

HYDRAULIC JUMP IN A DIVERGING CHANNEL WITH AN ADVERSE SLOPE*

M. ESMAEILI VARAKI^{1**}, A. KASI², J. FARHOUDI³ AND D. SEN⁴

¹Dept. of water Engineering, University of Guilan, Rasht, I. R. of Iran
Email: esmaeili@Guilan.ac.ir

^{2,3}Dept. of Irrigation and Reclamation Engineering, University of Tehran, Tehran, I. R. of Iran

⁴Dept. of Civil Engineering, Indian Institute of Technology, Kharagpur, West Bengal - 721 302 India

Abstract– In this research, the adverse hydraulic jump formed in a gradually expanding stilling basin of rectangular cross section is investigated both theoretically and experimentally. The experiments were conducted in a specially designed model for a wide range of bed slopes and diverging angles of the basin walls in addition to classic jump for a wide range of Froude numbers. A momentum-based theory is presented to determine the sequent depth ratio. The results show that there is good agreement between theoretical and experimental data. Comparison of important parameters of a diverging hydraulic jump on the adverse slope, such as length, sequent depth and energy loss, with those in the classic jumps indicated that any increase in the adverse bed slope and the diverging angle of basin wall would cause a reduction of the sequent depth and relative length of jump and increase in the relative energy loss in comparison to the classical hydraulic jump.

Keywords– Hydraulic jump, gradually diverging stilling basin, adverse bed slope, sequent depth, energy loss

1. INTRODUCTION

Stilling basins are commonly used in hydraulic structures to dissipate the kinetic energy of a supercritical flow by the formation of a hydraulic jump. The U.S. Bureau of Reclamation (USBR) surveyed the state of knowledge in this field and presented practical guidelines for the design of different types of stilling basins [1]. However, different geometries such as expanding the basin along the streamwise direction or adverse slope may be preferred with a simple transition structure to reduce cost without sacrificing performance. A diverging stilling basin can easily adapt itself to the upstream and downstream conditions in terms of depths and cross sections. A simple expansion of the basin width with straight walls is more economical than a costly transitional structure.

Experimental studies on the effect of gradually diverging stilling basin walls on the hydraulic jump parameters have shown that diverging walls cause reduction of the sequent depth up to 30 percent, length of the jump up to 22 percent and increase in the energy loss compared to the classic hydraulic jump [2-6]. Furthermore, in previous researches, an explicit momentum-based equation was developed for the estimation of the sequent depth. The main difference in the derivation of a theoretical expression in this case, compared with the classic hydraulic jump case, is the mathematical relationship used for the profile of the hydraulic jump; e.g. linear [2], elliptical [3 and 6] or second degree polynomial [4], which is used to estimate the hydrostatic forces on the side wall. In all cases, a satisfying agreement was observed between the experimental and theoretical results.

*Received by the editors July 18, 2012; Accepted April 16, 2013.

**Corresponding author

Investigations on characteristics of hydraulic jump on the adverse slope in the straight rectangular channel showed that the adverse bed slope decreases the sequent depth and relative length of the jump and increases the relative energy loss. In addition, with the application of an appropriate linear function to estimate the weight force of water in the hydraulic jump, an explicit momentum-based equation was developed for the estimation of the sequent depth. Comparison of the results showed that there was satisfactory agreement between the experimental and theoretical results [7-11].

In this study, the gradually diverging hydraulic jump in a rectangular channel on a wide range of adverse bed slopes is theoretically and experimentally investigated and the effects of the rate of expansion of the channel walls and adverse bed slope on the characteristics of the hydraulic jump are quantified.

2. THEORY OF DIVERGING HYDRAULIC JUMP ON ADVERSE BED SLOPE

To derive a theoretical expression for a gradually diverging hydraulic jump on the adverse slope, one may apply the equation of continuity, momentum and energy along with some basic assumptions. In this study, the analysis is based on the assumptions that (1) the streamlines are radial; (2) the pressure distributions are hydrostatic at the control volume surfaces; (3) the flow is steady; (4) the frictional force from the solid boundaries is negligible; and (5) the shape of the surface profile of the jump is assumed to be parabolic.

The flow of a gradually diverging hydraulic jump on the adverse slope is shown in Fig. 1. The body forces and surface forces acting on the control volume included the x-component of the upstream, downstream, and side wall hydrostatic pressure forces and the weight of water body within the hydraulic jump, i.e., F_{p1x} , F_{p2x} , F_{psx} and F_{B_x} , respectively. The momentum equation may be written between the Sections 1 and 2 in the flow direction along the center of the channel:

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t} \int_{CV} u \rho dV + \int_{CS} u \rho \vec{V} \cdot d\vec{A} \tag{1}$$

$$F_x = F_{p1x} + F_{p2x} + F_{psx} + F_{B_x} = \int_{CS} u \rho \vec{V} \cdot d\vec{A} \tag{2}$$

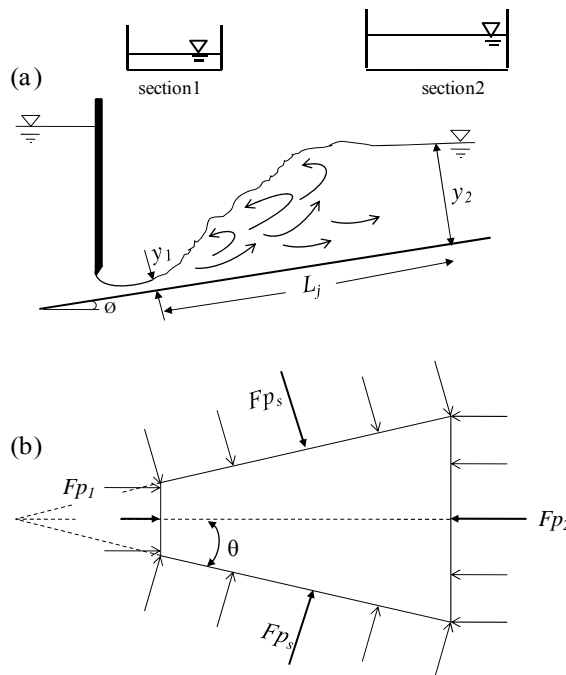


Fig. 1. Definition sketch of a hydraulic jump in expanding channel with adverse slope. (a) Hydraulic jump in longitudinal section. (b) Pressure forces acting on the surface of the control volume

The hydrostatic pressure forces are computed as follows:

$$F_{p1_x} = \int_0^{y_1} p dA = \int_0^{y_1} \gamma \cos \phi y b_1 dy = \frac{1}{2} \gamma y_1^2 b_1 \cos \phi \quad (3)$$

$$F_{p2_x} = \int_0^{y_2} p dA = \int_0^{y_2} \gamma \cos \phi y b_2 dy = \frac{1}{2} \gamma y_2^2 b_2 \cos \phi \quad (4)$$

In which γ is the specific weight of water, y is the depth of water perpendicular to bed, b is the bed width, ϕ is the bed slope angle with respect to the horizontal line and the subscripts 1 and 2 refer to the upstream and downstream sections of the jump in the x-direction, respectively.

The water surface profile is assumed to be parabolic with the horizontal axis being equal to the length of the jump, L_j and the vertical axis equal to the difference between the two sequent depths ($y_2 - y_1$), as shown in Fig. 2. The equation for the surface profile is:

$$\frac{y_x - y_1}{y_2 - y_1} = -\left(\frac{x}{L_j}\right)^2 + 2\left(\frac{x}{L_j}\right) \quad (5)$$

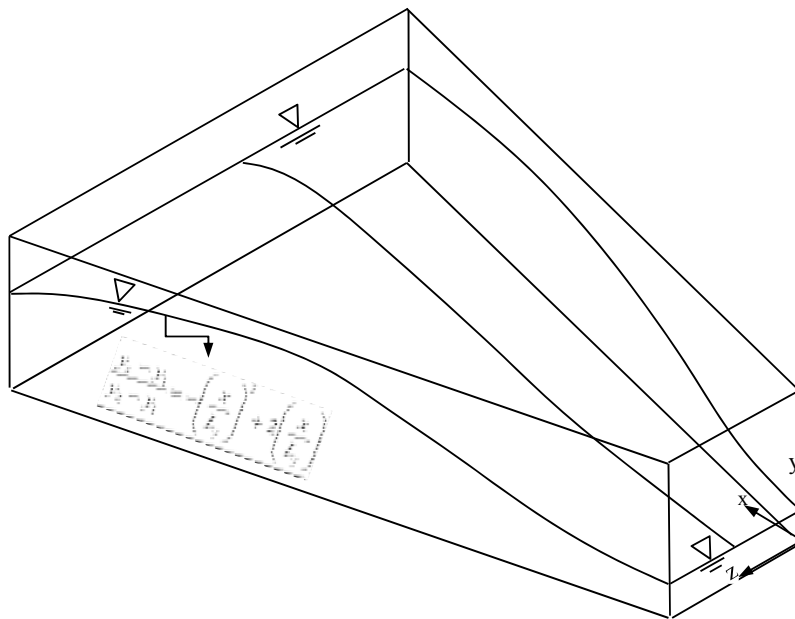


Fig. 2. Definition sketch of the parabolic jump profile

The weight force is derived by integrating over the control volume as follows:

$$F_{B_x} = \left[\gamma b_1 \int_0^{L_j} \int_0^y dy dx + 2\gamma \int_0^{L_j} \int_0^{(b_2-b_1)x/2L_j} \int_0^y dy dz dx \right] \sin \phi \quad (6)$$

$$F_{B_x} = \gamma L_j b_1 \left(\frac{1}{3} y_1 + \frac{2}{3} y_2 \right) \sin \phi + \gamma (b_2 - b_1) L_j \left(\frac{1}{12} y_1 + \frac{5}{12} y_2 \right) \sin \phi \quad (7)$$

The total force acting on the sides of the jump in the x-direction will be:

$$F_{ps_x} = 2 F_{ps} \sin \theta \quad (8)$$

$$F_{ps} = \gamma \int_0^{L_{js}} \frac{y^2}{2} dx = \frac{\gamma}{2} \int_0^{L_{js}} \left[\left(2 \left(\frac{x}{L_j} \right) - \left(\frac{x}{L_j} \right)^2 \right) (y_2 - y_1) + y_1 \right]^2 dx \quad (9)$$

$$F_{ps} = \gamma L_{js} \left(\frac{1}{2} y_1^2 + \frac{4}{15} (y_2 - y_1)^2 + \frac{2}{3} y_1 (y_2 - y_1) \right) = \gamma L_{js} \left(\frac{1}{5} y_1^2 + \frac{8}{15} y_2^2 - \frac{4}{15} y_1 y_2 \right) \quad (10)$$

In which, $L_{js} = L_j / \cos \theta$. Substituting Eqs. (3), (4), (7) and (10) into Eq. (1) yields:

$$F_{p1_x} - F_{p2_x} + F_{ps_x} - F_{B_x} = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right) \quad (11)$$

$$\begin{aligned} & \frac{1}{2} \gamma y_1^2 b_1 \cos \phi - \frac{1}{2} \gamma y_2^2 b_2 \cos \phi + 2 \gamma L_{js} \left(\frac{1}{5} y_1^2 + \frac{8}{15} y_2^2 - \frac{4}{15} y_1 y_2 \right) \sin \theta - \\ & \gamma L_j b_1 \left(\frac{1}{3} y_1 + \frac{2}{3} y_2 \right) \sin \phi - \gamma (b_2 - b_1) L_j \left(\frac{1}{12} y_1 + \frac{5}{12} y_2 \right) \sin \phi \\ & = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right) = \rho g F r_1^2 D_1 A_1 \cos \phi \left(\frac{A_1}{A_2} - 1 \right) = \rho g F r_1^2 y_1^2 b_1 \cos \phi \left(\frac{b_1 y_1}{b_2 y_2} - 1 \right) \end{aligned} \quad (12)$$

After some manipulation, the following equation is obtained:

$$Y^3 K_1 + Y^2 K_2 + Y K_3 - 2 F r_1^2 = 0 \quad (13)$$

$$\text{In which, } K_1 = \left(\frac{16 L_j \tan \theta}{15 b_1 \cos \phi} B - B^2 \right), \quad K_2 = \left(\frac{8 L_j \tan \theta}{15 b_1 \cos \phi} B - \frac{L_j \tan \phi}{y_1} \left(\frac{5}{6} B^2 + \frac{1}{2} B \right) \right),$$

$$K_3 = \left(B + \frac{2 L_j \tan \theta}{5 b_1 \cos \phi} B - \frac{L_j \tan \phi}{y_1} \left(\frac{1}{6} B^2 + \frac{1}{2} B \right) + 2 B F r_1^2 \right), \quad B = b_2 / b_1, \quad Y = y_2 / y_1.$$

Equation (13) is the general equation for the sequent depth ratio of a hydraulic jump in a diverging channel on an adverse slope.

The energy loss in the diverging hydraulic jump is calculated from the specific energy and continuity equations as follows:

$$\Delta E = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right) \quad (14)$$

$$\frac{\Delta E}{E_1} = \frac{E_1 - E_2}{E_1} = \frac{\left(y_1 + \frac{V_1^2}{2g} \right) - \left(y_2 + \frac{V_2^2}{2g} \right)}{\left(y_1 + \frac{V_1^2}{2g} \right)} \quad (15)$$

Where ΔE and $\frac{\Delta E}{E_1}$ are the energy loss and the relative loss of energy, respectively.

3. EXPERIMENTAL STUDY

The experimental setup is shown in Fig. 3. The model consisted of a stilling tank provided with a calibrated V-notch and a head tank ending at a sluice gate to obtain various flow depths and Froude

numbers. Experiments were conducted in a flume with Plexiglas bed and walls, which were 1 m wide, 1 m deep and 5.9 m long. The stilling basin had a length of 1.4 m. The side walls and bed were designed in such a way that a range of bed slopes and angles of divergence of the walls were achieved.

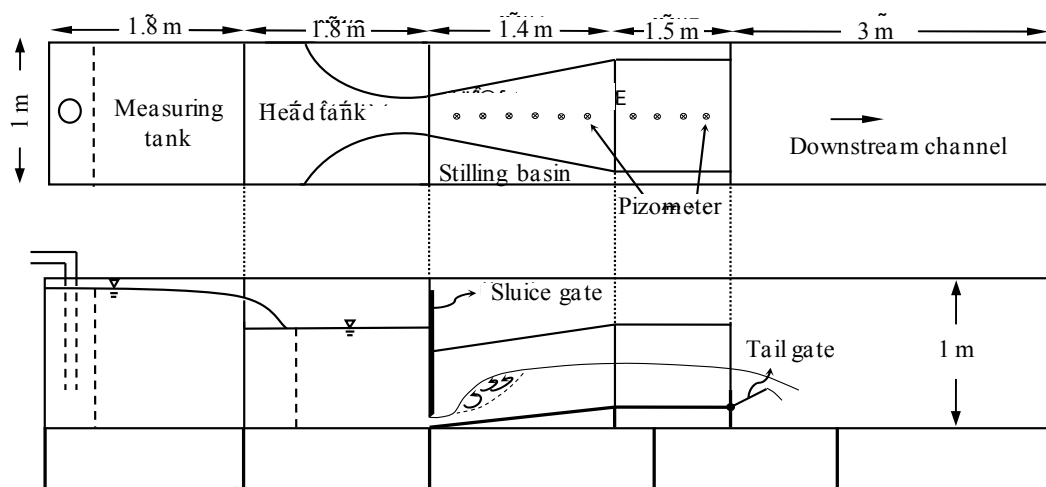


Fig. 3. Plan and side view of the experimental flume (not to scale)

A typical experiment started by setting the angle of divergence of the side walls and slope of the bed. Then the discharge was adjusted in a way that head of water in the head tank would be able to produce the desired approach Froude number. The supercritical jet was allowed to pass over the sloping apron. The tail water depth was adjusted by a manually controlled plane gate to maintain the sequent depth required to form the jump. The tail water was controlled so that the start of the jump was at least one to three times the approach depths downstream of the beginning of the adverse slope. As the jump stabilized, the depth of water along the jump profile was measured using both piezometers installed in the center plane of the basin and a point gauge at several points along the jump profile with a reading accuracy of ± 0.1 mm.

Altogether, 158 series of experiments were conducted for a wide range of Froude numbers, angles of divergence and slopes of bed. The experimental conditions are tabulated in Table 1.

Table 1. Experimental conditions

Parameter	Unit	Minimum	Maximum
y_1	cm	1.8	2.2
y_2	cm	4.4	21
Fr_1	Dimensionless	2.65	8.75
L_j	cm	11	121
Q	L/s	5.21	21.1
$S = \tan(\phi)$	Dimensionless	0 (Horizontal)	0.08
θ	degree	0	10

4. EXPERIMENTAL RESULTS

a) Stability of the hydraulic jump

The initial positioning of the diverging hydraulic jump on the adverse slope was achieved by continuous regulation of the tail gate at a low Froude number (less than 3.5). Experimental observations showed that, at each adverse slope, stability of the jump increased as divergence angle increased. However, at lower

Froude numbers, especially as the magnitude of the adverse bed slope increased, continuous adjustment was required even after the initial stabilization was obtained.

b) Hydraulic jump profile

In order to evaluate the theoretical equation for the jump profile, the measured water surface profiles are shown in Fig. 4 in terms of the depth y_x along the jump in dimensionless form as $(y_x - y_1) / (y_2 - y_1)$ versus x/L_j for all the experiments. In this figure, the parabolic jump profile given by Eq. (5) is also shown for comparison. The elliptical profile of Omid et.al (2007) [6], and the linear profile of Kouluseus and Ahmad (1961), [2] are also plotted. As seen in Fig. 4, the parabolic equation agrees reasonably well with the experimental water surface profiles of the jump in the diverging hydraulic jump on the adverse slope.

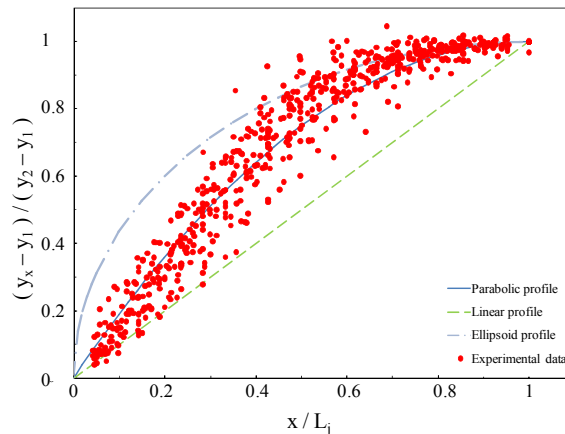


Fig. 4. Assumed and experimental dimensionless water-surface profiles for different divergence angles, adverse slopes, and approach Froude numbers

The validity of Eqs. (13) and (15) for the sequent depth ratio and the relative energy loss are assessed in terms of the ratio of the calculated to observed values of the sequent depth and the relative energy loss, respectively. Using subscripts “Exp” and “The” to describe the experimental and theoretical results, respectively, the ratios (y_{2Exp}/y_{2The}) and $((\frac{\Delta E}{E_1})_{Exp} / (\frac{\Delta E}{E_1})_{The})$ are plotted in Figs. 5 and 6 as functions of the approach Froude number for all of the experiments. Also, using the data series, the value of root mean square error (RMSE) was calculated. Figs. 5 and 6 show reasonable agreement between the calculated and the experimental results.

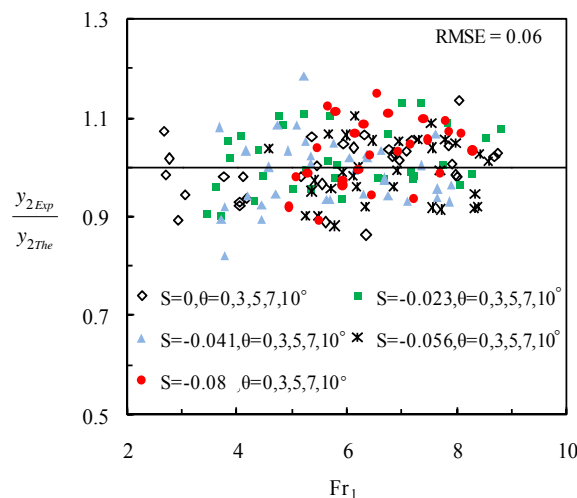


Fig. 5. Ratio of experimental and theoretical sequent depths as a function of the approach Froude number for different divergence angle (θ) and adverse bed slope (S)

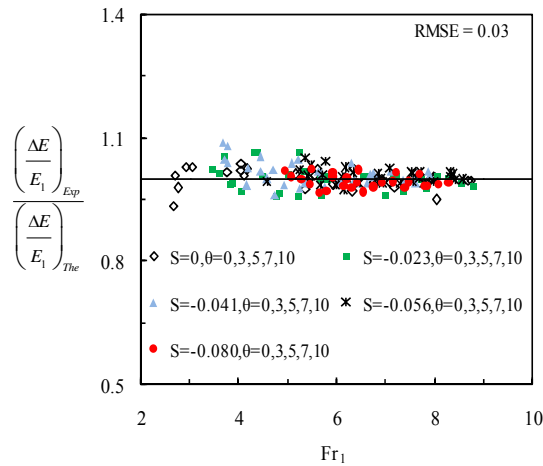


Fig. 6. Ratio of experimental and theoretical relative energy loss as a function of approach Froude number for different divergence angle (θ) and adverse bed slope (S)

c) Effect of the divergence angle and the adverse bed slope on the hydraulic jump profile

Figures 7a, 7b and 7c show, respectively, the experimental results of the sequent depth ratio, y_2/y_1 , relative length of the jump, L_j/y_1 , and the relative energy loss, R_L , as functions of the approach-Froude number, Fr_1 , for a divergence angle of 5° with different adverse slopes. Comparison of results shows that, for each divergence angle, increase in the bed slope results in reduction of the sequent depth and jump length and increase in relative energy loss compared to the classic jump.

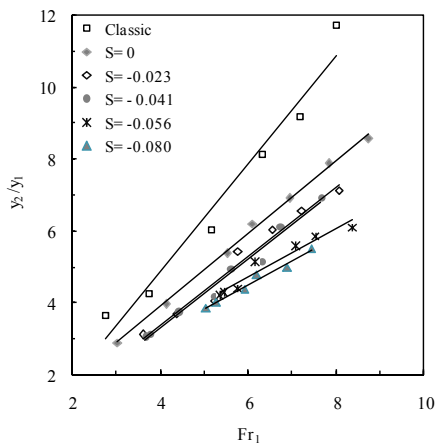


Fig. 7a. Sequent depth ratio versus approach Froude number for a divergence angle of 5°

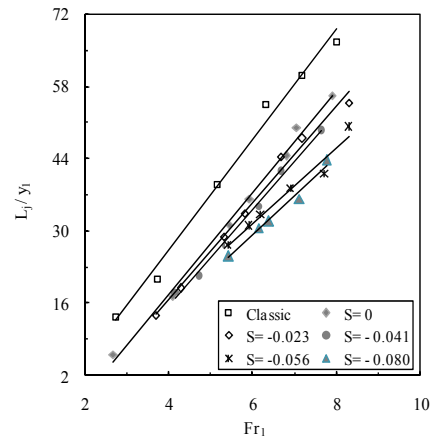


Fig. 7b. Relative length of the jump versus approach Froude number for a divergence angle of 5°

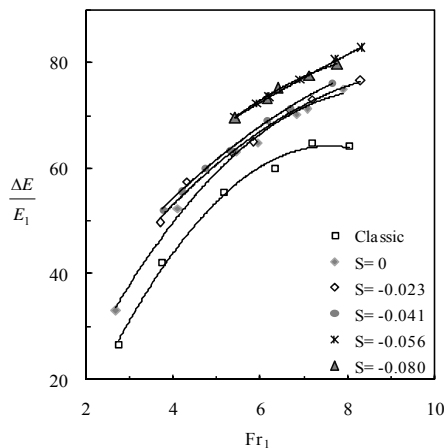


Fig. 7c. Relative energy loss versus approach Froude number for a divergence angle of 5°

Increase in the bed slope caused an increase in the contribution of the weight of the jump in the body force acting on the volume of the jump. Therefore, the required sequent depth for stabilization of the jump is decreased. As a result, the jump profile becomes shorter and the length of the jump reduces.

Figures 8a, 8b and 8c show, respectively, the experimental results of the sequent depth ratio, y_2/y_1 , relative length of the jump, L_j/y_1 , and the relative energy loss, $(\frac{\Delta E}{E_1})$, as functions of the approach Froude number, Fr_1 , for an adverse slope of 0.041 and all divergence angles. Comparison of the results showed that, for each adverse slope, increase in the divergence angle caused the reduction of the sequent depth and the jump length, and increased the relative energy loss in comparison with the classic jump.

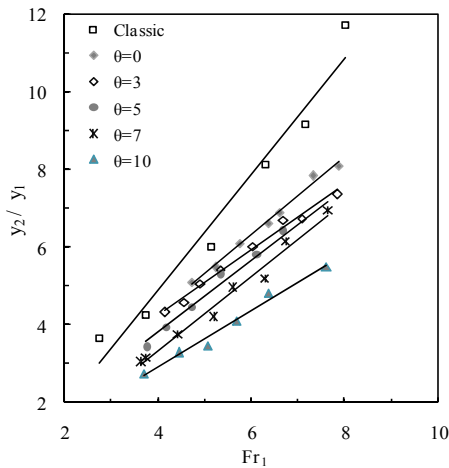


Fig. 8a. Sequent depth ratio versus approach Froude number for an adverse slope 0.041

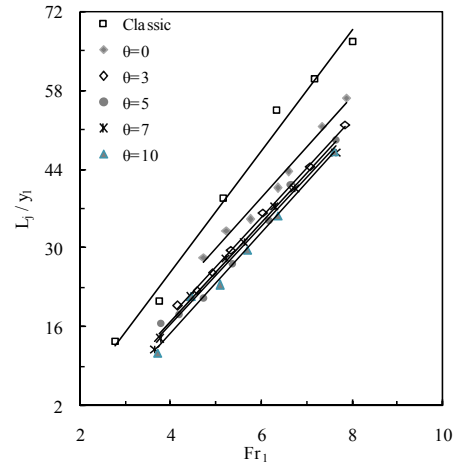


Fig. 8b. Relative length of the jump versus approach Froude number for an adverse slope 0.041

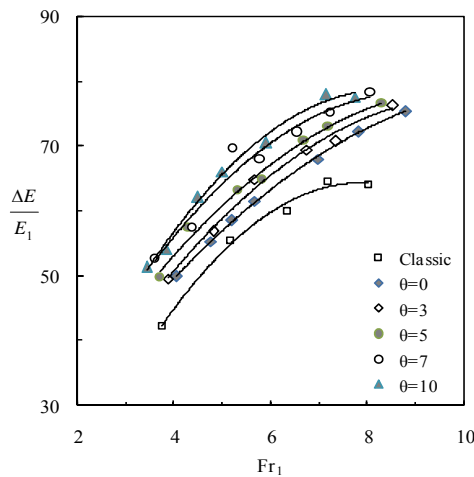


Fig. 8c. Relative energy loss versus approach Froude number for an adverse slope 0.041

5. DISCUSSION

One of the specific features of hydraulic jump on adverse slope is the tendency to submerge. Experimental observations have shown that formation of the free hydraulic jump could be improved by diverging the side walls of basin. Comparison of the inception Froude number, from which the hydraulic jump will form free afterwards, showed that at each adverse slope, the Froude number of inception decreases as diverging angle increases. The main cause of this behavior for formation of hydraulic jump is reduction of body force of the hydraulic jump. In Fig. 9 effect of both the angle of divergence and the adverse slope on stability of the hydraulic jump is shown.

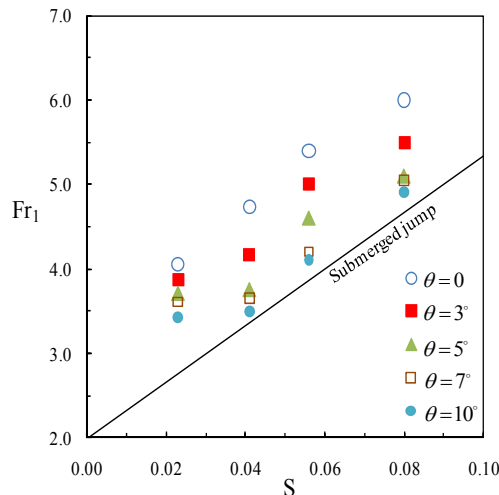


Fig. 9. Variation of the inception Froude number of submergence versus an adverse slope

It was found in this investigation that both the sequent depth and the relative length of jump in gradually diverging stilling basin on adverse slope are correspondingly less and the relative energy loss is more than those of the classic and also diverging hydraulic jump on horizontal bed. Hence such a structure could be more economical in dissipating the energy of the flow in comparison to the classical stilling basin. For the design of a gradually diverging stilling basin on adverse slope, the parameters y_1 / y_2 and L_j / y_1 are required to be known. However, with the help of the implicit relationship between these parameters of gradually diverging hydraulic jump, the regression model to predict the sequent depth and the relative length of the jump may be easy to design.

Important parameters to predict the sequent depth and the length of jump can be expressed as the following functional relationship:

$$f_1(Q, b_1, b_2, S, \theta, y_1, y_2, L_j, g) = 0 \tag{16}$$

In which, b_1 and b_2 are the widths of stilling basin upstream and downstream of jump, respectively. Using the Buckingham II theorem, the functional relationship (16) can be expressed in dimensionless form as:

$$f_2(y_2 / y_1, L_j / y_1, Fr_1, \theta, S) = 0 \tag{17}$$

Equation (17) may be used as the primary relation to derive the statistical models.

Because of the interrelation between the sequent depth and the relative lengths of the gradually diverging hydraulic jump on adverse slope, regression models were derived using the SAS Model in a manner which involved both parameters. Hence there can be two models with two unknown parameters that can be solved for obtaining the desired output.

$$\frac{y_2}{y_1} = 0.309(S+1)^{3.4542} (\theta+1)^{-0.1197} (Fr_1^{0.5175})(L_j/y_1)^{0.5444} + 1.4396 \tag{18}$$

$$\frac{L_j}{y_1} = 2.9607(S+1)^{3.7214} (\theta+1)^{-0.0774} (Fr_1^{1.4997}) \tag{19}$$

A comparison of models and corresponding experimental data is shown in Figs. 10a and b. As may be observed from these figures, the predictions by the models are quite satisfactory.

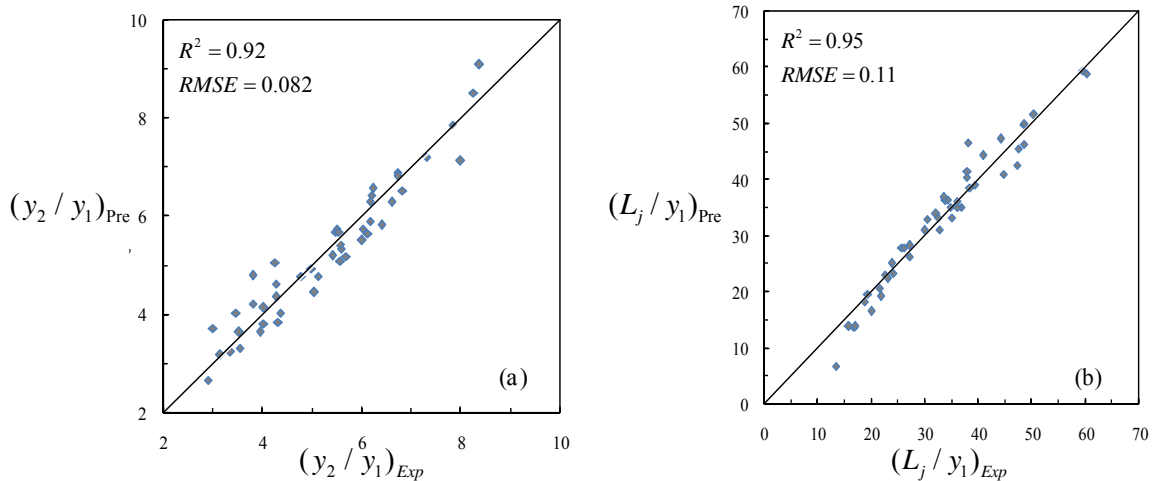


Fig. 10. Evaluation of regression model to predict hydraulic jump parameters:
a) sequent depth ratio and b) relative jump length

6. CONCLUSION

To reduce the construction costs of a stilling basin, a change in the plan and profile sections of the basin can be useful. Extensive measurements were conducted to study the effects of adverse slope and angle of divergence on the hydraulic jump in a gradually diverging channel. Experimental observations showed that, at low Froude numbers, the free hydraulic jump on an adverse slope is unstable. However, it was possible to maintain a relatively stationary limiting jump for Froude numbers between 4 and 9. The semi-theoretical momentum-based equation was derived to estimate the sequent depth of a diverging hydraulic jump on an adverse bed slope based on the parabolic equation which was used to calculate hydraulic jump profile. The results show a fairly good agreement between the experimental and theoretical values of the sequent depth.

Comparison of results showed that, for each diverging angle, an increase in the bed slope up to 8% caused the sequent depth and the relative length of jump to reduce 47% and 35%, respectively and the relative loss of energy increased 20% in comparison with the classic hydraulic jump. For each adverse bed slope, increasing the divergence angle to 10 degrees caused the sequent depth and the relative length of the jump to reduce 51% and 38%, respectively, and the relative loss of energy increased 23% in comparison with the hydraulic classic jump.

REFERENCES

1. Peterka, A. J. (1983). Hydraulic design of stilling basin and energy dissipaters. *Eng. Monograph No. 25, U.S. Bureau of Reclamation.*
2. Kouluseus, H. J. & Ahmad. D. (1961). Circular hydraulic jump. *J. Hydraulic. Eng.*, Vol. 95, No. 1, pp. 409-422.
3. Arabhabhirama, A. & Abela. A. (1971). Hydraulic jump within gradually expanding channel. *J. Hydraul. Eng.*, Vol. 97, No. 1, pp. 31-42.
4. Khlifia, A. M. & McCorquodale, A. M. (1979). Radial hydraulic jump. *J. Hydraulic. Eng.*, Vol. 105, No. 9, pp. 1065-1078.
5. Lawson, J. D. & Phillips, B. C. (1983). Circular hydraulic jump. *J. Hydraulic. Eng.*, Vol. 109, No. 4, pp. 505-518.
6. Omid, M. H., Esmaeili Varaki M. & Narayanan, R. (2007). Gradually expanding hydraulic jump in a trapezoidal channel. *J. Hydraulic. Res.*, Vol. 4, pp. 512-518.

7. McCorquodale, J. A. & Mohamed, M. S. (1994). Hydraulic jump on adverse slopes. *J. Hydraulic. Res.*, Vol. 31, No. 1, pp. 119–130.
8. Peruginelli, A. & Pagliara, S. (2000). Limiting and sill-controlled adverse-slope hydraulic jump. *J. Hydraulic. Eng.*, Vol. 126, No. 11, pp. 847-851.
9. Abrishami, J. & Saneie, M. (1991). Hydraulic Jump in adverse basin slopes. *M.Sc Thesis, University of Tehran, Iran.*
10. Beirami, M. K. & Chamani, M. R. (2006). Hydraulic jumps in sloping channels: sequent depth ratio. *Journal of Hydraulic Engineering*, Vol. 132, No. 10, pp. 1061-1068.
11. Beirami, M. K. & Chamani, M. R. (2010). Hydraulic jumps in sloping channels: roller length and energy loss. *Can. J. Civ. Eng.*, Vol. 37, No. 9, pp. 535-543.