suitable [3].
6. A short diversion shaft can be prepared [3].

b) Types of flow state in Morning-Glory

In two phase flow investigation, there is a need for robust methods capable of predicting interfaces, in addition to treating the traditional governing equations of fluid mechanics (Navier-Stokes Eqs.). Such methods in a finite volume approach can be classified into two typical categories called interface tracking and interface capturing methods. According to their abilities, interface capturing methods are of more interest in free surface modeling, especially when complex interface topologies such as wave breaking are included [4].

Morning-Glory spillway may be designed as "Free flow" or "Submerged". In free flow state it acts like normal direct spillway and the discharge changes have a relationship with the $3/2$ root of the head on top of the spillway, but in submerged state, the flow conditions are completely different. Discharge changes have a relationship with the square root of the depth on top of the spillway; in this situation when the water height on top of the spillway increases, the discharge increment will be very low. Thus if spillway is designed as submerged, a very high safety factor should be considered for its capacity [3].

Different states of flow in Morning-Glory spillway have been shown in Fig. 1. As it is seen in the figure, in the first state (Crest Control) the flow discharge is from point "a" to point "g", in the second state (Tube Control) the discharge is from point "g" to point "h", and in the third state (Conduit Control) the discharge is from point "h" upwards. Each of the above mentioned states may occur by varying spillway's dimensions and the head on the spillway [5].

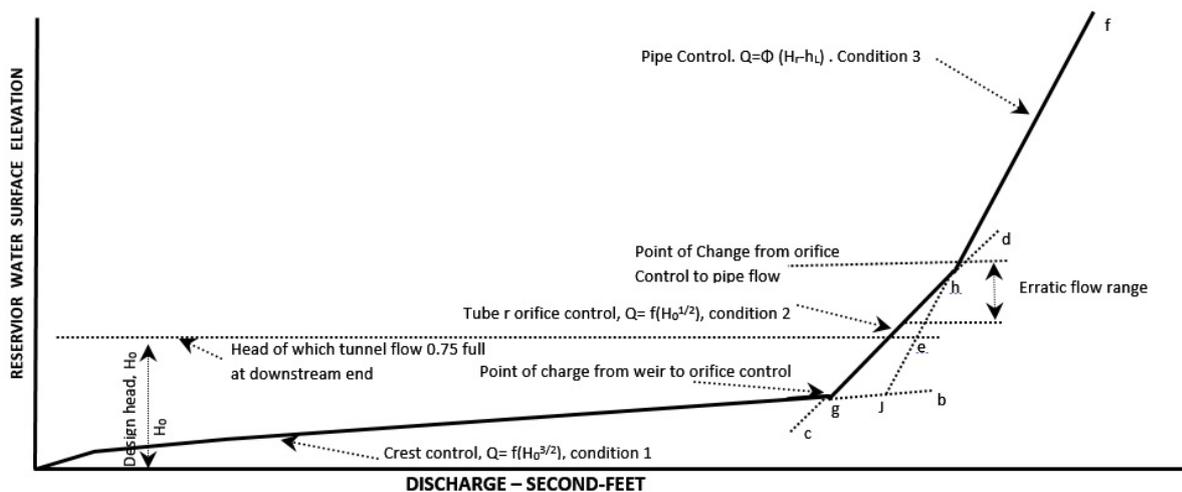


Fig. 1. Different states of flow in Morning-Glory spillways [4]

Due to the critical role of inlet shape of morning-glory spillways on discharge capability, many inlet shapes have been studied extensively by dam designers [6], but most of these works focus on physical models.

In this work the effect of inlet shape of spillway on the flow parameter is discussed with numerical method, for this purpose, three common inlet shapes are modeled by computational fluid dynamics (cfd) in the form of finite volume method.

The rest of the paper is organized as follows: section 2 briefly describes the parameters and methods for numerical modeling with software, subsections of section 2 describe each model and its results, section 3 compares the results and draws a concise conclusion.

2. NUMERICAL ANALYSIS WITH FINITE VOLUME METHOD

Because of the relative complex geometry of models, combined meshes including rectangular and triangular meshes have been used and the geometry of models has been presented as three-dimensional.

The number of used cells in this mesh was as follows: first, the mesh with multimillion cells has been used for the model. It was remarked that in comparison with the mesh having multi-thousand cells, there is a small difference among them regarding the specifications of flow. Therefore, because of the high number of considered exercises and their time-consuming analysis, the decision was made to use multi-thousand mesh and the finer meshes are merely used for comparison.

Because of high capability in modeling the flow in the state of free surface and conduit flows combination, the finite-volume software was used for modeling of spillways. Fluid volume method was applied because of its capability in forming the free surface, because the complexity of flow and presence of vortex, three dimensional model was preferred. Also, flow turbulences were modeled in the shape of both "k-ε" and "Reynolds stress" models, and after comparison, "k-ε" model was used because of its simplicity, its usage in similar works and also its prevalence. Two-phase flow including water and air has been used for flow simulation. The following have been considered for density and viscosity of water and air:

$$\begin{aligned} \text{Water:} & \quad P= 1000 \text{ Kg/m}^3 & \quad \mu= 0.001 \text{ Kg/m.s} \\ \text{Air:} & \quad P= 1.225 \text{ Kg/m}^3 & \quad \mu= 1.79 \text{ e-}5 \text{ Kg/m.s} \end{aligned}$$

A cylinder shaped volume was used on the spillway in order to develop a reservoir head on the spillway, and generally the boundary conditions are as shown in Table 1.

Table 1. The boundary conditions of numerical model

Model inlet boundary	Pressure-inlet
Outlet of spillway	Pressure-outlet
Walls of spillway	Wall

The water level in reservoir was considered from 0 to 50 meters in 10 stages with 0.5 meter variations. Discharge, cavitation index and other specifications of the flow have been studied in each stage.

It should be mentioned that vortex phenomenon has not been considered in the first stage, because the reservoir's head in this stage is less than submerged limit, so that only the inlet funnel shape determines the rate of discharge (Crest Control).

In this paper the morning-glory is modeled using different types of equations including WAGNER equation ($Y=a*X^{3.88}$)[7], CREAGER equation ($Y=a*X^{1.87}$)[7] and CIRCULAR form for their funnel profile shapes.

a) Model 1 (Circular model)

1. Geometry and meshing of model 1: The geometry of model 1 is shown in Fig. 2. This spillway consists of three Parts:

- 1- Inlet orifice with 13 meter initial diameter and 8 meter final diameter; this cross section variation in profile follows the CIRCULAR equation.
- 2- 44.66 meter prismatic vertical shaft.
- 3- 90 degree bending with 8 meter diameter.

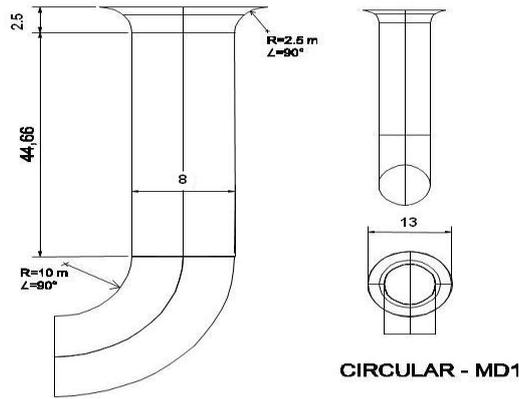


Fig. 2. Geometrical specifications of model (1)

Irregular combined mesh has been used. The number of used cells is 214483, Mesh Grid and boundary conditions are shown in Fig. 3.

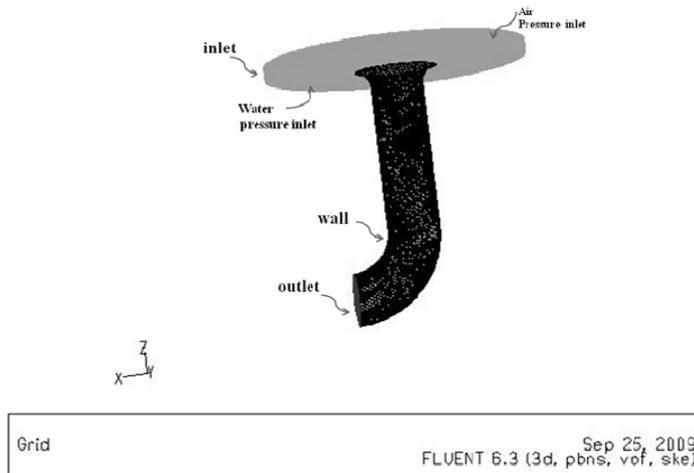


Fig. 3. Meshing & boundary conditions of Model 1

2. The results of model 1: After making the geometry, meshing and modeling the spillway for 10 different depths, the rate of discharge and flow specifications were studied in each stage. Figure 4 shows the spillway's vertical section (Y=0).

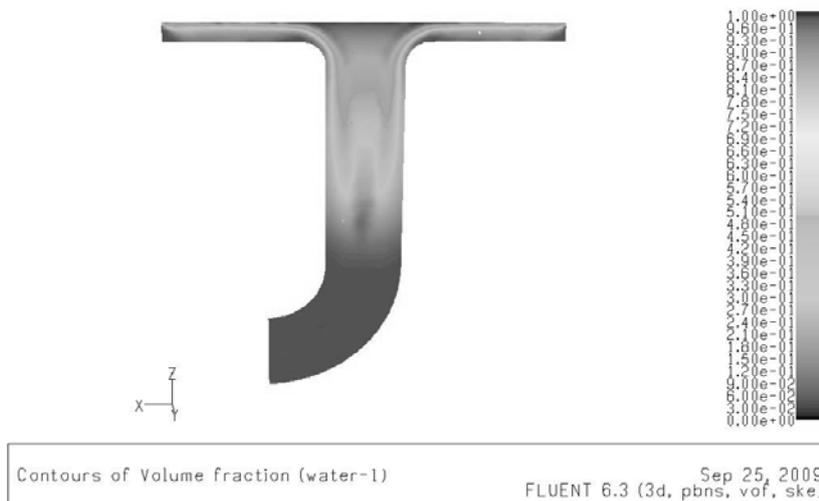


Fig. 4. Vertical section of model 1 for h=2m

Figure 5 shows the chart for relationship of discharge variations to the head on spillway.

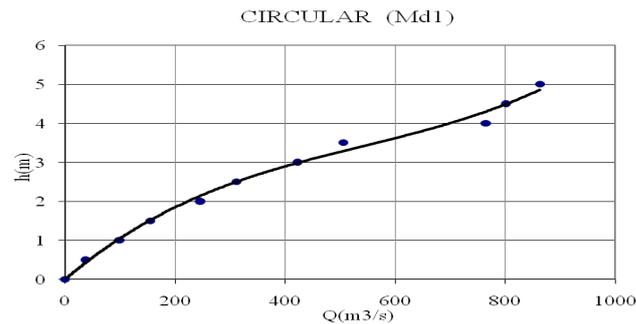


Fig. 5. Relationship of discharge variations to the head for model 1

Figure 6 also shows the chart for relationship of discharge coefficient to h/R , where h is head of water and R is radius of inlet.

Since three modeled spillways are different in scales because of special geometric form, discharge coefficient parameter versus dimensionless number is used for comparing the reservoir's discharge capability. Discharge coefficient in Crest control state is:

$$C = \frac{Q}{\frac{3}{Lh^2}} \quad (1)$$

Where C is the Discharge coefficient, Q is the Discharge, L is the perimeter of funnel edge and h is the head on spillway [8].

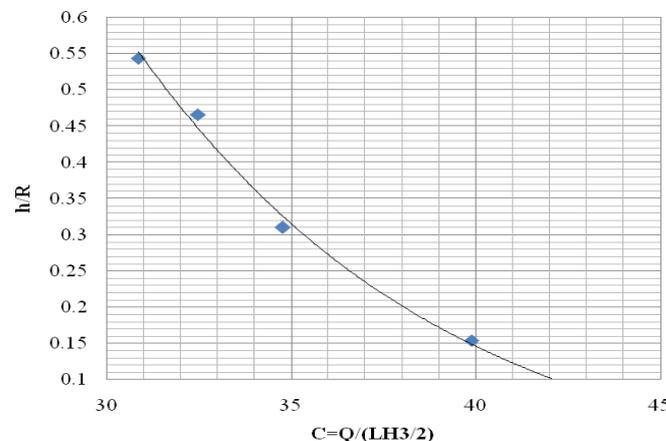


Fig. 6. Chart for relationship of discharge coefficient to h/R for model 1

As it can be seen in Fig. 6, the range of h/R is from 0.15 to 0.55, this range is selected because of the adherence between the water and the spillway's wall under 0.15 in this model, and flow's profile would become unstable after 0.55.

As it was mentioned before, one of the important factors in designing Morning-Glory spillway is the possibility of cavitation. Cavitation index is the most important tool used for cavitation estimation, and is obtained by the following equation:

$$\sigma = \frac{P - P_v}{\frac{\rho V^2}{2}} \quad (2)$$

P and Pv are flow absolute pressure and vapor pressure, respectively, and V is the velocity in each point of flow [4].

Figure 7 shows the variation of cavitation index with respect to h. In this state, the velocity in all spots of the model is less than 9 meters per second, in which the cavitation index is always more than 0.25 critical rates, and the possibility of cavitation occurrence is very small.

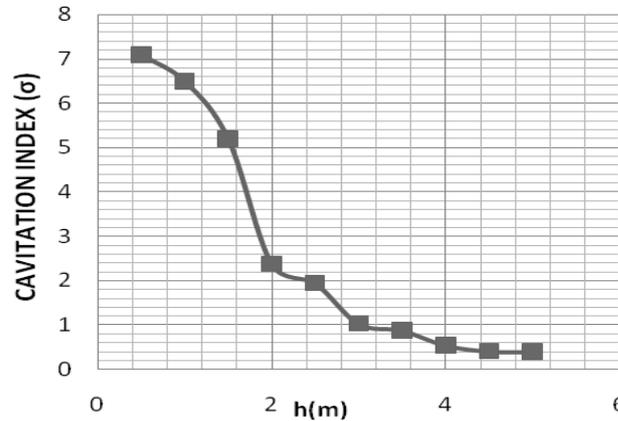


Fig. 7. The variation of cavitation index with respect to h

In Circular model, the point which is considered as the critical for cavitation in spillway's inlet section is located in the junction of inlet funnel and vertical shaft, in this point there is high possibility of flow separation from the wall surface.

b) Model 2 (Wagner model)

1. Geometry and meshing of model 2: The geometry of model 2 is shown in Fig. 8. This model also consists of three main parts:

- 1- Inlet orifice which follows the WAGNER function: $Y = 0.0102 * X^{3.818}$
- 2- Prismatic vertical shaft with 10 meter length and 7 meter diameter.
- 3- 90 degree bending with 19.73 meter central radius and 7 meter diameter.

Meshing of WAGNER model has 369571 cells, and as the previous model, irregular mesh has been used. After creating initial and boundary conditions, the spillway was examined for 10 different depths, and then the results were studied.

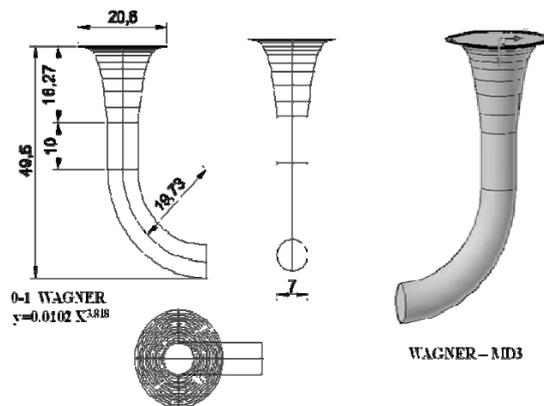


Fig. 8. Geometrical specifications of model 2

2. The results of model 2(Wagner): Figure 9 shows discharge variations chart for different heads on model 2.

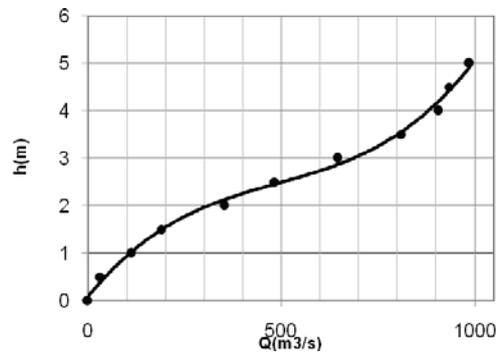


Fig. 9. The chart for relationship of discharge variations to head for model 2

Figure 10 shows the relationship of discharge coefficient variations to h/R for WAGNER spillway.

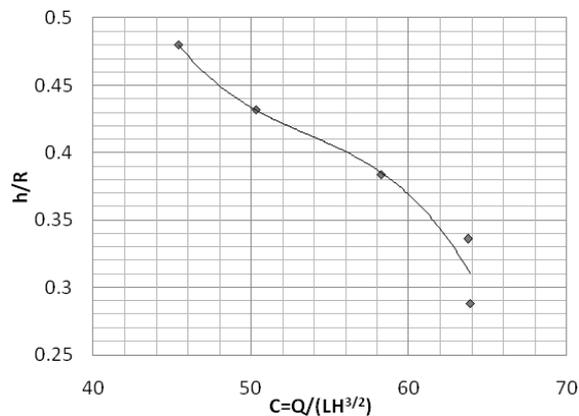


Fig. 10. Relationship of discharge coefficient variations to h/R for spillway in model 2

From the results of model 2, the most critical point for cavitation occurrence is in the first part of spillway with WAGNER function curve. The relationship of Cavitation index variations to the head for the most critical point in model 2 has been shown in Fig. 11.

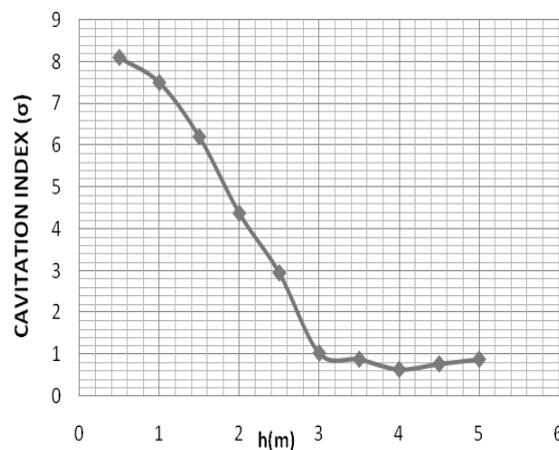


Fig. 11. Relationship of cavitation index variations to the head for the most critical point in model 2

C) Model 3 (Creager)

1. Geometry and meshing of model 3: Figure 12 shows the geometry of model 3. This spillway consists of three parts:

Part 1: Inlet part which is a funnel that varies its radius by CREAGER equation ($Y=a*X^{1.87}$) and with 9.68 meter initial radius.

Part 2: Non-prismatic vertical shaft with linear variation in its section (begins with an 8.52 meter radius and ends in 7.12 meter radius).

Part 3: Non-prismatic 90 degree bending with linear variation in its section which begins with a 7.12 meter radius and ends in 2.25 meters.

As it is shown in Fig. 12, in this model, the curvature of inlet is less than WAGNER and CIRCULAR models, So as it is clear in Fig. 13, compared to previous models, this spillway shows less variation in discharge coefficient by increasing the head. Increasing the slope in the end of the chart in Fig. 13 confirms this action.

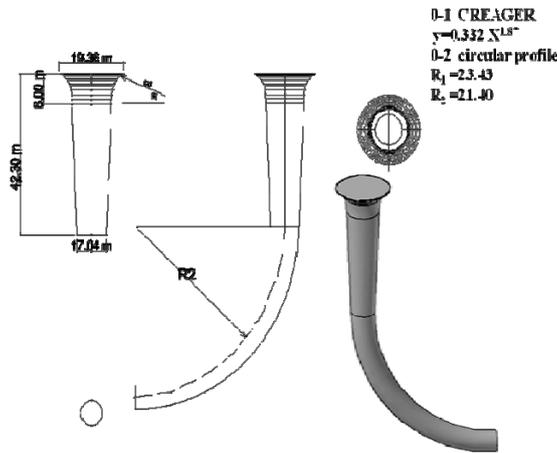


Fig. 12. Geometrical specifications of model 3

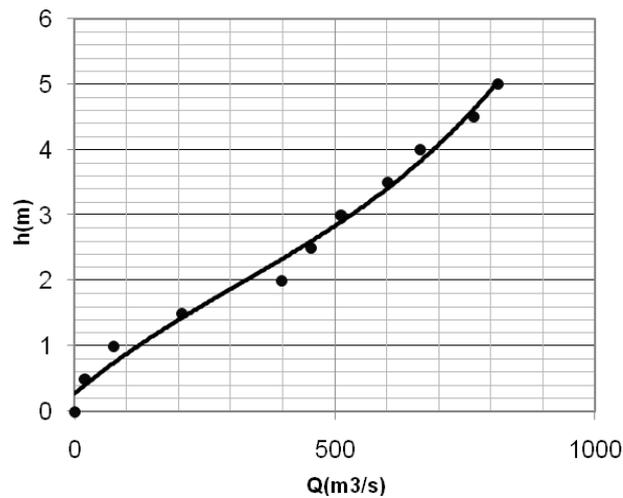


Fig. 13. Chart for relationship of discharge variations to the head in model 3

2. The results of Model 3(CREAGER): Study of this model for different heads illustrates that the most critical point for spillway is the conjunction point of non-prismatic vertical shaft to the bending.

In the chart shown in Fig. 14, the relationship of discharge coefficient to depth for model 3 can be seen. This spillway shows a more steady action to the head's variations, whereas more slope of this chart means that by increasing the head, the spillway would have more capability in discharging the reservoir. This proper action of spillway can be seen in discharge coefficient chart.

Figure 15 also shows the relationship of cavitation index to the head.

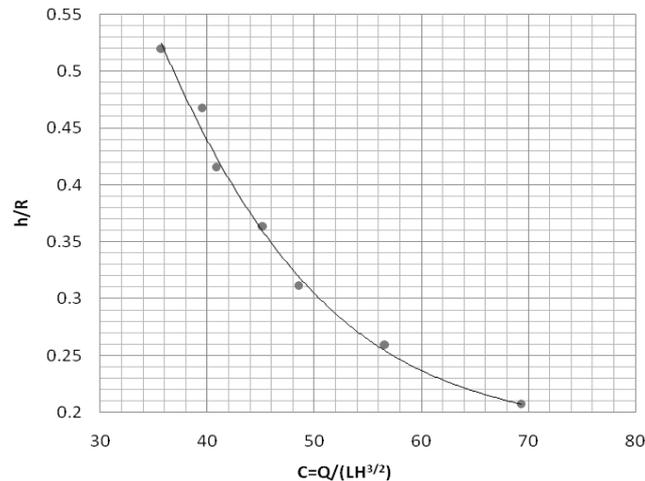


Fig. 14. Relationship of discharge coefficient variations to h/R for spillway in model 3

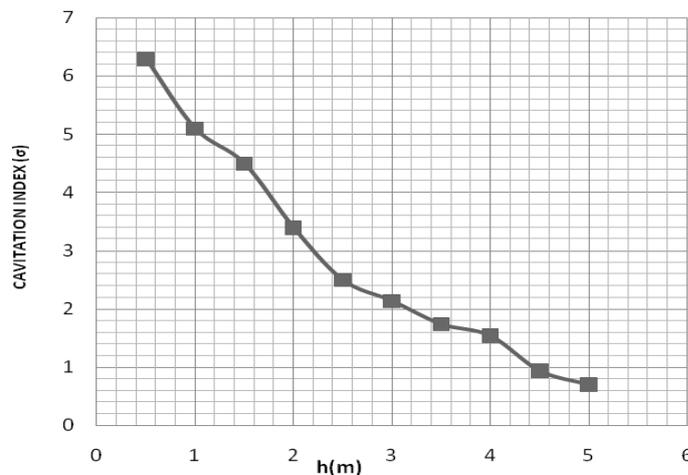


Fig. 15. Relationship of cavitation index variations to the head for the most critical point in model

3. COMPARING THE MODELS

a) The study of discharge-head chart

The chart shown in Fig. 16 illustrates the relationship of discharge rate of all three models with respect to the head. As it is shown, each of three curves consists of three parts:

- 1- The first part shows the crest control.
- 2- The second part is tube control, in this state the spillway is in semi-submerged status.
- 3- The third part shows the spillway is in completely submerged state and the spillway's trap is in high-pressure flow state.

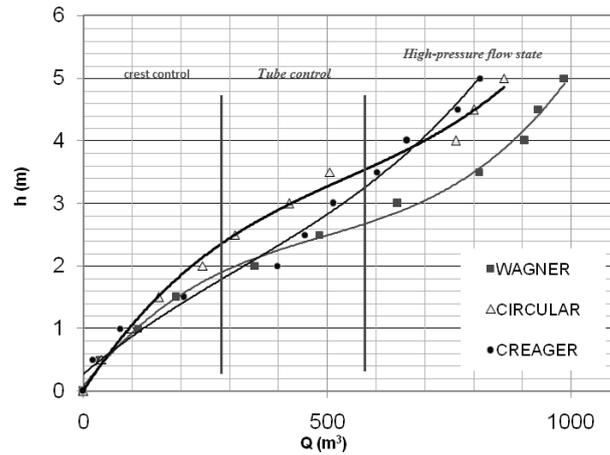


Fig. 16. Relationship of discharged rate to the head for three models

The tilt of curve in CIRCULAR model is greater in the first part of the chart than the other parts, but it is smoother in the third part compared to the three other models. So, it shows this spillway has a higher discharge capability in high relative heads.

The spillway in WAGNER model has less tilt in the first part of its curve, but it increases in the final part. It shows that the spillway for low relative heads has a great discharge capability, but once the head increases and the spillway is submerged, the rate of outlet discharge will incline to a static rate. WAGNER and CREAGER model have a rather similar behavior, but the CREAGER model has less tilt at first, while it has a more tilt in the middle part compared to WAGNER model. Therefore, it is concluded that the CREAGER model has more discharge capability in Crest control state and WAGNER model has more discharge capability in Tube control state. So, inlet funnel in WAGNER model is more suitable, but because the spillway has been modeled with different dimensions, the proper tool for comparing their discharge capability is discharge coefficient, a chart for the relationship of which to the relative head's variations has been shown in Fig. 17.

b) The study of relationship of discharge coefficient to the relative head ((h/R)-C)

According to the chart in Fig. 16, CIRCULAR model has a lower discharge coefficient than the two other models. Therefore, the optimal form of the inlet should be selected between WAGNER and CREAGER model. Discharge coefficient in WAGNER model is higher than CREAGER.

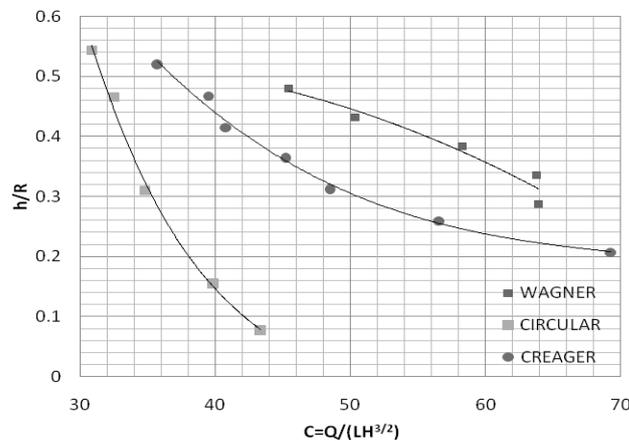


Fig. 17. Relationship of discharge coefficient to relative head for 3 models

c) The study of relationship of cavitation index to the head

Figure 18 shows the chart for relationship of cavitation index to depth for all three models. It should be mentioned that there are more critical points in the bending parts of the models, but this study focuses on the inlet shape, so the critical points have been chosen from funnel section. In CIRCULAR and WAGNER model, the cavitation probability is higher because of the high curvature in the point of section change. As it is shown in this figure, in heads more than 3 meters, the cavitation index of CIRCULAR model and CRAEGER model is less than WAGNER, but in CRAEGER model the cavitation probability is less because of the lower curvature of the funnel.

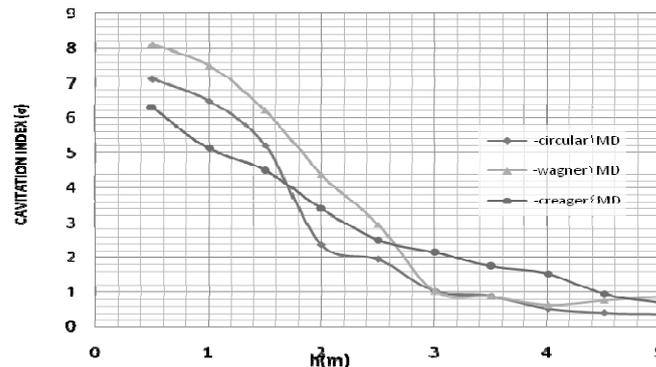


Fig. 18. Relationship of cavitation index to the relative head for three models

4. CONCLUSION

1. Computational Fluid Dynamics (CFD) is a suitable method to investigate the head versus discharge and cavitation index for various inlet shapes of morning glory in comparison to physical models with regard to time and cost.
2. It is clear that each of the inlet shapes is appropriate for its own discharge and reservoir conditions. However, according to discharge index, WAGNER and CRAEGER models are more economical in comparison to CIRCULAR model due to construction cost.

Comparing the cavitation index, CRAEGER model performed better than WAGNER model.

REFERENCES

1. Odgaard, A. J. (1986). Free-surface air core vortex. *ASCE J Hydraul Div.*, Vol. 112, pp. 610–620.
2. Vischer, D. L. & Hager, W. H. (1998). *Dam hydraulics*. Zurich, John Wiley & sons.
3. Bradley, J. N (1945). *Prototype behavior*. TRANSACTION, asce, No. 2802, 1852, pp. 311-333.
4. Panahi, R., Jahanbakhsh, E. & Seif, M. S. (2006). *Comparison of interface capturing methods in two phase flow*. *IJST*, Vol. 29, No. B6, pp.539-548.
5. Peterka, A. J. (1954). Performance tests on prototype and model. *Transaction, ASCE*, No. 2802, 1852, pp. 385-409.
6. Knauss, J. (1987). *Swirling flow problems at intakes*. Hydraulic Structures Design Manual, 1.A. A. Rotterdam: Balkema.
7. Wagner, W. E. (1954). Determination of pressure –Controlled profiles. *Transaction, ASCE*, No. 2802, 1852, pp. 345-385.
8. Blazek, J. (2001). *Computational fluid dynamics: Principales and applications*. Alstom Power Ltd., Baden-Daettwil, Switzerland.