COST OPTIMIZATION OF CASTELLATED BEAMS USING CHARGED SYSTEM SEARCH ALGORITHM*

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Abstract—Castellated beam is formed by modifying a standard rolled beam through creating a regular pattern of holes in the web. The main goal of manufacturing these beams is to increase the moment of inertia and section modulus, which results in greater strength and rigidity. In this study, the charged system search algorithm is used for obtaining the solution of the design problem. Here, castellated beams with hexagonal and circular openings are considered as design problems. The minimum cost is taken as the design objective function. The design methods used in this study are consistent with BS5950 part 1 and 3 and Euro code 3. A number of design examples are considered to demonstrate the efficiency of the presented algorithm. It is observed that optimization results obtained by the ECSS algorithm for three castellated beams with hexagonal openings have less cost in comparison to the cellular beams. Also, the results of these examples illustrate the capability of the ECSS algorithm in finding the optimum solution in less number of iterations.

Keywords—Castellated beams, cellular beams, steel structures, design optimization, charged system search (CSS)

1. INTRODUCTION

In design of steel structures, beams with web-openings are widely used to pass the under floor services ducts such as water pipes and air ducts [1]. Castellated beams are varieties of girders that are manufactured by cutting a hot rolled beam lengthwise using computer control plasma arc torches, often in half-circle or half-hexagon patterns. The split halves are then offset and welded back together to form a deeper beam with full circular or hexagonal shaped web openings. The resulting castellated beam is approximately 50% deeper and much stronger than the original hot rolled beam [1-3]. Castellated beams can be used in a great variety of applications. The most common applications for castellated beams are for long-span floor system [4]. A cellular beam is the modern version of the traditional castellated beam, with a far wider range of applications for floor beams. Cellular beams are steel sections with circular openings that are made by cutting a rolled beam web in a half circular pattern along its centerline and re–welding the two halves of hot rolled steel sections as shown in Fig. 1. This opening increases the overall beam depth; moment of inertia and section modulus, without increasing the overall weight of the beam [5, 6].

In the first part of this paper, the design of castellated beam is introduced. In the second part, optimum designs of these beams are formulated based on The Steel Construction Institute Publication Number 100 and Euro code 3. In the third part, the CSS algorithm is introduced, and finally in the last part, the cost of castellated beam as the design objective function is minimized.

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2. THE DESIGN OF CASTELLATED BEAMS

Beams must be sufficiently strong to carry the bending moments and shear forces of the applied loads. The performance of any beam depends upon the cross-section geometry, the physical dimensions, and the shape. At present, there is not a generally accepted design method due to the complexity of the behavior of castellated beams and their associated modes of failure [1]. The strength of a beam with various web opening is determined by considering the interaction of the flexure and shear at the openings [9]. There are many failure modes to be considered in the design of a beam with web opening including Vierendeel mechanism, flexural mechanism, lateral-torsional buckling, rupture of welded joints and web post buckling. In the design of castellated beams, these criteria should be considered [7-13]:

a) **Overall beam flexural capacity**

The maximum moment under applied external loading $M_U$ should not exceed the plastic moment capacity $M_P$ of the castellated beam.
where $A_{LT}$ is the area of lower tee, $P_y$ is the design strength of steel and $H_U$ is distance between center of gravities of the upper tee and lower tee.

**b) Beam shear capacity**

In the design of castellated beams, it is necessary to control two modes of shear failure. The first one is the vertical shear capacity and the upper and lower tees should undergo that. Sum of the shear capacity of the upper and lower tees are checked using Eq. (2).

$$P_{YY} = 0.6P_y (0.9A_{WUL}) \quad \text{For circular opening}$$

$$P_{YY} = \frac{\sqrt{3}}{3} P_y (A_{WUL}) \quad \text{For hexagonal opening}$$

The second one is the horizontal shear capacity. It is developed in the web post due to change in axial forces in the tee-section as shown in Fig. 2. Web post with too short mid-depth welded joints may fail prematurely when horizontal shear exceed the yield strength. The horizontal shear capacity is checked using Eq. (3).

$$P_{YH} = 0.6P_y (0.9A_{WP}) \quad \text{For circular opening}$$

$$P_{YH} = \frac{\sqrt{3}}{3} P_y (A_{WP}) \quad \text{For hexagonal opening}$$

where $A_{WUL}$ is the total area of webs of tees and $A_{WP}$ is the minimum area of web post.

**Fig. 2. Horizontal shear in the web post of castellated beams, (a) hexagonal opening, (b) cellular beam**
c) Flexural and buckling strength of web post

It is assumed the compression flange of the castellated beam is restrained by the floor system. Thus, the overall buckling strength of the castellated beam is omitted from the design consideration. The web post flexural and buckling of the capacity of the castellated beam is given by:

$$M_{\text{MAX}} = [C_1\alpha - C_2\alpha^2 - C_3]$$  \(\text{(4)}\)

where $M_{\text{MAX}}$ is the maximum allowable web post moment and $M_E$ is the web post capacity at critical section A-A shown in Fig. 2. $C_1$, $C_2$ and $C_3$ are constants obtained by the following expressions:

$$C_1 = 5.097 + 0.1464(\beta) - 0.00174(\beta)^2$$  \(\text{(5)}\)

$$C_2 = 1.441 + 0.0625(\beta) - 0.000683(\beta)^2$$  \(\text{(6)}\)

$$C_3 = 3.645 + 0.0853(\beta) - 0.00108(\beta)^2$$  \(\text{(7)}\)

where $\alpha = \frac{S}{d}$ for the hexagonal openings, and $\alpha = \frac{S}{D_0}$ for the circular openings, also $\beta = \frac{2d}{t_w}$ for the hexagonal openings, and $\beta = \frac{D_0}{t_w}$ for the circular openings, $S$ is the spacing between the centers of holes, $d$ is the cutting depth of hexagonal opening, $D_0$ is the holes diameter, and $t_w$ is the web thickness.

d) Vierendeel bending of upper and lower tees

This mode of failure is associated with high shear forces acting on the beam and it is critical. The Vierendeel bending stresses in the circular opening are obtained by using the Olander’s approach, as shown in Fig. 3. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as follows:

$$\frac{P_0}{P_U} + \frac{M}{M_P} \leq 1.0$$  \(\text{(8)}\)

$$P_0 = T\cos\theta - \frac{V}{2}\sin\theta$$  \(\text{(9)}\)

$$M = T(x_{50} - x_0) + \frac{V}{2}\left(\frac{H_2}{2} - x_{50}\right)$$  \(\text{(10)}\)

where $P_0$ and $M$ are the force and the bending moment on the section, respectively. $P_U$ is equal to the area of the critical section $\times P_f$, $M_P$ is calculated as the plastic modulus of critical section $\times P_f$ in plastic section or elastic section modulus of critical section $\times P_f$ for other sections.

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Fig. 3. Olander's curved beam approach
The plastic moment capacity of the tee-sections in castellated beams with hexagonal opening are calculated independently. The total plastic moment is equal to the sum of the Vierendeel resistances of the top and bottom tee-sections. The interaction between Vierendeel moment and shear forces should be checked by the following expression:

\[ V_{OMAX} e - 4 M_{TP} \leq 0 \]  

where \( V_{OMAX} \) and \( M_{TP} \) are the maximum shear force and the moment capacity of tee-section, respectively.

e) Deflection of castellated beam

Serviceability checks are of high importance in the design, especially in beams with web opening where the deflection due to shear forces is significant. The deflection of castellated beam under applied load combinations should not exceed span/360. In castellated beams with circular opening, the deflection at each point is calculated by the following expressions [9, 10]:

Deflection due to the bending moment in tee is

\[ Y_{MT} = \frac{0.091 \left( \frac{D_o}{2} \right)^3}{3 E I_T} (V_i V_i) \]  

Deflection due to the bending moment in web post of beam is

\[ Y_{WP} = \frac{13.15}{E I_W} \left[ \log \left( \frac{S - 0.90 \left( \frac{D_o}{2} \right)}{S - 2.0 \left( \frac{D_o}{2} \right)} \right) + 2 \left( \frac{S - 2.0 \left( \frac{D_o}{2} \right)}{S - 0.90 \left( \frac{D_o}{2} \right)} \right) - 0.50 \left( \frac{S - 2.0 \left( \frac{D_o}{2} \right)}{S - 0.90 \left( \frac{D_o}{2} \right)} \right)^2 \right] \frac{V_i V_i}{V_H} \]  

Deflection due to the axial force in tee is

\[ Y_{AF} = \frac{2 S}{E A_T} (T_i T_i) \]  

Deflection due to the shear in tee is

\[ Y_{ST} = \frac{0.45 \left( \frac{D_o}{2} \right)}{G A_{PWEB}} (V_i V_i) \]  

Deflection due to the shear in web post is

\[ Y_{SWP} = \frac{1.63}{G t_W} X \log \left( \frac{S - 0.90 \left( \frac{D_o}{2} \right)}{S - 2.0 \left( \frac{D_o}{2} \right)} \right) \frac{V_H V_H}{V_H} \]  

where \( E \) is the elasticity modulus of steel, \( I_T \) is the moment of inertia of tee-section, \( G \) is the shear modules of steel and \( X \) is the web post form factor. The total deflection of a single opening under applied load is obtained by summing the deflections given by Eq. (17).
For a castellated beam with hexagonal opening and length $L$ subjected to transverse loading, the total deflection is composed by two terms: the first term corresponding to pure moment action $f_b$, and the second one corresponding to shear action $f_s$ [11]. Thus, the total deflection can be calculated by the following expression:

$$f = f_b + f_s$$

(18)

$f_b$ and $f_s$ are illustrated in Ref. [11].

### 3. OPTIMUM DESIGN OF CASTELLATED BEAMS

The main initiative for producing and using castellated beam is to suppress the cost of material by applying more efficient cross sectional shapes made from standard profiles in combination with aesthetic and architectural design considerations. Also, the web holes can be utilized for cross passing utility systems in building floors. There are many factors that should be considered when estimating the cost of castellated steel beams, such as man-hours of fabrication, weight, price of web cutting and welding process. At this study, it is assumed that the cost associated with man-hours of fabrication for hexagonal and circular opening is the same. Therefore, the objective function includes three parts: The beam weight, price of cutting and price of the welding. The formulation of the objective function is shown in Eq. (19).

$$F_{\text{cost}} = \rho A_{\text{initial}} (L + \frac{S}{2})P_1 + L_{\text{cut}} P_2 + L_{\text{weld}} P_3$$

(19)

where $P_1$, $P_2$, and $P_3$ are the price of the weight of the beam per unit weight, length of cutting and welding for per unit length, $\rho$ is the density of steel, $A_{\text{initial}}$ is the area of the selected universal beam section, $L_{\text{cut}}$ and $L_{\text{weld}}$ are the cutting and welding length, respectively.

**a) Design of castellated beam with circular opening**

The design of a cellular beam consists of three parts: The selection of a rolled beam, the selection of a suitable circular hole diameter, and the spacing between the center of holes or the total number of the holes in the beam as shown in Fig. 1 [9, 10]. Hence the sequence number of the rolled beam section in the standard steel sections tables, the circular holes diameter and the total number of holes are taken as design variables in the optimum design problem. The optimum design problem formulated by considering the constraints explained in the previous sections as shown in the following expression:

Find an integer design vector $\{X\} = \{x_1, x_2, x_3\}^T$ where $x_1$ is the sequence number of the rolled steel profile in the standard steel section list, $x_2$ is the sequence number for the hole diameter which contains various diameter values, and $x_3$ is the total number of holes for the cellular beam [9]. Hence the design problem turns out to be as follows:

Minimize

$$F_{\text{cost}} = \rho A_{\text{initial}} (L + \frac{S}{2})P_1 + L_{\text{cut}} P_2 + L_{\text{weld}} P_3$$

(20)

Subject to

$$g_1 = 1.08 \times D_0 - S \leq 0$$

(21)
where $D_0$ is the hole diameter, $NH$ is the number of holes, $t_{w_f}$ is the web thickness, $H_S$ and $L$ are the overall depth and the span of the cellular beam and $S$ is the distance between centers of holes. $M_U$ is the maximum moment under the applied loading, $M_P$ is the plastic moment capacity of the cellular beam, $V_{MAXSAPV}$ is the maximum shear at the support, $V_{OMAX}$ is the maximum shear at the opening, $H_{MAXV}$ is the maximum horizontal shear, $A_{MAXA}$ is the maximum moment at A-A section shown in Fig. 2, $W_{MAXM}$ is the maximum allowable web post moment, $V_{TEE}$ represent the vertical shear on the tee at $\theta = 0$ of web opening, $P_0$ and $M$ are the internal forces on the web section as shown in Fig. 3, and $Y_{MAX}$ denotes the maximum deflection of the cellular beam [9,13].

\[ g_2 = S - 1.60 \times D_0 \leq 0 \]  
\[ g_3 = 1.25 \times D_0 - H_S \leq 0 \]  
\[ g_4 = H_S - 1.75 \times D_0 \leq 0 \]  
\[ g_5 = M_U - M_P \leq 0 \]  
\[ g_6 = V_{MAXSAPV} - P_{Y} \leq 0 \]  
\[ g_7 = V_{OMAX} - P_{YY} \leq 0 \]  
\[ g_8 = V_{HMAX} - P_{YH} \leq 0 \]  
\[ g_9 = M_{A-MAXA} - M_{WMAX} \leq 0 \]  
\[ g_{10} = V_{TEE} - 0.50 \times P_{YY} \leq 0 \]  
\[ g_{11} = \frac{P_0}{P_U} + \frac{M}{M_P} - 1.0 \leq 0 \]  
\[ g_{12} = Y_{MAX} - \frac{L}{360} \leq 0 \]  

**b) Design of castellated beam with hexagonal opening**

In design of castellated beams with hexagonal openings, the design vector includes four design variables: The selection of a rolled beam, selection of a cutting depth, the spacing between the center of holes or total number of holes in the beam and the cutting angle as shown in Fig. 1. Hence the optimum design problem formulated by the following expression:

Find an integer design vector $\{x\} = \{x_1, x_2, x_3, x_4\}^T$, where $x_1$ is the sequence number of the rolled steel profile in the standard steel section list, $x_2$ is the sequence number for the cutting depth which contains various values, $x_3$ is the total number of holes for the castellated beam and $x_4$ is the cutting angle. Therefore, the design problem turns out to be as follows:

Minimize

\[ F_{cost} = \rho A_{\text{initial}} \left( L + \frac{S}{2} \right) P_1 + L \cdot P_2 + L \cdot P_3 \]  

\[ (33) \]
Subject to

\[ g_1 = d - \frac{3}{8} (H_S - 2t_f) \leq 0 \quad (34) \]
\[ g_2 = (H_S - 2t_f) - 10 \times (d_T - t_f) \leq 0 \quad (35) \]
\[ g_3 = \frac{2}{3} d \cot \phi - e \leq 0 \quad (36) \]
\[ g_4 = e - 2d \cot \phi \leq 0 \quad (37) \]
\[ g_5 = 2d \cot \phi + e - 2d \leq 0 \quad (38) \]
\[ g_6 = 45^\circ - \phi \leq 0 \quad (39) \]
\[ g_7 = \phi - 64^\circ \leq 0 \quad (40) \]
\[ g_8 = M_U - M_P \leq 0 \quad (41) \]
\[ g_9 = V_{\text{MAXSUP}} - P_V \leq 0 \quad (42) \]
\[ g_{10} = V_{\text{OMAX}} - P_{YY} \leq 0 \quad (43) \]
\[ g_{11} = V_{\text{HMAX}} - P_{VH} \leq 0 \quad (44) \]
\[ g_{12} = M_{A-\text{AMAX}} - M_{WMAX} \leq 0 \quad (45) \]
\[ g_{13} = V_{\text{TEE}} - 0.50 \times P_{YY} \leq 0 \quad (46) \]
\[ g_{14} = V_{\text{OMAX}} \cdot e - 4M_{TP} \leq 0 \quad (47) \]
\[ g_{15} = Y_{\text{MAX}} - \frac{L}{360} \leq 0 \quad (48) \]

where \( d \) is the cutting depth, \( t_f \) is the flange thickness, \( d_T \) is the depth of the tee-section, \( \phi \) is the cutting angle in hexagonal opening, \( e \) is the length of horizontal cutting of web, \( M_P \) is the plastic moment capacity of the castellated beam, \( M_{A-\text{AMAX}} \) is the maximum moment at A-A section shown in Fig. 2, \( M_{WMAX} \) is the maximum allowable web post moment, \( V_{\text{TEE}} \) represent the vertical shear on the tee, \( M_{TP} \) is the moment capacity of tee-section, and \( Y_{\text{MAX}} \) denotes the maximum deflection of the castellated beam with hexagonal opening [1,7].

4. THE CHARGED SYSTEM SEARCH ALGORITHM

a) The standard CSS

The CSS algorithm is developed by Kaveh and Talatahari [14] and has been successfully utilized in many optimization problems. Charged System Search is a population-based search method, where each agent (CP) is considered as a charged sphere with radius \( a \), having a uniform volume charge density which
can produce an electric force on the other CPs. The force magnitude for a CP located in the inside of the sphere is proportional to the separation distance between the CPs, while for a CP located outside the sphere it is inversely proportional to the square of the separation distance between the particles. The resultant forces or acceleration and the motion laws determine the new location of the CPs. The pseudo-code for the CSS algorithm can be summarized as follows:

**Step 1: Initialization.** The initial positions of CPs are determined randomly in the search space and the initial velocities of charged particles are assumed to be zero. The values of the fitness function for the CPs are determined and the CPs are sorted in an increasing order. A number of the first CPs and their related values of the fitness function are saved in a memory, so called charged memory (CM).

**Step 2: Determination of the forces on CPs.** The force vector is calculated for each CP as

$$F_j = \sum_{i \neq j} \left( \frac{q_i}{a^2} r_{i,j} + \frac{q_i}{a^2} r_{i,j} \right) a r_{i,j} P_{i,j} (X_i - X_j)$$

where $F_j$ is the resultant force acting on the $j$th CP; $N$ is the number of CPs. The magnitude of charge for each CP ($q_i$) is defined considering the quality of its solution as

$$q_i = \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}}$$

where $\text{fitbest}$ and $\text{fitworst}$ are the best and the worst fitness of all particles, respectively; $\text{fit}(i)$ represents the fitness of the agent $i$; and $N$ is the total number of CPs. The separation distance $r_{i,j}$ between two charged particles is defined as follows:

$$r_{i,j} = \frac{\|X_i - X_j\|}{\|X_i - X_j\|/2 - X_{best}}$$

where $X_i$ and $X_j$ are respectively the positions of the $i$th and $j$th CPs, $X_{best}$ is the position of the best current CP, and $\epsilon$ is a small positive number. Here, $P_{i,j}$ is the probability of moving each CP towards the others and is obtained using the following function:

$$P_{i,j} = \begin{cases} 1 & \text{if } \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}} > \text{rand} \land \text{fit}(j) < \text{fit}(i) \\ 0 & \text{else} \end{cases}$$

In Eq. (30), $ar_{i,j}$ indicates the kind of force and is defined as

$$ar_{i,j} = \begin{cases} 1 & \text{rand} > 0.80 \\ 0 & \text{else} \end{cases}$$

where $\text{rand}$ represents a random number.

**Step 3: Solution construction.** Each CP moves to the new position and the new velocity is calculated

$$X_{j,\text{new}} = \text{rand}_{j,3} K_a F_j + \text{rand}_{j,2} K_v V_{j,\text{old}} + X_{j,\text{old}}$$

$$V_{j,\text{new}} = X_{j,\text{new}} - X_{j,\text{old}}$$
where $K_a$ is the acceleration coefficient; $K_v$ is the velocity coefficient to control the influence of the previous velocity; $rand_{1,i}$ and $rand_{2,i}$ are two random numbers uniformly distributed in the range, $(0, 1)$; $K_a$ and $K_v$ are taken as

$$
K_a = 0.5 \times (1 + \frac{iter}{iter_{max}}),\quad K_v = 0.5 \times \left(1 - \frac{iter}{iter_{max}}\right)
$$

(56)

where $iter$ is the iteration number and $iter_{max}$ is the maximum number of iterations.

**Step 4:** Updating process. If a new CP exits from the allowable search space, a harmony search-based handling approach is used to correct its position. In addition, if some new CP vectors are better than the worst ones in the CM, these are replaced by the worst ones in the CM.

**Step 5:** Termination criterion control. Steps 2-4 are repeated until a termination criterion is satisfied.

b) An enhanced CSS

In the standard CSS algorithm, when the calculations of the amount of forces are completed for all CPs, the new locations of agents are determined. Also, CM updating is fulfilled after moving all CPs to their new locations. All these conform to discrete time concept. In the optimization problems, this is known as an iteration. On the contrary, in the enhanced CSS, time changes continuously and after creating just one solution, all updating processes are performed. Using this enhanced CSS, the new position of each agent can affect the moving process of the subsequent CPs while in the standard CSS unless an iteration is completed, the new positions are not utilized. All other aspects of the enhanced CSS are similar to the original one [14-16].

5. DESIGN EXAMPLES

In order to compare the fabrication cost of the castellated beams with circular and hexagonal holes, three examples are selected. Among the steel section list of British Standards, 64 Universal Beam (UB) sections starting from $254 \times 102 \times 28UB$ to $914 \times 419 \times 388UB$ are selected to constitute the discrete set for steel sections from which the algorithm selects the sectional designations for the castellated beams. In the design pool of holes diameters 421 values are arranged which vary between 180 and 600 mm with an increment of 1 mm. Also, for cutting the depth of hexagonal opening, 351 values are considered which vary between 50 and 400 mm with increment of 1 mm and cutting angle changes from 45 to 64. Another discrete set is arranged for the number of holes [9, 10]. Also, in all the examples, the coefficients $P_1$, $P_2$ and $P_3$ in the objective function are considered 0.85, 0.30 and 1, respectively.

a) Castellated beam with 4-m span

As the first design example, a simply supported beam with 4-m span is selected and shown in Fig. 4. The beam is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 kN also acts at mid-span of the beam and the allowable displacement of the beam is limited to 12 mm. The modulus of elasticity is taken as $205 \ \text{kN/mm}^2$ and Grade 50 is selected for the steel of the beam which has the design strength 355 MPa. The number of CPs is taken as 50 and maximum number of iterations is limited to 200. The radius of each agent is considered to be one.
Table 1 represents the design variables and the cost of the castellated beam with 4m span obtained by ECSS method. It is observed that the castellated beam with hexagonal opening gives better results than the cellular beam. Figure 5 shows the convergence of the ECSS algorithm for design of the castellated beams with different openings.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Hexagonal opening</th>
<th>Circular opening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum UB-section selected</td>
<td>UB 305<em>102</em>25</td>
<td>UB 305<em>102</em>25</td>
</tr>
<tr>
<td>Hole diameter or cutting depth (mm)</td>
<td>125</td>
<td>248</td>
</tr>
<tr>
<td>Total number of holes</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Cutting angle</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>Minimum cost ($)</td>
<td>89.78</td>
<td>96.32</td>
</tr>
</tbody>
</table>

Fig. 5. Variation of Minimum cost versus the number of iterations for 4 m span castellated beam.
(a) Castellated beam with hexagonal opening, (b) Castellated beam with circular opening

b) Castellated beam with 8-m span

As the second example, ECSS algorithm is used to design a simply supported castellated beam with a span of 8m. The beam carries a uniform dead load 0.40 kN/m, which includes its own weight. The beam is subjected to two concentrated loads; dead load of 70 kN and live load of 70 kN as shown in Fig. 6. The allowable displacement of the beam is limited to 23 mm. The modulus of elasticity is taken as...
205 kN/mm² and Grade 50 is selected for the steel of the beam which has the design strength of 355 MPa. The number of CPs is taken as 50. The maximum number of iterations is considered 200 and the radius of agent is considered to be one.

![Figure 6](image)

**Fig. 6. A simply supported beam with 8m span length**

**Table 2. Optimum designs of the castellated beams with 8m span obtained by the ECSS algorithm**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Hexagonal opening</th>
<th>Circular opening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum UB-section selected</td>
<td>UB 610<em>229</em>101</td>
<td>UB 610<em>229</em>113</td>
</tr>
<tr>
<td>Hole diameter or cutting depth (mm)</td>
<td>246</td>
<td>484</td>
</tr>
<tr>
<td>Total number of holes</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Cutting angle</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>Minimum cost ($)</td>
<td>719.47</td>
<td>803.72</td>
</tr>
</tbody>
</table>

The corresponding cost obtained by the ECSS is equal to 719.47$ for hexagonal opening, while the cost of circular opening is equal to 803.72$. The difference between minimum cost shows that cellular beams under concentrate loading are not an appropriate option. Figure 7 shows the convergence of the ECSS algorithm for design of these beams with 8m span length.

![Figure 7](image)

**Fig. 7. Variation of minimum cost versus the number of iterations for 8m span beam. (a) Castellated beam with hexagonal opening,(b) Castellated beam with circular opening**

c) **Castellated beam with 9-m span**

The castellated beam with 9m span length is considered as the last example of this study. The beam carries uniform load of 40 kN/m including its own weight and two concentrated loads of 50 kN as shown.
in Fig. 8. The allowable displacement of the beam is limited to 25 mm. The modulus of elasticity is taken as $205 \text{kN/m}^2$ and grade 50 is selected for the steel of the beam which has the design strength of 355 MPa. Similar to the previous examples the number of CPs is taken as 50, the maximum number of iterations is considered 200 and the radius of agent is considered to be one.

In the optimum design of castellated beam with hexagonal hole, ECSS algorithm selects $684*254*125$ UB profile, 13 holes, and 277 mm for the cutting depth and 56 for the cutting angle. The minimum cost of design is equal to 995.97$. Also, in the optimum design of cellular beam, the represented algorithm selects $684*254*125$ UB profile, 14 holes and 539 mm for the holes diameter. The optimum result is given in Table 3.

![Fig. 8. Simply supported beam with 9m span length](image)

The convergence of the ECSS algorithm for design of castellated beam with 9m span length is shown in Fig. 9. Also, the optimum shapes of the hexagonal and circular openings are demonstrated separately in Fig. 10.

![Fig. 9. Variation of Minimum weight versus the number of iterations for 9m span beam. (a) Castellated beam with hexagonal opening, (b) Castellated beam with circular opening](image)

**Table 3. Optimum designs of the castellated beams with 9m span length obtained by the ECSS algorithm**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Hexagonal opening</th>
<th>Circular opening</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum UB-section selected</td>
<td>UB 684<em>254</em>125</td>
<td>UB 684<em>254</em>125</td>
</tr>
<tr>
<td>Hole diameter or cutting depth (mm)</td>
<td>277</td>
<td>539</td>
</tr>
<tr>
<td>Total number of holes</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>Cutting angle</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>Minimum cost ($)</td>
<td>995.97</td>
<td>998.94</td>
</tr>
</tbody>
</table>
Fig. 10. Optimum profiles of the castellated beams with cellular and hexagonal openings

The method of this paper can be applied to other structural optimization problems such as those of Refs. [17-21].

6. CONCLUDING REMARKS

In this paper, three castellated beams are selected from literature. The hexagonal and circular openings are considered as web-opening of the castellated beams. The cost of the beam is considered as the objective function. In this paper, a new objective function is defined and the sequence number of Universal Beam section, the hole diameter, cutting depth, cutting angle and the total number of holes in the beam are considered as design variables. To optimize the these beams the ECSS algorithm are utilized. It is observed that optimization results obtained by the ECSS algorithm for three castellated beams with hexagonal openings have less cost in comparison to the cellular beams. Also, the results of these examples illustrate the capability of the ECSS algorithm in finding the optimum solution in less number of iterations.

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REFERENCES


