IJST, Transactions of Civil Engineering, Vol. 39, No. C1, pp 65-79
Printed in The Islamic Republic of Iran, 2015

EVALUATING TOPOLOGY DESIGN OF MATERIAL LAYOUT IN STEEL PLATE STRUCTURES WITH HIGH STIFFNESS AND EIGENFREQUENCY*

D. LEE[1], S. SHIN[2**] AND S. PARK[3]
[1]Dept. of Architectural Engineering, Sejong University, Seoul, 143-747, Korea
[2]Research Institute of Industrial Technology, Pusan National University, Busan, 609-735, Korea
Email: shinsumi82@pusan.ac.kr
[3]Dept. of Architectural Engineering, Pusan National University, Busan, 609-735, Korea

Abstract– This study presents optimal distributions of steel materials in steel thin plate structures determined by using a classical element-wise and the present node-wise topology optimization design methods for a dynamic problem. More specifically, the present article describes an application of a node-wise topology optimization technique to the problem of maximizing fundamental frequency for plane structure. The terms element-and node-wise indicate the use of element and node densities, respectively, as design parameters on a given design space. For a dynamic free vibration problem, the objective function in general is to achieve maximum eigenfrequency with first-order eigenmode subject to a given limited material, since structures with a high fundamental frequency have a tendency to be reasonably stiff. For both static and dynamic problems SIMP (Solid Isotropic Microstructure with Penalization for Intermediate Density) material artificially penalizing the relation between density and stiffness is used in this study, and an implemented optimization technique is the method of moving asymptotes usually used for topology optimization. Numerical applications topologically maximizing the first-order eigenfrequency and depending on element or node densities as design parameters and varied boundary conditions to verify the present optimization design method provide appropriate manufacturing information for optimally form-finding of steel materials with Poisson's ratio of 0.3 into thin plates.

Keywords– Topology optimization, material layout, stiffness, eigenfrequency, plate, SIMP

1. INTRODUCTION

Numerical techniques based on finite element model are commonly used for structural analyses, and these provide basic information to determine appropriate structural systems "optimally" resisting horizontal forces. Therefore, the finite element model is linked to the so-called optimization of structures. "Optimization," derived from the term "optimally," is a mathematical discipline concerned with finding the minimum and maximum of needed functions, subject to so-called constraints. For example, rigidity and lightness are two opposite goals; however, optimization strategy tries to practically improve both of these two opposites.

The numerical and mathematical step beyond the pure simulation of the mechanical behavior of structures is to optimize their response in advance of physical production and to fit it to the specific needs. Therefore, almost all finite element method codes have incorporated at least basic structural optimization, such as sizing [1-3], shape [4-6], and topology [7-9] capabilities, in order to support the design analyst.

For both sizing and shape optimization the first design proposal is provided, such as a given initial sizing and shape, and is utilized as the start design. The arbitrary assumption of initial design in general

*Received by the editors December 4, 2013; Accepted July 9, 2014.
**Corresponding author

may result in unstable optimal solutions, which are absolutely dependent on initial design conditions. Thus far, classical optimization methods have mostly been based on the expertise and imagination of the designer, and thus solutions are not necessarily ideal. Topology optimization aims at solving this problem, and may be regarded as an applied tool for conceptual "imagination" information. Topology optimization has been developed to make the design stage easier and to find new design concepts. Topology optimization can be used with ease at the beginning of the design stage. The only required information is the design space, i.e. positions into which material is contained, the boundary conditions and the finite element model for discretization, which may be in contact with the optimized structure. No assumption about the number, the kind or the connectivity of structural elements is required, and therefore the efforts for modeling and preparation are extremely low and simple, in contrast to shape or sizing optimization.

The topology optimization problem presented in this study is to determine the layout of material of specified volume in a continuous design space, which maximizes or minimizes objective function for a given set of loads and boundary conditions. This differs from a ground-structure approach [10] for discrete topology optimization, in which needless elements of all potential members are removed by using optimality criteria. In the classical area of material topology optimization methods of continuous structures, one main optimization model has been mainly studied thus far, which is termed "element-based topology optimal design" due to implementing constant densities within elements as design parameters. A well-known homogenization method developed by Bendsøe and Kikuchi [9] in 1988 is based on this design, and is the most popular microscopic approach. Solid Isotropic Microstructure with Penalty for Intermediate Density (SIMP) [11-13] is a simple macroscopic isotropic model and roughly approximates the material stiffness-density relationship of porous materials. The element-based design is an efficient formulation because it operates on a fixed mesh with a small number of design parameters. More recently, another model conforming to the main element-based design model in the area of continuous material topology optimization methods was introduced by Kumar and Gossard [14] in 1996, and this is denoted as "node-based design," in which node densities are used as design parameters. For finite element analyses, a material property within each element is defined by a constant element density by arithmetically averaging node densities into one element. Since constant element densities are utilized for finite element analyses, contour parameterization and the SIMP penalization, this model can be considered a typical element-based design.

In order to overcome the academic and conceptual treatment of topology optimization designs, in this study the SIMP method, which produces superior solutions with respect to engineering, is treated for the classical area of material topology optimization. In addition, for more practical applications, a dynamic problem such as free vibration [15] [16] using optimal material distribution is considered for topology optimization. The knowledge and experience of the dynamic behavior of structures caused by natural phenomena such as earthquakes and winds is often of primary importance in many engineering applications, particularly in the field of large-scale structural components, such as buildings and bridges. Prediction of modal parameters such as resonance frequencies and mode-shapes is an essential step in order to discover structural behaviors or responses for the dynamic design.

The key point in this study is that the SIMP method for dynamic problems is implemented for both the present node-based and classical element-based designs. In particular, the present article describes an application of a node-wise topology optimization technique into the maximization problem of fundamental frequency for plane structure. Wang and Ni [17] treated nodal density as interpolant but restricted it to a linear elastic problem. Huang et al. [18] introduced evolutionary topology optimization based on element density variables under the condition of free vibration problems. The idea of applying the classical element-based topology optimization method for the dynamic problem is not new. There are many studies that have introduced topology optimization with respect to eigenfrequencies which are used for different objectives; fundamental eigenfrequency maximization [19-21], lowest eigenfrequencies maximization

[22] [23] [15]. The problem of minimization of structural responses caused by given dynamic load is also known as dynamic compliance minimization [24] [25].

Please note that a node-based SIMP method for dynamic problems is firstly presented in this study. The dynamic response is calculated using the Jacobi method [26] [27] of modal analyses. The method of moving asymptotes (MMA) [28] [17] is used as an optimizer. For static problems, numerical solutions based on comparisons between the element-based and node-based SIMP methods have already been investigated by Lee et al. [29, 30, 34-36].

This study is arranged as follows: in Section 2, the SIMP-based optimization formulation for dynamical problems is considered, Section 3 presents element-based and node-based design problems. In Section 4, numerical process comparisons of the element-based and node-based topology optimization are presented. Examples of evaluating topologically optimal shapes of steel plates for structural reinforcement are presented in Section 5. Section 6 presents the conclusions of this study.

2. FORMULATIONS OF SIMP-BASED MATERIAL TOPOLOGY OPTIMIZATION PROBLEMS

a) Optimization problem for dynamic structural free vibration system

In continuous formulations of the material topology optimization problem, the design is given by a continuous scalar function Φ from the fixed design space $\Omega_x \subseteq \Re^n \; (n=2)$ to the allowed material density $0 \le \Phi \le 1$. The schematic of the continuous material topology optimization of a solid structure with specified field and boundary conditions is shown in Fig. 1. This study concentrates on the free vibration and eigenvalue problem without considering loads. Therefore, loading conditions for the specific dynamic problem need not be considered in Fig. 1.

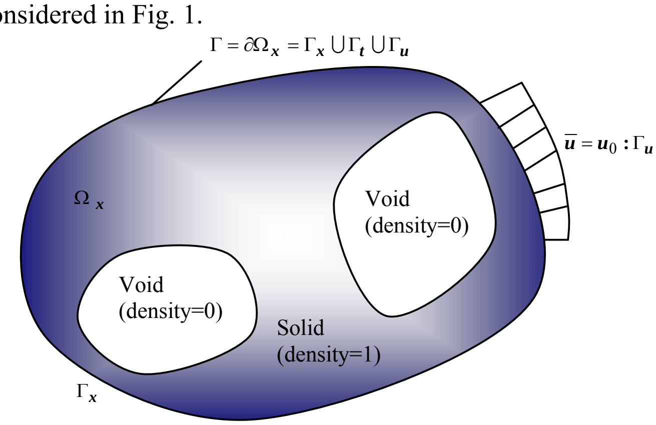


Fig. 1. Schematic for the material topology optimization of continuous structures

Eigenvalue optimization designs are profitable for mechanical structural systems subjected to dynamic loading conditions like earthquakes and wind loads. The dynamic behaviors of structural systems can be estimated by eigenfrequency, which describes structural stiffness. In general, maximizing first-order eigenfrequency can be an objective for dynamic topology optimization problems, as the stiffness of structures also increases when eigenfrequency increases. Problems of topology optimization for maximizing natural eigenfrequencies of vibrating elastostatic structures have been considered in the studies [9] [10] [12].

Assuming that damping can be neglected, such a dynamic design problem can be formulated as follows.

$$\underset{\Phi}{max} : \; \omega_1^2(\Phi) = \frac{\boldsymbol{u}_1^{\mathrm{T}} \boldsymbol{K} \boldsymbol{u}_1}{\boldsymbol{u}_1^{\mathrm{T}} \boldsymbol{M} \boldsymbol{u}_1} \tag{1}$$

$$Subject\ to: \frac{V(\Phi)}{V_0} \le g \tag{2}$$

$$: \lfloor \boldsymbol{K} - \omega_i^2 \boldsymbol{M} \rfloor \boldsymbol{u}_i = \underline{0} \tag{3}$$

$$: 0 < \Phi_{min} \le \Phi \le \Phi_{max} \tag{4}$$

where ω_1 denotes the first-order eigenfrequency, i.e. a given objective function, depending on design variable Φ. V and V_0 are actual and nominal volume fraction, respectively. g denotes material volume limitation. $\boldsymbol{K}$ and $\boldsymbol{M}$ are the global stiffness matrix and mass matrix. Both matrices depend on the penalization of design variable Φ, as shown in Section 2.2 which follows. A consistent mass, a lumped mass, and a combination of consistent and lumped mass such as in the present study can be used for $\boldsymbol{M}$. The inequality optimization constraint is $0 < \Phi_{min} \le \Phi \le \Phi_{max}$ of Eq. (4). In order to escape numerical singularity, the limit of Φ is given as $\Phi_{min} = 0.001$ and $\Phi_{max} = 1.0$. Equality constraints are provided by the dynamic free vibration equation of Eq. (3) and the limit on the required amount of material in terms of the constant volume V_0 of the design domain of Eq. (2). g is the ratio between an obtained volume and a given volume constraint.

b) Principles of the SIMP method

The goal of topology optimization is to provide the optimal material distribution into a restricted space, i.e. the design space. For this purpose, the principle is to cut the design space into small finite elements and to determine which ones belong to the solution.

Optimization variables correspond to the densities of each finite element. The relative density may take any value between 0 and 1, and an artificial material law (SIMP) is implemented to link together stiffness and density as

$$E_i^h\left(\Phi_i^h\right) = E_0\left(\Phi_i^h\right)^k, \quad k \ge 1,\ 0 \le \Phi_i^h \le 1,\ i = 1 \cdots N_e \tag{5}$$

where E_0 and Φ_0 denote nominal values of Young's modulus and material density of elements, respectively, and N_e is the number of elements. k is the penalization factor and Φ_i is the relative density of element i.

On the other hand, for node-based SIMP method penalization the formulation between Young's modulus and node density can also be written as follows.

$$E_{ij}^h\left(\Phi_{ij}^h\right) = E_0\left(\Phi_{ij}^h\right)^k, \quad k \ge 1,\ 0 \le \Phi_{ij}^h \le 1,\ i = 1 \cdots N_e,\ j = 1 \cdots N_n \tag{6}$$

where E_0 and Φ_0 denote nominal values of Young's modulus and material density of nodes into elements, respectively, and N_n is the number of nodes. k is the penalization factor and Φ_{ij} is the relative density of node j on element i.

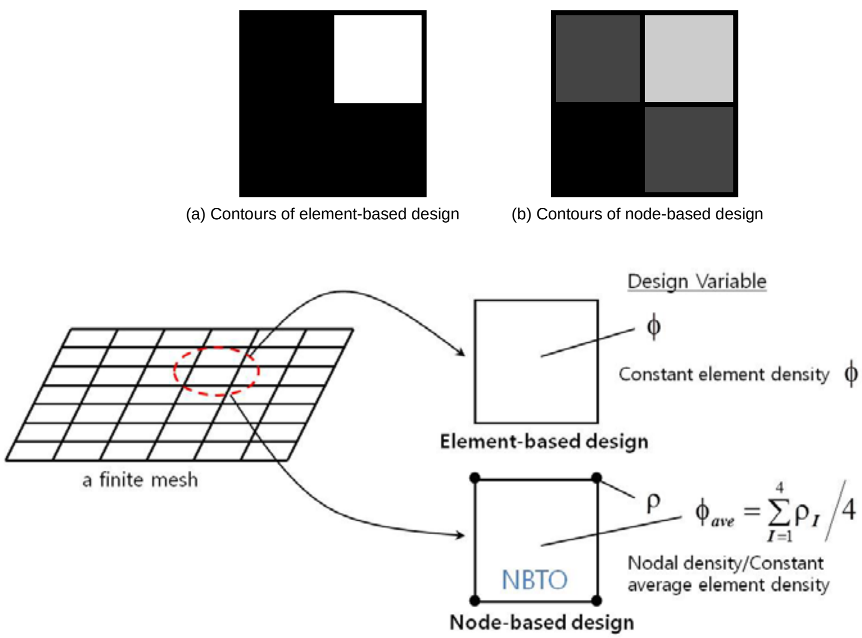
Equations (5) and (6) show that intermediate densities are allowed but that factor k penalizes their use. This artifice allows the optimization problem to be solved more easily. In this study, these penalization formulations are used for dynamic optimal design as in Section 2b as well as for static optimal designs as in Section 2a.

3. COMPARISONS BETWEEN ELEMENT- BASED AND NODE-BASED DESIGNS

According to comparisons between element- and node-based designs for static problems represented by Lee et al. [29], the characteristics can be also applied equally for the present dynamic node-based SIMP method. For example, Fig. 2 shows density distributions into four elements with four nodes per element for element in Fig. 2a and for node in Fig. 2b based designs. Each element takes constant density, and it should be noted that the constant element density for the node-based design is an arithmetic mean value of the densities of four nodes. It can be seen that optimal topology solutions for both element- and node-based designs are evaluated by the same element-wise density distribution contours as shown in Fig. 2. Therefore, jagged boundaries occur between voids (0) and solids (1), which leads to the tendency of a conceptual design. This is a shortcoming of classical element-based topology optimization methods, not a practical design. As can be seen in Fig. 2, the jagged boundaries are more smoothed in the node-based design than in the element-based design.

However, as can be seen in Fig. 3, density values between neighboring elements are compatible in node-based design, while density incompatibility occurs in element-based design. The density incompatibility indicates that the material density distribution is discontinuous among neighboring elements, as shown in Fig. 4b in Section 4.

Substantially, the density information is not compatible between neighboring elements. Notably, this density incompatibility problem may result in a material boundary’s distortion near neighboring elements, like a checkerboard pattern or a loss of material volume when the density information is evaluated in elements using a contour line of specific level sets. Therefore, a material topology optimization method that can directly obtain material information at nodes, i.e. node-based design, is an ideal alternative in order to resolve the density incompatibility.



(a) Contours of element-based design (b) Contours of node-based design

(c) Design variables of element and node-based design

Fig. 2. Comparisons between element- and node-based material topology optimization designs

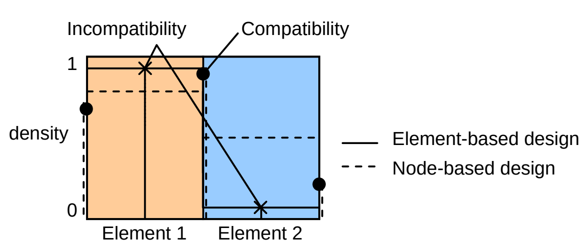


Fig. 3. Density compatibility of element- and node-based designs in one dimension

4. NUMERICAL ALGORITHMS OF ELEMENT- AND NODE-BASED MATERIAL TOPOLOGY OPTIMIZATION FOR DYNAMIC PROBLEMS

The element- and node-based SIMP material topology optimization algorithms associated with FEM have the numerical steps of the following Subsections, and are compared in Fig. 4. Here, a MATLAB code introduced by Sigmund [11] is an element-based SIMP program for static problems, and the program was extended for dynamic topology optimization for node-based design.

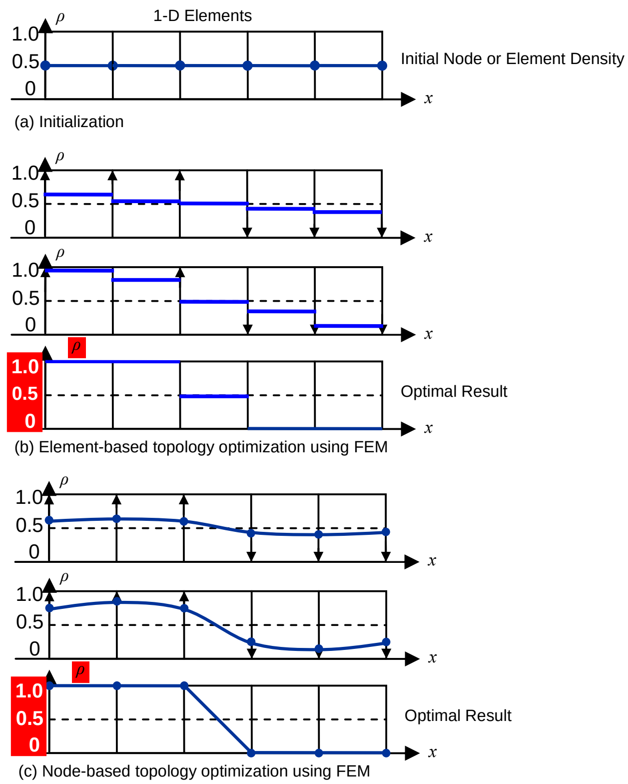


Fig. 4. Element- and node-based topology optimization processes in a given volume constraint of 50%

a) Initialization

For node-based design, node densities are given as design parameters, while element densities are assigned for element-based design. Their initial values are assigned by a reference volume fraction value. The objective function and constraints are defined to be an optimization model. Geometry of a structure, boundary, and loading conditions are defined in the design domain.

b) SIMP method A: Classical element-based topology optimization using FEM

Constant element density distributions of the initial design domain defined by constant material properties make almost voids (0.001) and solids (1) during the optimization procedures as shown in Fig. 4b. Material properties within elements are constant and then material properties of neighboring elements are discontinuously distributed in the design domain. After some iterations, optimal density distributions of 0.001 and 1 are produced.

c) SIMP method B: Node-based topology optimization using FEM

Continuous nodal density distributions of the initial design domain are arithmetically averaged in order to be defined by constant material properties, and they make voids (almost 0) and solids (1) during the optimization procedures, as shown in Fig. 4c, and continuous material properties within elements are distributed in the design domain. After some iterations, optimal density distributions of 0 and 1 are produced.

d) Comparisons of element- and node-based topology optimization using FEM

Figure 5 shows algorithm comparisons of element- and node-based design in material topology optimization. As can be seen, in element-based design, constant element densities are used for design variables and material properties during whole optimization processes. Unlike element-based design, node-based design uses nodal densities as design variables during every optimization process, but constant element densities linearly interpolated by nodal densities are assigned to each finite element for structural analysis of FEM.

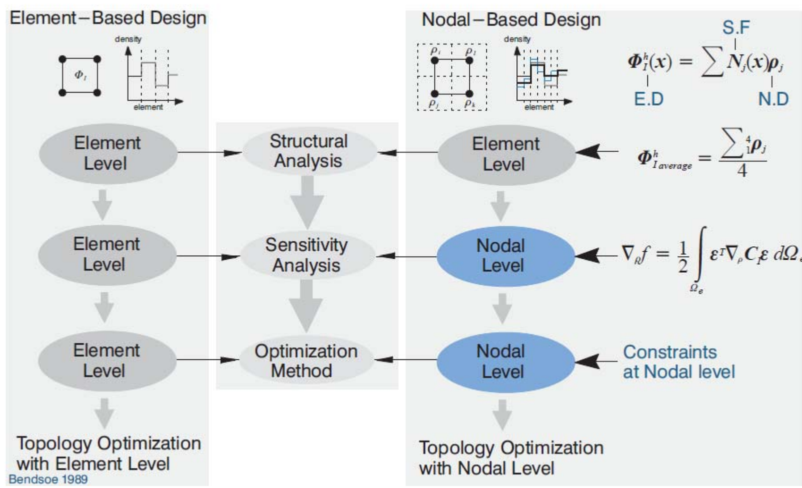


Fig. 5. Algorithm comparisons of element- and node-based design in material topology optimization

Figure 6 describes the duality of sensitivity analysis in element- and node-based design for material topology optimization of maximal stiffness or minimal strain energy. Here, Φ and ρ denote element and nodal density, respectively.

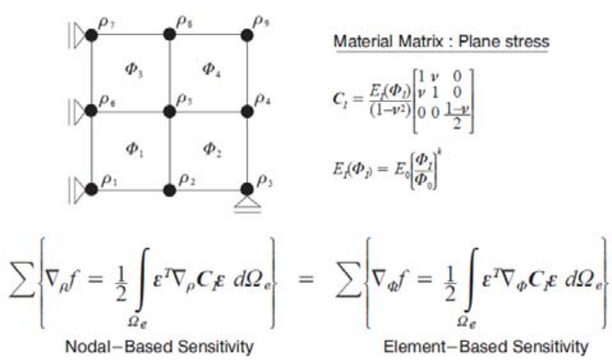


Fig. 6. Duality of sensitivity analysis in element- and node-based design in material topology optimization

5. NUMERICAL APPLICATIONS AND DISCUSSION

a) Topological optimal material layout extraction of element-based and nodal-based design

This numerical application describes the principle of material layouts between element- and node-based designs in a given 3×3 design space. Figure 7 shows the geometrical material layout representations of (a) element-based and (b) node-based topology optimization. The material layouts are Eulerian types which use fixed meshs. The layouts are presented by rectangular finite elements. The element-based design produces one material discontinuity line of 0 or 1 values of density design variables, while node-based design makes two material discontinuity lines due to intermediate density value regions.

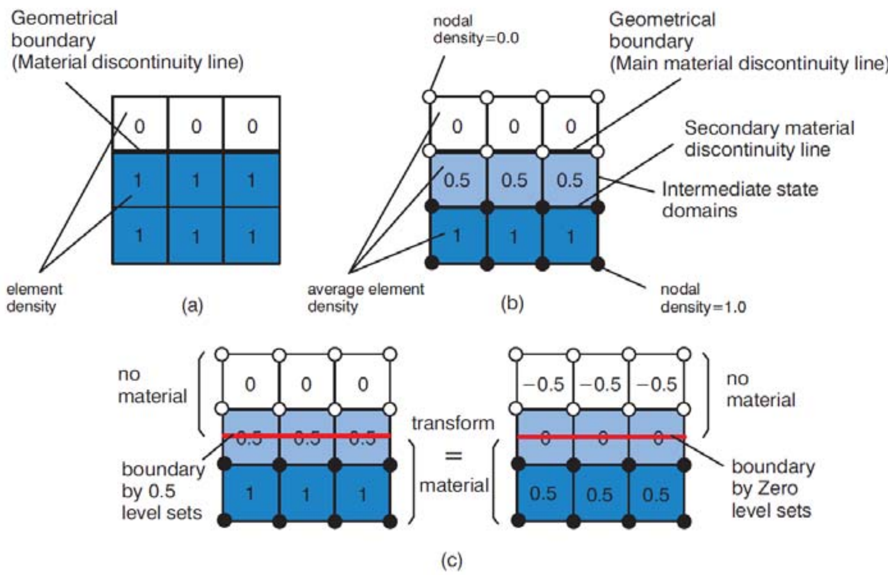


Fig. 7. Representations of material layouts of element- and node-based design: (a) element-based design, (b) node-based design, (c) the use of zero or 0.5 isoline (level sets) for describing material layouts Analysis models

Figure 8 presents material layouts of a circle with curved material boundaries. As can be seen, node-based design produces smoothness of material boundaries, while element-based design makes strong material discontinuities between solids and voids. The strong material discontinuities lead to omitting layout information detailed near the material layout. The use of density information on each node provides detained material layout information near boundaries between solids and voids.

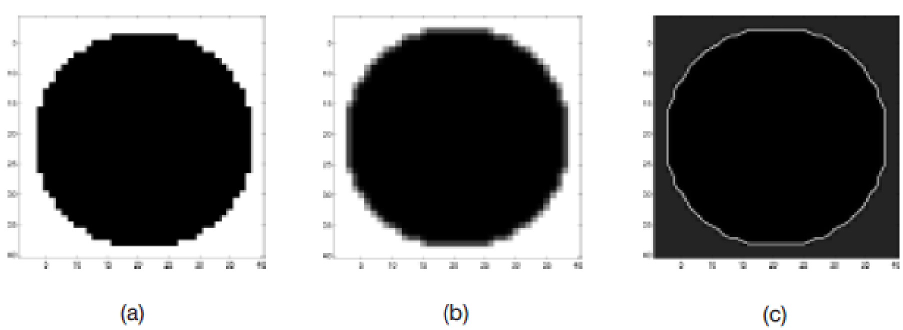
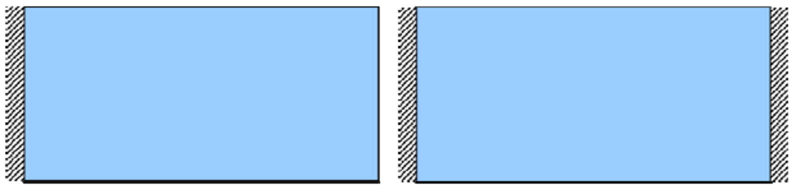


Fig. 8. Material layouts of a circle: (a) element-based design, (b) node-based design, (c) 0.5 isoline layout of node-based design

b) Optimal material layout extraction for steel plate structures resisting dynamic behaviors: Deep beam of cantilever-type

According to material layout principle of element- and node-based designs, this numerical application involves solution comparisons between element- and node-based designs for two-phase dynamic material topology optimization designs of 2D wall steel plate with a fixed support at the left side and a free support at the right side. Loading condition is not considered, because the example structures use simple free vibration problem. The design domain (3m×2m) is defined in Fig. 9a. 30×20 finite elements are discretized in the design domain. Material properties are nominal Young's modulus E=1.0 GPa and Poisson's ratio ν=0.3 for steel material. Isotropic material of plane stress is assumed. The objective function is a maximal first-order eigenfrequency dynamic problem. Total volumes are fixed to 30%, 40%, 50%, 60% and 70% during every optimization procedure. MMA is used for element- and node-based optimization methods.



(a) cantilever beam space (b) clamped beam space

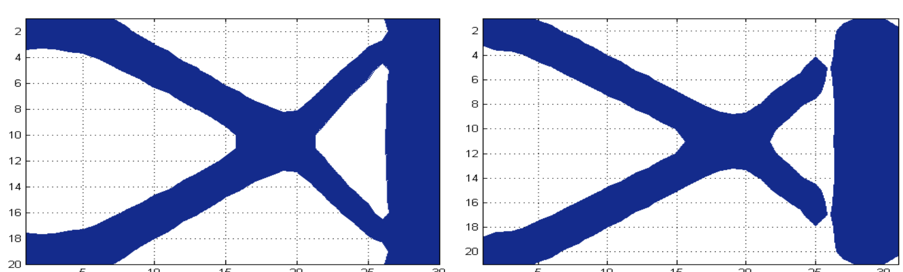
Fig. 9. Analysis models: 2D wall structures

Figure 10 shows the density distribution contours for element- and node-based designs in volume of 50%. It can be seen that the results obtained through the node-based design are smoother than those obtained through the element-based design with respect to the material boundaries between solids and voids.

Volume	Node-based design	Element-based design
50%		

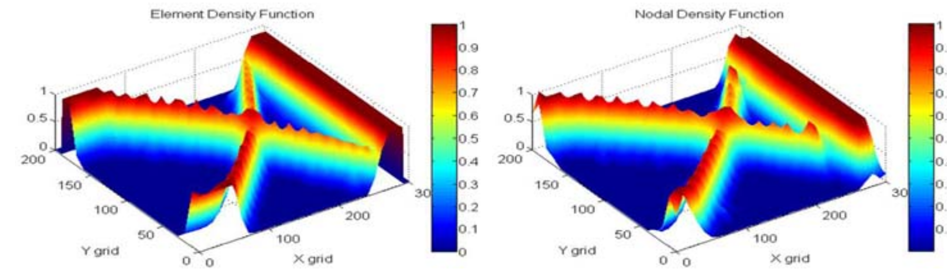
Fig. 10. Optimal topologies for dynamic topology optimization design: density distribution contours

Figure 11 shows the 0.5 density isoline topology contours [31, 32] designed using element- and node-based designs for dynamic eigenfrequency-based topology optimization methods. Please note that non-connectivity between members does not occur in the node-based design. The optimal layout results from the use of a specific 0.5 isoline as shown in Fig. 12 that describes three dimensional density distribution by using element and nodal densities. As can be seen, material density distribution in the node-based design is significantly below 0.5 when a 0.5 isoline is used, in comparison with the distribution in the element-based design.

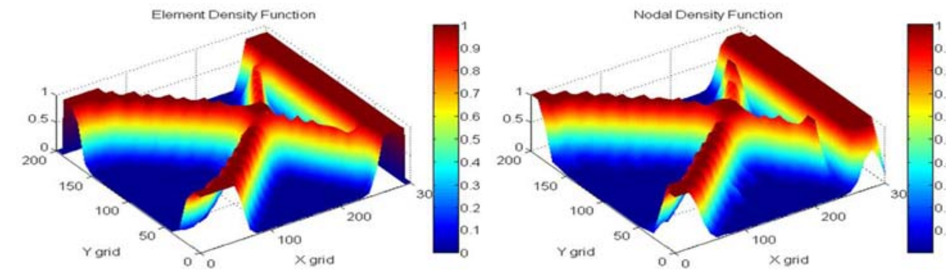


(a) Element-based design (b) Node-based design

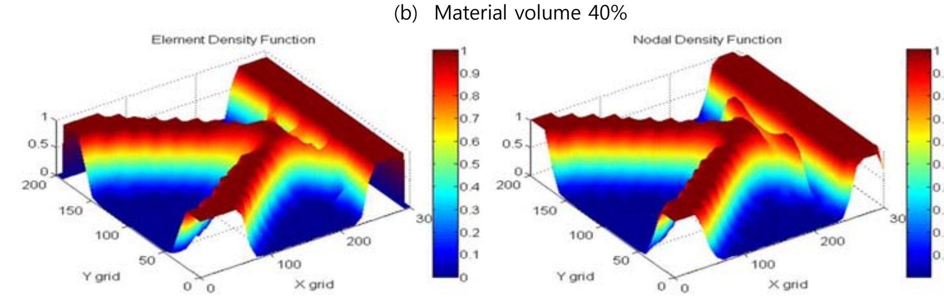
Fig. 11. Optimal topologies for dynamic topology optimization design under material volume 40%: 0.5 isolines



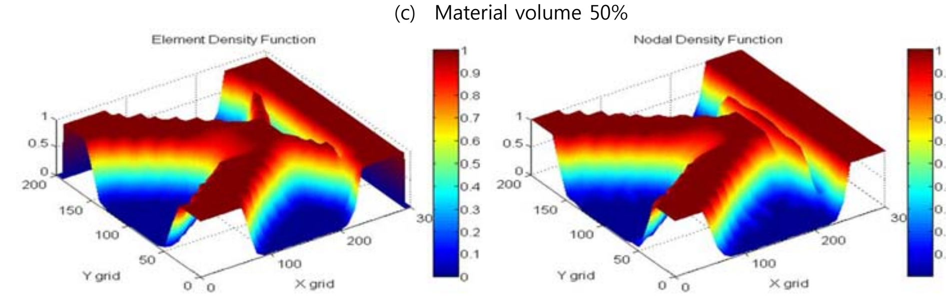
(a) Material volume 30%



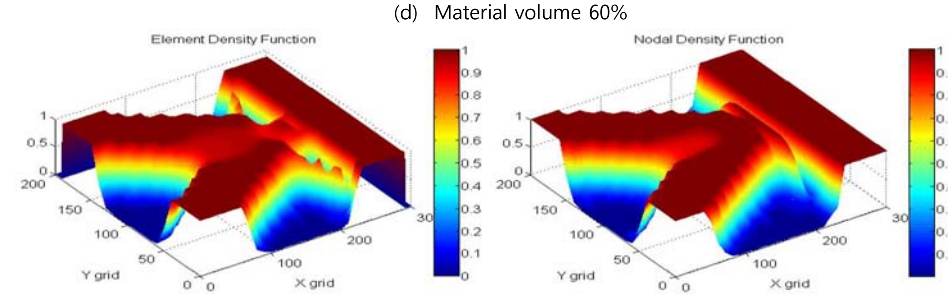
(b) Material volume 40%



(c) Material volume 50%



(d) Material volume 60%



(e) Material volume 70%

Fig. 12. Optimal topologies for dynamic topology optimization design: three-dimensional density function

c) Optimal material layout extraction for steel plate structures resisting dynamic behaviors: Clamped-type

The goal of this numerical application is the same as in Section 5.2. The structure is a 2D wall clamped structure with a fixed support at both the left and right sides. Loading condition is not considered, because the example structures use a simple free vibration problem. The design domain (4m×2m) is defined in Fig. 9b. 40×20 finite elements are discretized in the design domain. Material properties are nominal Young's modulus E =1.0 GPa and Poisson's ratio ν =0.3 for steel material. Isotropic material of plane stress is assumed. The dynamic behaviors of structural systems can be estimated by eigenfrequency, which describes structural stiffness. In general, maximizing the first eigenfrequency can be an objective for dynamic topology optimization problems, since the stiffness of structures also increases when eigenfrequency increases. Therefore, the objective function is maximal first-order eigenfrequency for the dynamic problem. Total volumes are fixed to 30%, 40%, 50%, 60% and 70% during every optimization procedure. MMA is used for element- and node-based optimization methods.

Figure 13 shows the density distribution contours for element- and node-based designs with a volume of 60%. It can be found that results of the node-based design are smoother than those of the element-based design near material boundaries between solids and voids.

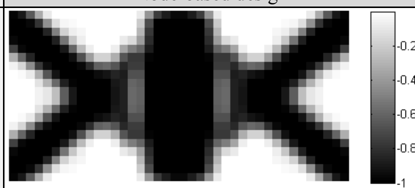
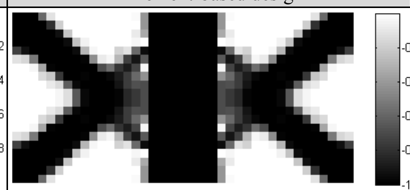
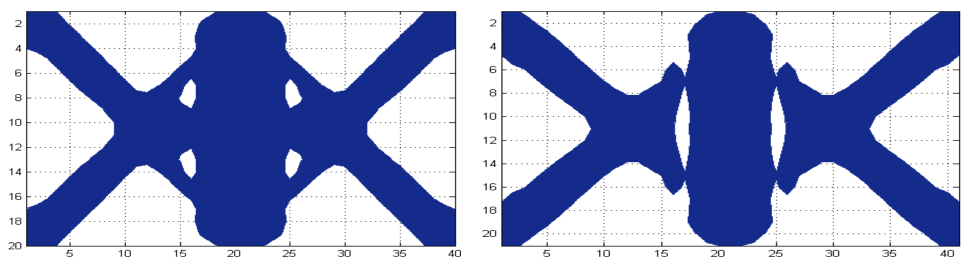
Volume	Node-based design	Element-based design
60%		

Fig. 13. Optimal topologies for dynamic topology optimization design: density distribution contours



(a) Element-based design (b) Node-based design

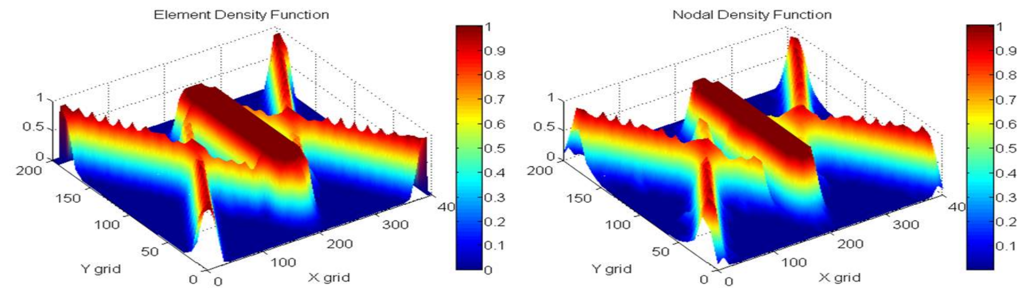
Fig. 14. Optimal topologies for dynamic topology optimization design under material volume 50%: 0.5 isolines

Figure 14 shows the 0.5 density isoline topology contours [31, 32] designed using element- and node-based designs for dynamic eigenfrequency-based topology optimization methods. As can be seen, the checkerboard patterns with enforcing material discontinuities of voids and solids at the node-based designs is more relieved than that in Section 5.2. Please note that the checkerboard patterns with enforcing material discontinuities of voids and solids do not occur in the node-based design. The optimal layout results from the use of a specific 0.5 isoline, as shown in Fig. 13, which describes three-dimensional density distribution by using element and nodal densities.

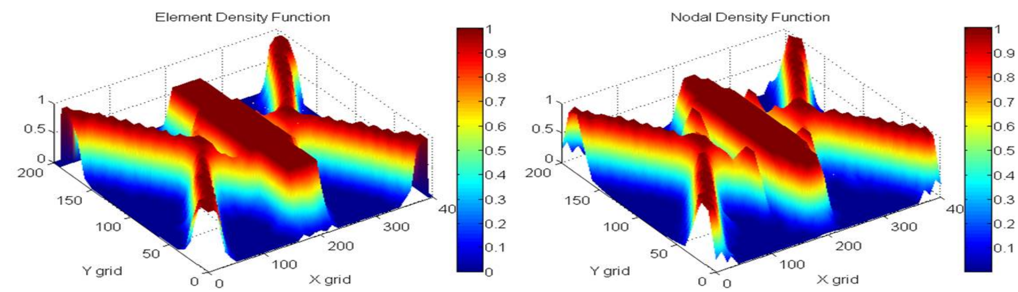
Figures 10-15 provide efficient information of optimal shapes and deposition of the steel plates, when reinforcement design of steel plate is considered in the given design space, such as a cantilever and a clamped beam.

Please note that the two examples of Fig. 9a, i.e., a cantilever beam design space, and Fig. 9b, i.e., a clamped beam design space, are verified, since the optimal topologies of a clamped beam join exactly at the symmetric half, i.e., those of a cantilever beam.

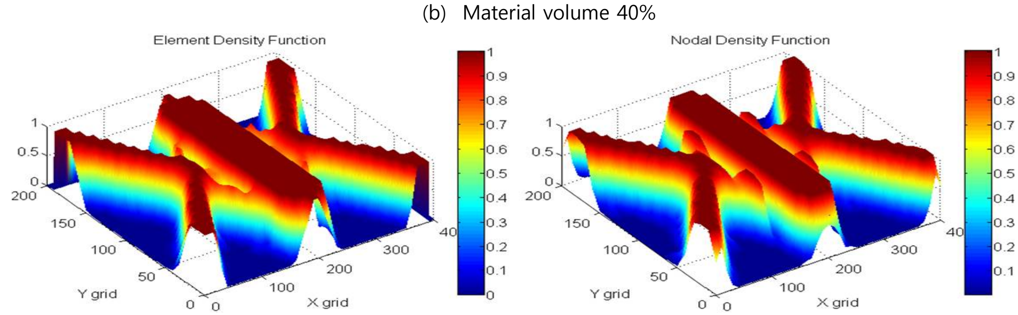
As can be seen in Figures 11 and 13, the result of the node-based method does not seem to be connected with some members. For this reason a 0.5 isoline contour is used. Density distributions below 0.5 values exist near the regions of non-connectivity.



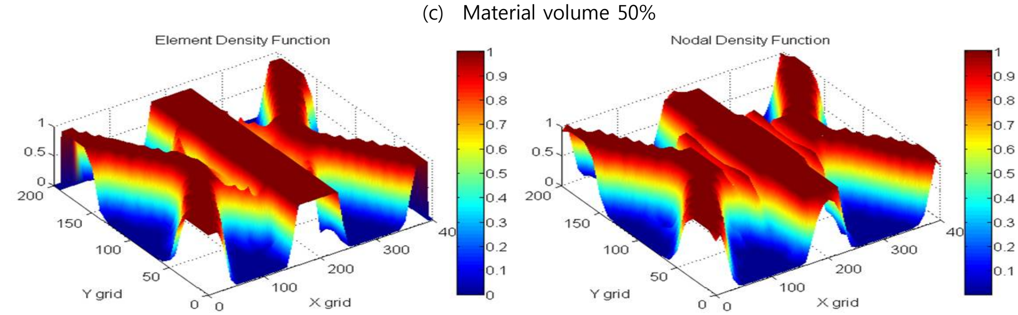
(a) Material volume 30%



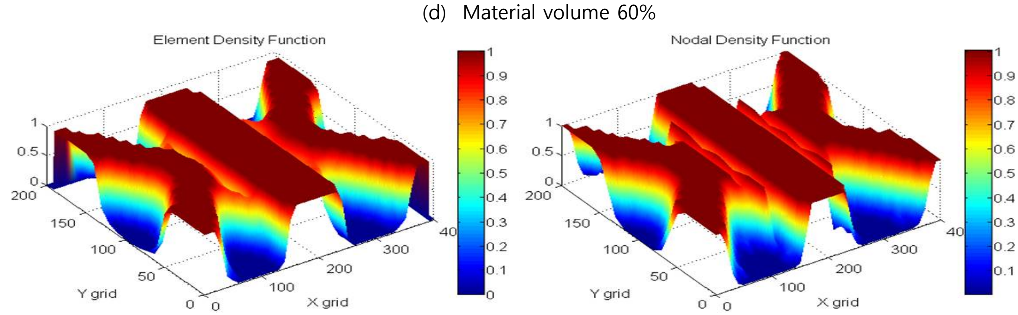
(b) Material volume 40%



(c) Material volume 50%

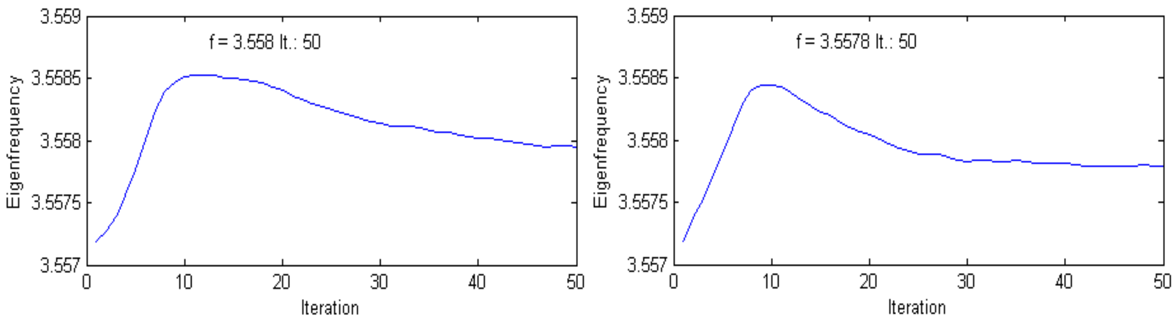


(d) Material volume 60%



(e) Material volume 70%

Fig. 15. Optimal topologies for dynamic topology optimization design: three-dimensional density function



(a) Element-based design (b) Node-based design

Fig. 16. Convergence curves of element- and node-based design in volume fraction 70% in Fig. 5b

Figure 16 shows convergence histories of element- and node-based design under a volume of 80% in clamped structures. As can be seen, node-based design produces convergence of objective function faster than element-based design, although the number of design parameters in node-based design is 7.6% more than that of element-based design. It seems that density compatibility or continuity among elements, as shown in node-based design in Fig. 3, enforces the sensitivity of design parameters, and results in fast convergence.

6. CONCLUSION

This study presents an alternative approach, i.e. the so-called node-based design method, to classical element-based SIMP material topology optimization for dynamics problems, and also makes an effort for practical applications of topology optimization for optimal form finding of dynamic structures. For static problems, the methodology of node-based design in comparison with the classical element-based design has already been introduced by the author [29] [30]. In the preceding we have demonstrated that node-based design can effectively solve a wide range of eigenfrequency topology optimization problems. Compared with the existing classical element-based topology optimization techniques, the greatest advantage of the node-based design method is smoothness of material distribution contours by using the arithmetic mean of node densities and density compatibility, i.e., material continuity, between neighboring elements. In this study, these advantages were also verified for dynamic problems.

Two examples on optimal form finding of steel thin plate structures for dynamic free vibration problem which are deposited into a cantilever and a clamped type verified the superiority and the efficiency of the topology optimization design method for structural design.

Acknowledgements: This research was supported by the MKE (The Ministry of Knowledge Economy), Korea, under the Convergence-ITRC (Convergence Information Technology Research Center), supervised by the NIPA(National IT Industry Promotion Agency), (NIPA-2013-H0401-13-1003) and grants (code# 2010-0019373, 2012R1A2A1A01007405, NRF-2013R1A1A2057502, and 2014R1A1A3A04051296) from the National Research Foundation of Korea (NRF) funded by the Korea government, including research funds provided by Pusan National University in Korea.

REFERENCES

1. Chamoret, D., Qiu, K. & Domaszewski, M. (2009). Optimization of truss structures by a stochastic method. *International Journal for Simulation and Multidisciplinary Design Optimization*, Vol. 3, No. 1, pp. 321-325.

2. Jaberzadeh, E., Azhari, M. & Boroomand, B. (2014). Free vibration of moving laminated composite plates with and without skew roller using the element-free Galerkin method, *Iranian Journal of Science & Technology, Transactions of Civil Engineering*, Vol. 38, No. C2, pp. 377-393.
3. Parezzan, J. C. & Hernandez, S. (1999). Size and shape optimization of long portal bridge. *Computer Aided Optimum Design of Structures*, Vol. 40, pp. 416-429.
4. Camprubi, N., Bischoff, M. & Bletzinger, K. U. (2004). Shape optimization of shells and locking. *Computers & Structures*, Vol. 82, No. 29-30, pp. 2551-2561.
5. Song, X. & Baldwin, J. D. (1999). A novel node-based structural shape optimization algorithm. *Computers & Structures*, Vol. 70, No. 5, pp. 569-581.
6. Rozvany, G. I. N., Zhou, M. & Birker T. (1992). Generalized shape optimization without homogenization. *Structural Optimization*, Vol. 4, pp. 250-252.
7. Bruggi, M. (2009). Generating strut-and-tie patterns for reinforced concrete structures using topology optimization. *Computers & Structures*, Vol. 87, No. 23-24, pp. 1483-1495.
8. Yang, X. Y., Xie, Y. M. & Steven, G. P. (2005). Evolutionary methods for topology optimization of continuous structures with design dependent loads. *Computers & Structures*, Vol. 83, No. 12-13, pp. 956-963.
9. Bendsøe, M. P. & Kikuchi, N. (1988). Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering*, Vol. 71, pp. 197-224.
10. Dorn, W. S. & Greenberg, H. J. (1964). Automatic design of optimal structures. *J. Mec*., Vol. 3, pp. 25-52.
11. Sigmund, O. (2001). A 99 topology optimization code written in Matlab. *Structural and Multidisciplinary Optimization*, Vol. 21, pp. 120-127.
12. Tcherniak, D. (2002). Topology optimization of resonating structures using SIMP method. *International Journal for Numerical Methods in Engineering*, Vol. 54, No. 11, pp. 1605-1622.
13. Walls, R. & Elvin, A. (2010). Mass and stiffness distributions in optimized ungrouped unbraced frames. *International Journal of Steel Structures*, Vol. 10, No. 3, pp. 233-242.
14. Kumar, A. V. & Gossard, D. C. (1996). Synthesis of optimal shape and topology of structures. *Journal of Mechanical Design*, Vol. 118, pp. 68-74.
15. Ma, Z. D., Kikuchi, N., Hagiwara, I. & Cheng, H. C. (1995). Topological optimization technique for free vibration problems. *Journal of Applied Mechanics*, Vol. 62, No. 1, pp. 200-207.
16. Kaveh, A. & Shokohi, F. (2014). Cost optimization of castellated beams using charged system search algorithm. *Iranian Journal of Science & Technology, Transactions of Civil Engineering*, Vol. 38, No. $C1^+$, pp 235-249.
17. Wang, H. and Ni, Q. (2008). A new method of moving asymptotes for large-scale unconstrained optimization. *Applied Mathematics and Computation*, Vol. 203, No. 1, pp. 62-71.
18. Huang X., Zuo Z. H. & Xie, Y. M. (2010). Evolutionary topological optimization of vibrating continuum structures for natural frequencies. *Computers and Structures*, Vol. 88, pp. 357-364.
19. Yoon, G. H. (2010). Maximizing the fundamental eigenfrequency of geometrically nonlinear structures by topology optimization based on element connectivity parameterization. *Computers & Structures*, Vol. 88, No. 1-2, pp. 120-133.
20. Pedersen, N. L. (2004). Optimization of holes in plates for control of eigenfrequencies. *Structural Multidisciplinary Optimization*, Vol. 28, No. 1, pp. 1-10.
21. Tenek, L. H. & Hagiwara, I. (1994). Eigenfrequency maximization of plates by optimization of topology using homogenization and mathematical programming. *JSME International Journal*, Vol. 37, No. 4, pp. 667-677.
22. Du, J. & Olhoff, N. (2007). Topological design of freely vibrating continuum structures for maximum values of simple and multiple eigenfrequencies and frequency gaps. *Struct. Multidiscip. Optimiz*., Vol. 34, pp. 91-110.
23. Jensen, J. S. & Pedersen, N. L. (2004). On maximal eigenfrequency separation in two-material structures: the 1D and 2D scalar cases. *Journal of Sound and Vibration*, Vol. 289, No. 4-5, pp. 967-986.

24. Sandberg, G. & Ohayon, R. (2009). *Computational Aspects of Structural Acoustics and Vibration*. International Center for Mechanical Sciences: Courses and Lectures, 505, Springer.
25. Min, S., Kikuchi, N., Park, Y. C., Kim, S. & Chang, S. (1999). Optimal topology design of structures under dynamic loads. *Structural Optimization*, Vol. 17, pp. 208-218.
26. Zhou, X. & Yu, R. (2007). Development of a seismic design method based on response spectra for building structures. *Frontiers of Architecture and Civil Engineering in China*, Vol. 1, No. 2, pp. 129-141.
27. Lehoucq, R. B., Sorensen, D. C. & Yang, C. (1998). *ARPACK Users' Guide: Solution of large-scale eigenvalue problems with implicitly restarted Arnoldi methods*. SIAM Publications, Philadelphia.
28. Svanberg, K. (1987). The method of moving asymptotes - a new method for structural optimization. *Int. J. Numer. Meth. Engng*., Vol. 24, pp. 359-373.
29. Lee, D. K., Lee, J. H., Kim, J. H. & Starossek, U. (2014). Investigation on material layouts of structural diagrid frames by using topology optimization. *KSCE Journal of Civil Engineering*, Vol. 18, No. 2, pp. 549-557.
30. Lee, D. K. & Shin, S. M. (2014). Advanced high strength steel diagrid using TRIZ and nonlinear pushover analysis. *Journal of Constructional Steel Research*, Vol. 96, pp. 151-158.
31. Youn, S. K. & Park, S. H. (1997). A study on the shape extraction process in the structural topology optimization using homogenized material. *Computers and Structures*, Vol. 62, No. 3, pp. 527-538.
32. Victoria, M., Marti, P. & Querin, O. M. (2008). Topology design of two-dimensional continuum structures using isolines. *Computers & Structures*, Vol. 87, No. 1-2, pp. 101-019.
33. Lee, D. K., Lee, J. H. & Ahn, N. S. (2014). Generation of structural layout in use for '0-1' material considering *n*-order eignefrequency dependence. *Materials Research Innovations*, Vol. 18, No. S2, pp. 833-839.
34. Lee, D. K., Lee, J. H., Lee, K. H. & Ahn, N. S. (2014). Evaluating topological optimized layout of building structures by using nodal material density based bilinear interpolation. *Journal of Asian Architecture and Building Engineering*, Vol. 13, No. 2, pp. 421-428.
35. Lee, D. K., Ha, T. H., Jung, M. Y. & Kim, J. H. (2014). Evaluating high performance steel tube-framed diagrid for high-rise buildings. *Steel and Composite Structures*, Vol. 16, No. 3, pp. 289-303.