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AN ANALYTICAL SOLUTION FOR RELIABILITY ASSESSMENT OF PSEUDO-STATIC STABILITY OF ROCK SLOPES USING JOINTLY DISTRIBUTED RANDOM VARIABLES METHOD*

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Abstract– Reliability analysis of rock slope stability has received considerable attention in the literature. It has been used as an effective tool to evaluate uncertainty so prevalent in variables. In this research the application of the jointly distributed random variables method for probabilistic analysis and reliability assessment of rock slope stability with plane sliding is investigated. In a recently published paper, the authors showed the dependency of the numerator and denominator of the safety factor relationship and argued that, as a result of this dependency, the method could not assess the reliability correctly. In the current research the authors present a new approach to solve this problem. In this approach, using the basic relations in this method, the safety factor relationship is obtained directly without separation of its numerator and denominator. Furthermore, in addition to friction angle of sliding surface, apparent cohesion, depth of water in tension crack, and earthquake acceleration ratio, in the present work the unit weight of rock is also considered as a stochastic parameter. The results are compared with the Monte Carlo simulation. Comparison of the results indicates good performance of the proposed approach for assessment of reliability. The new results of parametric analysis using the jointly distributed random variables method show that the friction angle of sliding surface is the most effective parameter in rock slope stability with plane sliding.

Keywords– Reliability, Jointly distributed random variables, Monte Carlo simulation, Rock slope stability

1. INTRODUCTION

The application of probability analysis for reliability assessment of complex engineering systems was first introduced in the 1940s in the fields of structural and aeronautical engineering. Among its early uses in geotechnical engineering was the application in reliability assessment of open pit mine slope design where a certain risk of failure is acceptable [1-4]. Probabilistic evaluation of slope failures is increasingly seen as the most appropriate framework for accounting for uncertainties in design. Herein rock slope stability analysis often involves a considerable uncertainty.

Generally, two main observations can be made concerning the existing body of work on this subject. The first common approach accounts for the uncertainty in the geometrical properties of the joint/fracture network in the slope, and the second one considers uncertainties in the slope performance. Uncertainty in the geometrical characterization of fractures within the rock mass has led to the development of stochastic fracture network models which are widely discussed and well documented in the literature [5-9]. In the second approach which is frequently used in rock slope stability analyses, the aim is to find the probability of slope failure given uncertain input parameters in the stability analysis model. In this category, the

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output of slope stability analysis is a probability distribution of either factor of safety for a fixed slope height, or a probability distribution of critical height for a fixed level of factor of safety. However, due to incomplete information, some uncertainties cannot be managed satisfactorily by probability theory.

Rock slope stability is one of the most widely reported geotechnical applications of reliability. A few recent examples are cited herein.

Some research on the application of finite or distinct element method in slope stability analysis has been documented in the literature [10-12]. Li et al. [13] demonstrated the use of a new form of stability number for rock slope designs that has been recently developed from finite element analysis methods and provided guidance for its use in probabilistic assessments.

Additionally, several attempts have been made to analyze the stability of slopes using Monte Carlo simulation (MCs) [14, 15]. Wang et al. [16] presented a reliability-based Robust Geotechnical Design (RGD) approach and demonstrated this approach using a rock slope design example. In Park et al. [17] the uncertain parameters in rock slope stability analysis were expressed as fuzzy numbers using the fuzzy set theory. The MCs technique and reliability index approach were implemented with fuzzy set theory in order to take into account the fuzzy uncertainties in the evaluation of probability of failure. Li et al. [18] proposed a stochastic response surface method for reliability analysis involving correlated non-normal random variables, in which the Nataf transformation is adopted to effectively transform the correlated non-normal variables into independent standard normal variables.

Lee et al. [19] developed a Knowledge-based Clustered Partitioning (KCP) technique for determining reliability index and failure probability of rock wedge. In this approach, reliability index is analyzed and optimized using KCP technique.

A number of recent attempts have been made to apply the response surface method to assess the reliability of slope stability analysis [e.g. 20]. Also, some attempts have been made to apply approximate methods such as Point Estimate Method (PEM) [e.g. 21–23], First Order Second Moment reliability method (FOSM) [e.g. 24] and First Order Reliability Method (FORM) [25, 26] in reliability analysis of stability of slopes.

Some researchers have used analytical methods to slope stability analysis [27, 28]. Nomikos and Sofianos [29] obtained an analytical solution for the probability density of the factor of safety in underground rock mechanics. In analytical methods, the probability density functions of input variables are expressed mathematically. They are then integrated analytically into the adopted rock slope stability analysis model to derive a mathematical expression of the density function of the factor of safety. The Jointly Distributed Random Variables (JDRV) method that is used in this research lies in this category [30-32].

Recently, Johari et al. [28] presented the results of an investigation into application of the JDRV method in reliability assessment of rock slope stability. This work showed the dependency of the numerator and denominator of the safety factor relationship and argued that, as a result of this dependency, the JDRV method could not assess the reliability correctly. In the current research, for solving this problem, instead of calculating the numerator and denominator of the safety factor equation and dividing them (as was done in the previous paper) the authors used a new approach. For this purpose, using the basic relations in the JDRV method, the safety factor relationship was obtained directly without separation of its numerator and denominator.

2. ROCK SLOPE WITH PLANE SLIDING

In the present study, planar sliding rock slope is considered. The rock slope is an unweathered granite with exfoliation or sheet joints. These joints are parallel to the surface of the granite and the spacing between successive joints increases with increasing distance into the rock mass. Furthermore, the sheet joint surface continues under the potentially unstable slope, which is the potential triggering factor of the slope failure. Figure 1 shows this type of sliding and its parameters.

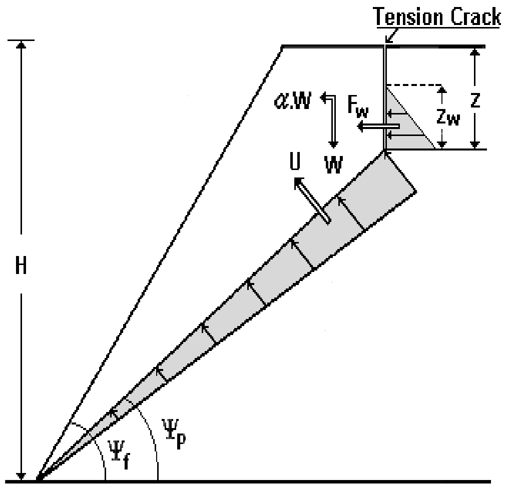

Fig. 1. Geometrical definition of the rock slope stability with plane sliding

For this type of sliding failure the equation for determination of safety factor can be expressed as [33]:

$$FS = \frac{cA + \left(W\left(\cos\psi_P - \alpha\sin\psi_P\right) - U - F_w\sin\psi_P\right)\tan\varphi}{W\left(\sin\psi_P + \alpha\cos\psi_P\right) + F_w\cos\psi_P} \quad (1)$$

Where:

$$W = \frac{\gamma_r H^2}{2}\left[\left(1 - \left(\frac{Z}{H}\right)^2\right)\cot\psi_p - \cot\psi_f\right] \quad (2)$$

$$Z = H\left(1 - \sqrt{\cot\psi_f \tan\psi_p}\right) \quad (3)$$

$$U = \frac{\gamma_w Z_w A}{2} \quad (4)$$

$$A = \frac{H - Z}{\sin\psi_p} \quad (5)$$

$$F_w = \frac{\gamma_w {Z_w}^2}{2} \quad (6)$$

FS : Factor of safety against sliding
Z : Depth of tension crack
Z_w : Depth of water in tension crack
A : Area of wedge
W : Weight of rock wedge resting on failure surface
H : Height of the overall slope
U : Uplift force due to water pressure on failure surface
F_w : Horizontal force due to water in crack
α: Horizontal earthquake acceleration
c : Cohesive strength along sliding surface
φ: Friction angle of sliding surface
Ψ_p: Angle of failure surface, measured from horizontal
Ψ_f : Angle of slope face, measured from horizontal
γ_r : Unit weight of rock
γ_w: Unit weight of water

In this research, two sets of parameters are considered in equation (1), fixed parameters H, Ψ_p , Ψ_f and Z and stochastic parameters Z_w, α, c, φ and γ_r which were assumed to be uncorrelated.

3. STOCHASTIC PARAMETERS

To account for the uncertainties in rock slope stability, 5 input parameters have been defined as stochastic variables. The selected parameters are friction angle of sliding surface (φ), apparent cohesion (c), depth of water in tension crack (Z_w), and earthquake acceleration ratio (α) and unit weight of rock (γ_r). Friction angle, apparent cohesion and unit weight are modeled using a normal probability density function and depth of water in tension crack and earthquake acceleration ratio are modeled by truncated exponential parameters. It should be noted that although in reality cohesion and friction angle are negatively correlated, they are considered as uncorrelated variables for mathematical convenience. The parameters related to geometry are regarded as constant parameters.

4. JOINTLY DISTRIBUTED RANDOM VARIABLES METHOD

The JDRV method is an analytical probabilistic method. In this method, density functions of input variables are expressed mathematically and joined together by statistical relations. The available statistical and probabilistic relations between parameters are given in this section [30-32].

Theorem 1

Let X_1, X_2, …, X_n be continuous random variables having marginal densities $f_{X1}(x_1)$, $f_{X2}(x_2)$,…, $f_{Xn}(x_n)$. Then X_1, X_2, …, X_n are independent if and only if for all $x_1,x_2,\ldots,x_n$ the joint pdf $f_{X1,X2,\ldots,Xn}(x_1, x_2,\ldots, x_n)$ is defined by:

$$f_{X_1,X_2,\ldots,X_n}(x_1,x_2,\ldots,x_n)=f_{X_1}(x_1).f_{X_2}(x_2)\ldots.f_{X_n}(x_n) \tag{7}$$

Theorem 2

Suppose that $X_1,X_2,\ldots,X_n$ are continuous random variables with joint probability density $f_{X1,X2,\ldots,Xn}(x_1, x_2,\ldots, x_n)$ and let $Y_1=g_1(X_1,X_2,\ldots,X_n)$, $Y_2=g_2(X_1,X_2,\ldots,X_n)$, …, $Y_n=g_n(X_1,X_2,\ldots,X_n)$ define a one-to-one transformation between the points $(x_1,x_2,\ldots,x_n)$ and $(y_1,y_2,\ldots,y_n)$. Let $y_1=g_1(x_1,x_2,\ldots,x_n)$, …, $y_n=g_n(x_1,x_2,\ldots,x_n)$ be uniquely solved for $x_1,x_2,\ldots,x_n$ in terms of $y_1,y_2,\ldots,y_n$ as $x_1=h_1(y_1,y_2,\ldots,y_n)$, $x_2=h_2(y_1,y_2,\ldots,y_n)$, …, $x_n=h_n(y_1,y_2,\ldots,y_n)$. Then the joint probability of $y_1,y_2,\ldots,y_n$ is:

$$f_{Y_1,Y_2,\ldots,Y_n}(y_1,y_2,\ldots,y_n)=\left|J(y_1,y_2,\ldots,y_n)\right|.f_{X_1,X_2,\ldots,X_n}\left(h_1(y_1,y_2,\ldots,y_n),\ldots,h_n(y_1,y_2,\ldots,y_n)\right) \tag{8}$$

Where:

$$J(y_1,y_2,\ldots,y_n)=\begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} & \cdots & \frac{\partial x_1}{\partial y_n} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} & \cdots & \frac{\partial x_2}{\partial y_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial x_n}{\partial y_1} & \frac{\partial x_n}{\partial y_2} & \cdots & \frac{\partial x_n}{\partial y_n} \end{vmatrix} \tag{9}$$

Where $\left|J(y_1,y_2,\ldots,y_n)\right|$ is the absolute value of $J(y_1,y_2,\ldots,y_n)$.

Theorem 3

If the joint probability density function of continuous random variables X_1, X_2, …, X_n is $f_{X1,X2,\ldots,Xn}(x_1, x_2,\ldots, x_n)$ the marginal probability density function of X_i is:

$$f_{X_i}(x_i) = \iint\limits_{R_{Xi}} \ldots \int f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)\,dx_1 dx_2 \ldots dx_{i-1} dx_{i+1} \ldots dx_n \tag{10}$$

Theorem 4

If *X* is a random variable with the probability density of $f_X(x)$, and *Y* is a function of *X* in the form Y = g(x), the probability density of *Y* can be determined as:

$$f_Y(y) = f_x\left(g^{-1}(y)\right) \times \left|\frac{dg^{-1}(y)}{dy}\right| \tag{11}$$

This relation is valid for monotonically increasing or decreasing function g(x).

5. RELIABILITY ASSESSMENT OF THE ROCK SLOPE STABILITY WITH PLANE SLIDING

In this research the terms of the safety factor Eq. (1) are grouped together in the following form (Eq. 12) and the probability density equation of each group is derived separately using Eqs. (7) to (11). The derivation of these equations is presented in the Appendix 1.

$$F.S. = \frac{k_1 + \left(k_5(a - b.k_2) - k_3(\chi - \chi_s.k_3)\right).k_4}{k_5(b + a.k_2) + \chi_c.k_3{}^2} \tag{12}$$

Where:

$$\begin{cases} k_1 = c.A \\ k_2 = \alpha \\ k_3 = Z_w \\ k_4 = \tan\left(\dfrac{\pi.\varphi}{180}\right) \\ k_5 = \gamma_r \end{cases} \tag{13}$$

And

$$\begin{cases} S = \dfrac{H^2}{2}\left[\left(1 - \left(\dfrac{Z}{H}\right)^2\right)\cot\psi_p - \cot\psi_f\right] \\ a = S.\cos(\psi_p) \\ b = S.\sin(\psi_p) \\ \chi = \dfrac{\gamma_w.A}{2} \\ \chi_c = \dfrac{\gamma_w.\cos(\psi_p)}{2} \\ \chi_s = \dfrac{\gamma_w.\sin(\psi_p)}{2} \end{cases} \tag{14}$$

Using the obtained mathematical functions k_1 to k_5, $f_{K1}(k_1)$ to $f_{K5}(k_5)$ and equations A20 to A32 a computer program, coded in MATLAB, was developed to determine the probability density function of factor of safety. However the integrals in equation A31 do not have analytical solutions and must be solved numerically. In addition, for comparison, determination of the safety factor for rock slopes using the MCs was also coded in the same computer program.

6. EXAMPLE

To demonstrate the efficiency and accuracy of the proposed method in determining the probability density distribution of the safety factor in rock slopes with plane sliding, an illustrative example is presented. For this purpose, the Sau Mau Ping slope in Hong Kong, which is analyzed probabilistically in Chapter 8 of Hoek [33], is used. In this example, all rock parameters are considered as stochastic parameters.

The stochastic parameters with normal and truncated exponential distributions are shown in Tables 1 and 2 respectively, and the deterministic parameters are given in Table 3.

Table 1. Stochastic parameters with normal distribution

Parameters	Mean	Standard deviation	Minimum	Maximum
c (kPa)	10.0	2.0	2.0	18.0
Φ (Degree)	35.0	5.0	15.0	55.0
γ_r (kN/m^3)	24.5	0.875	21	28

Table 2. Stochastic parameters with truncated exponential distribution

Parameters	λ	Mean=1/ λ	Standard deviation	Minimum	Maximum
Zw (m)	0.1428	7.0	3.6795	0.0	14.0
α	12.5	0.08	0.0420	0.0	0.16

Table 3. Deterministic parameters

Height of the slope (m)	Ψ_p (Degree)	Ψ_f (Degree)	γ_w (kN/m^3)
60.0	35.0	50.0	10.0

In order to verify the results of the presented method with those of the MCs, the final probability density functions for the factor of safety are determined using the same data for both methods. For this purpose, 10,000,000 generation points are used for the MCs. The results are shown in Fig. 2. As it can be seen in this figure, the results obtained using the developed method are very close to those of the MCs. Figure 3 shows the cumulative distribution curve of safety factor. It can be seen that the probability of failure (FS≤1) for this slope is about 78.826%, indicating that the slope will most likely be unstable. It shows that with these stochastic parameters, the maximum probability value of FS for this site is about 1.5.

On the other hand, a deterministic calculation using the mean values of the stochastic parameters shows that, the safety factor of failure is about 0.7492. This demonstrates that in this situation the slope would fail, but the probability of failure is not specified. Therefore, the designer cannot have an engineering judgment. In fact, reliability assessment and engineering judgment are employed together to develop risk and decision analyses.

7. PARAMETRIC ANALYSIS

To evaluate the response of stability of slope with plane sliding (equation 1) to changes in parameters, a parametric analysis is carried out following two different approaches using the JDRV method. In the first approach, the means of the five stochastic input parameters are increased based on their standard deviation (new mean = old mean + 2.0×std). The results are shown in Fig. 4. Additionally, the cumulative distribution curves are plotted in Fig. 5. To evaluate the influence of changes in each parameter, the parameter is increased while the ranges of the other stochastic input parameters are kept constant. Using Fig. 5 the amounts of changes in probability of failure corresponding to 2×std increase in the pdf of the input parameters are calculated and given in Table 4. Similar to the previous approach, it can be seen that the friction angle of sliding surface is the most sensitive parameter in the safety factor (stability) of rock slopes.

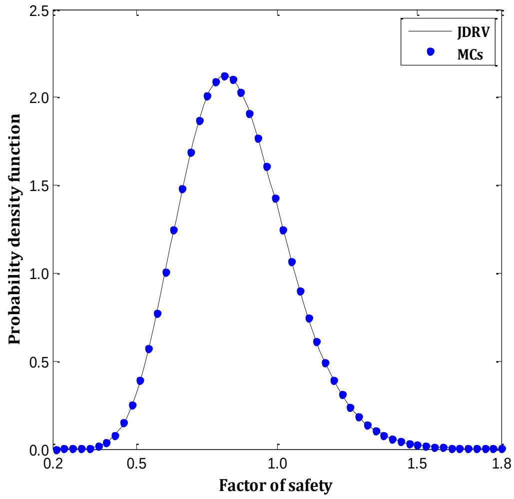

Fig. 2. Comparison of probability density functions of the factor of safety by MCs and JDRV method

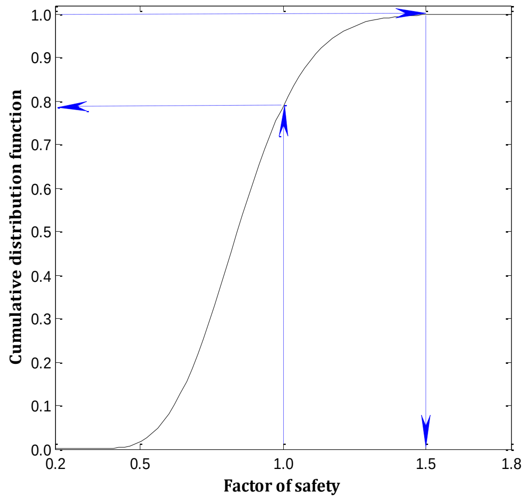

Fig. 3. Cumulative distribution function of failure in the example problem

In the second approach, the mean of each stochastic input parameter is kept constant and their variances are changed. For this purpose, the variance of the variable is increased (1.0×std and 1.5×std) while the variances of the other stochastic input parameters are kept constant. Figures 6 and 7 show the probability density functions and cumulative distribution curves related to 1.0×std increase in variance and Figs. 8 and 9 show these curves for 1.5×std increase. Using Figs. 7 and 9 the changes in probability of failure corresponding to 1.0×std and 1.5×std increase in the pdf of input parameters are calculated and the results are given in Tables 5 and 6. It can be seen that, as in the previous approach, the friction angle of sliding surface is the most effective parameter in safety factor (stability) of rock slopes.

For this analysis, the standard deviation of truncated exponential distribution is determined using the following procedure. The pdf of variable x is defined as Eq. (15).

$$f_X(x) = \frac{\lambda_x}{c_k}.\exp(-\lambda_x x) \qquad a \le x \le b \tag{15}$$

where a, b are lower and upper truncated values of x. c_k is the area under exponential distribution between a and b which is calculated using Eq. (16). Additionally, the mean and variance of truncated exponential distribution can be calculated through Eqs. (17) to (19).

$$c_k = \int_a^b \lambda_x.\exp(-\lambda_x x)dx = \exp(-\lambda_x a) - \exp(-\lambda_x b) \tag{16}$$

$$\mu_x = E(X) = \frac{\left(a + \frac{1}{\lambda_x}\right).\exp(-a\lambda_x) - \left(b + \frac{1}{\lambda_x}\right).\exp(-b\lambda_x)}{\exp(-a\lambda_x) - \exp(-b\lambda_x)} \tag{17}$$

$$E(X^2) = \frac{\left(a^2 + \frac{2}{\lambda_x}\left(a + \frac{1}{\lambda_x}\right)\right).\exp(-a\lambda_x) - \left(b^2 + \frac{2}{\lambda_x}\left(b + \frac{1}{\lambda_x}\right)\right).\exp(-b\lambda_x)}{\exp(-a\lambda_x) - \exp(-b\lambda_x)} \tag{18}$$

$$\sigma_x = \left[E(X^2) - (E(X))^2 \right]^{\frac{1}{2}} = \frac{\left[\exp(2a\lambda_x) + \exp(2b\lambda_x) - \left(2 + (\lambda_x (a-b))^2\right).\exp(\lambda_x (a+b)) \right]^{\frac{1}{2}}}{\lambda_x.(\exp(a\lambda_x) - \exp(b\lambda_x))} \quad (19)$$

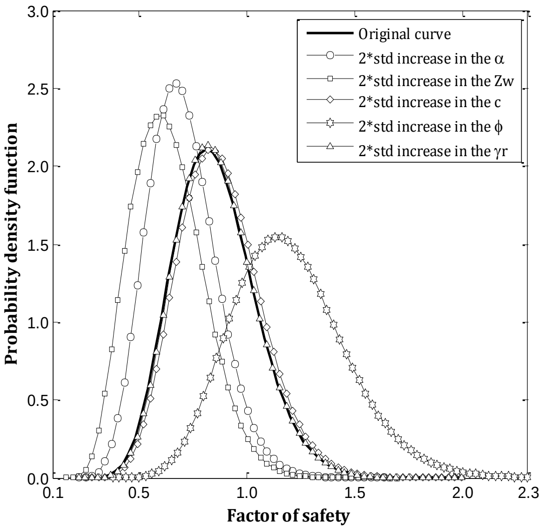

Fig. 4. Variation of probability density functions of the factor of safety in parametric analysis

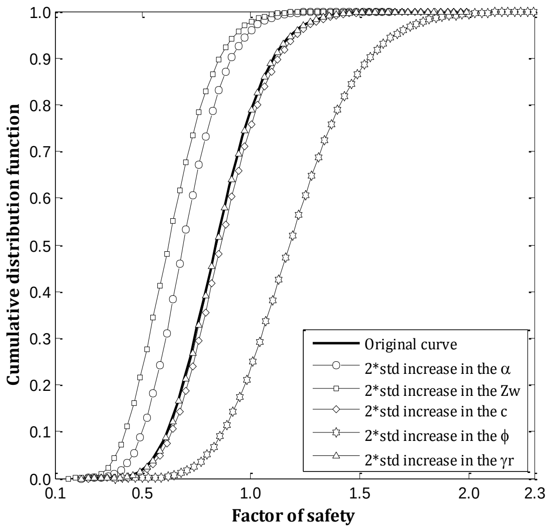

Fig. 5. Parametric analysis to determine the most effective parameter

Table 4. Changes in probability of failure corresponding to 2×std increase (shift rightward) in the pdf of input parameters

Stochastic parameter	Shift in the φ	Shift in the c	Shift in the γ_r	Shift in the α	Shift in the Zw
Change (%)	-69.96	-4.61	-0.98	+20.80	+23.11

Table 5. Changes in probability of failure corresponding to 1×std increase (shift rightward) in the pdf of input parameters

Stochastic parameter	Shift in the φ	Shift in the c	Shift in the γ_r	Shift in the α	Shift in the Zw
Change (%)	-35.55	-2.32	-0.67	+13.94	+15.95

Table 6. The amounts of changes in probability of failure corresponding to 1.5×std increase (shift rightward) in the pdf of input parameters

Stochastic parameter	Shift in the φ	Shift in the c	Shift in the γ_r	Shift in the α	Shift in the Zw
Change (%)	-7.91	-1.09	-0.43	+4.45	+5.43

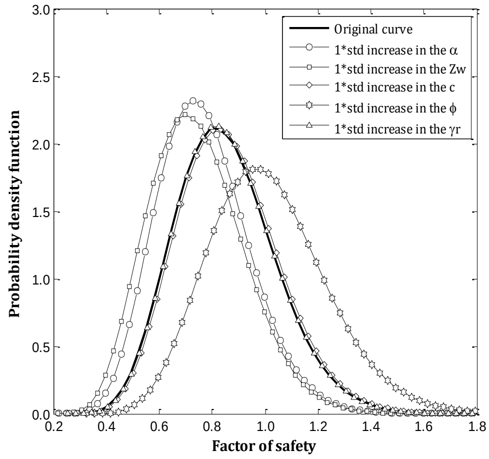

Fig. 6. Variation of probability density functions of the safety factor in parametric analysis based on 1×std Increase of variance

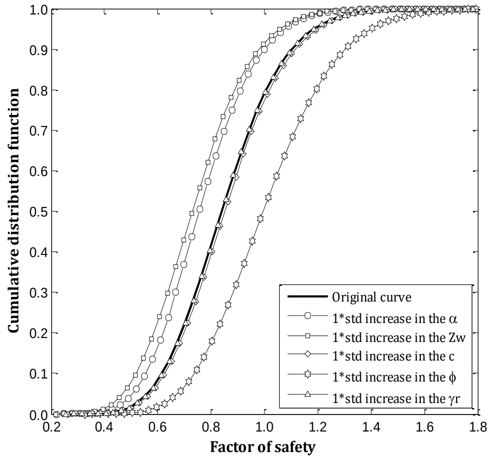

Fig. 7. Variation of cumulative distribution curve of the safety factor in parametric analysis based on 1×std increase of variance

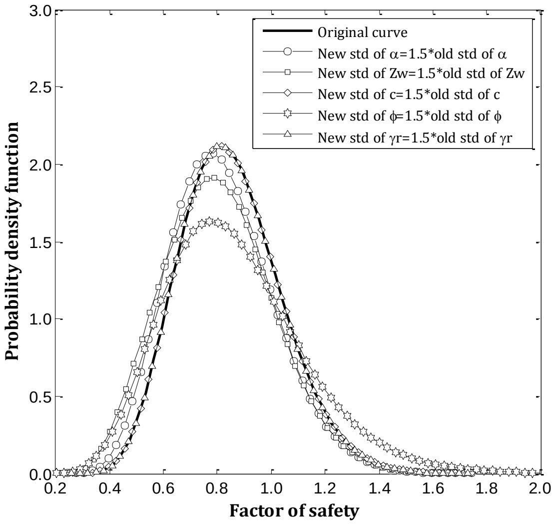

Fig. 8. Variation of probability density functions of the safety factor in parametric analysis based on 1.5×std increase of variance

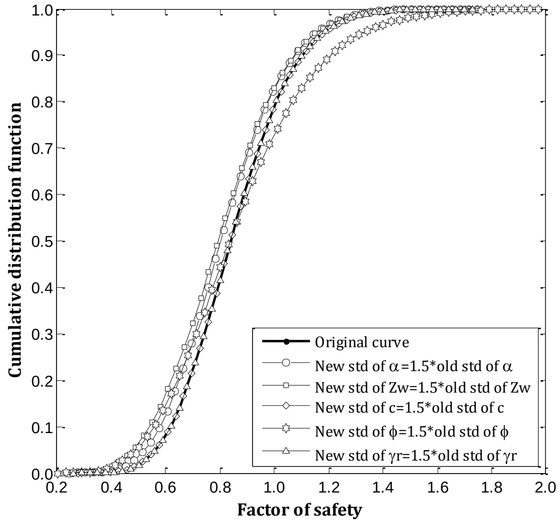

Fig. 9. Variation of cumulative distribution curve of the safety factor in parametric analysis based on 1.5×std increase of variance

8. CONCLUSION

Analysis of stability of rock slopes is a probabilistic problem due to the inherent uncertainties in the geometrical properties and geotechnical parameters. In this paper, the application of the jointly distributed random variables method in assessing the reliability of analysis of rock slope stability with plane sliding was investigated. Recent research by the authors showed the dependency of the numerator and denominator of the safety factor relationship and argued the fact that, as a result of this dependency, the JDRV could not assess the reliability correctly. In the current paper, a new approach was presented to overcome this problem. For this purpose, using the basic relations in the JDRV method, the safety factor relationship was obtained directly without separation of its numerator and denominator. Comparison of the results of the jointly distributed random variables method and the MCs showed good performance of the proposed method. A parametric analysis was carried out to assess the influence of various stochastic parameters using the JDRV method. The results showed that the friction angle of sliding surface is the most effective parameter in influencing the safety factor of rock slopes.

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APPENDIX 1

Derivation of mathematical functions k_1 to k_7 and F.S and their domains is presented in this Appendix:

$$f_{k_1}(k_1)=\frac{1}{A.\sigma_c\sqrt{2\pi}}\exp\left(-0.5\left(\frac{k_1-A.c_{mean}}{A.\sigma_c}\right)^2\right) \qquad c_{min}.A\le k_1\le c_{max}.A \tag{20}$$

$$f_{k_2}(k_2)=\frac{\lambda_{k_2}.\exp\left(-\lambda_{k_2}.k_2\right)}{1-\exp\left(-\lambda_{k_2}.k_{2max}\right)} \qquad 0\le k_2\le k_{2max} \tag{21}$$

$$f_{k_3}(k_3)=\frac{\lambda_{k_3}.\exp\left(-\lambda_{k_3}.k_3\right)}{1-\exp\left(-\lambda_{k_3}.k_{3max}\right)} \qquad 0\le k_3\le k_{3max} \tag{22}$$

$$f_{k_4}(k_4)=f_{\varphi}(k_4)\times\left|\frac{d}{dk_4}\left(\frac{180}{\pi}\tan^{-1}(k_4)\right)\right|=\frac{180}{\pi\left(1+k_3{}^2\right)\sigma_{\varphi}\sqrt{2\pi}}\exp\left(-0.5\left(\frac{\frac{180}{\pi}\tan^{-1}(k_3)-\varphi_{mean}}{\sigma_{\varphi}}\right)^2\right) \tag{23}$$

$$\tan(\varphi_{min})\le k_4\le\tan(\varphi_{max})$$

$$f_{k_5}(k_5)=\frac{1}{\sigma_{\gamma_r}\sqrt{2\pi}}\exp\left(-0.5\left(\frac{k_5-\gamma_{r_{mean}}}{\sigma_{\gamma_r}}\right)^2\right) \qquad \gamma_{r_{min}}\le k_5\le\gamma_{r_{max}} \tag{24}$$

Using Theorem 2 the following equations can be written:

$$\begin{cases} u = FS = g_1(k_1,k_2,k_3,k_4,k_5) = \dfrac{k_1 + \left(k_5(a - b.k_2) - k_3(\chi - \chi_s.k_3)\right).k_4}{k_5(b + a.k_2) + \chi_c.k_3^{\,2}} \\ v = g_1(k_1,k_2,k_3,k_4,k_5) = k_2 \\ w = g_1(k_1,k_2,k_3,k_4,k_5) = k_3 \\ z = g_1(k_1,k_2,k_3,k_4,k_5) = k_4 \\ p = g_1(k_1,k_2,k_3,k_4,k_5) = k_5 \end{cases} \quad (25)$$

$$\begin{cases} k_1 = h_1(u,v,w,z,p) = u\left(p(b + a.v) + \chi_c.w^2\right) - z\left(p(a - b.v) - w(\chi - \chi_s.w)\right) \\ k_2 = h_2(u,v,w,z,p) = v \\ k_3 = h_3(u,v,w,z,p) = w \\ k_4 = h_4(u,v,w,z,p) = z \\ k_5 = h_5(u,v,w,z,p) = p \end{cases} \quad (26)$$

Using Theorem 1 the following equation can be written:

$$f_{K_1,K_2,K_3,K_4,K_5}(k_1,k_2,k_3,K_4,K_5) = f_{K_1}(k_1).f_{K_2}(k_2).f_{K_3}(k_3).f_{K_4}(k_4).f_{K_5}(k_5) \quad (27)$$

Using Theorem 2 the following equation can be written:

$$f_{U,V,W,Z,P}(u,v,w,z,p) = \\ \left|J(u,v,w,z,p)\right|.f_{K_1,K_2,K_3,K_4,K_5}\left(h_1(u,v,w,z,p),h_2(u,v,w,z,p),h_3(u,v,w,z,p),h_4(u,v,w,z,p),h_5(u,v,w,z,p)\right) \quad (28)$$

Where:

$$J(u,v,w,z,p) = \begin{vmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} \end{vmatrix} = \begin{vmatrix} \frac{\partial k_1}{\partial u} & \frac{\partial k_1}{\partial v} & \frac{\partial k_1}{\partial w} & \frac{\partial k_1}{\partial z} & \frac{\partial k_1}{\partial p} \\ \frac{\partial k_2}{\partial u} & \frac{\partial k_2}{\partial v} & \frac{\partial k_2}{\partial w} & \frac{\partial k_2}{\partial z} & \frac{\partial k_2}{\partial p} \\ \frac{\partial k_3}{\partial u} & \frac{\partial k_3}{\partial v} & \frac{\partial k_3}{\partial w} & \frac{\partial k_3}{\partial z} & \frac{\partial k_3}{\partial p} \\ \frac{\partial k_4}{\partial u} & \frac{\partial k_4}{\partial v} & \frac{\partial k_4}{\partial w} & \frac{\partial k_4}{\partial z} & \frac{\partial k_4}{\partial p} \\ \frac{\partial k_5}{\partial u} & \frac{\partial k_5}{\partial v} & \frac{\partial k_5}{\partial w} & \frac{\partial k_5}{\partial z} & \frac{\partial k_5}{\partial p} \end{vmatrix} = J_{11} = p(b + a.v) + \chi_c.w^2 \quad (29)$$

Where:

$$\begin{cases} J_{11} = \dfrac{\partial k_1}{\partial u} = p(b + a.v) + \chi_c.w^2 \\ J_{12} = \dfrac{\partial k_1}{\partial v} = p(a.u + b.z) \\ J_{13} = \dfrac{\partial k_1}{\partial w} = 2w(u.\chi_c + z\chi_s) + z\chi \\ J_{14} = \dfrac{\partial k_1}{\partial z} = p(b.v - a) + w(\chi + \chi_s.w) \\ J_{15} = \dfrac{\partial k_1}{\partial p} = u(b + a.v) - z(a + bv) \\ J_{ij} = 1 \quad \text{if } i = j \quad (i \geq 2 \text{ and } j \geq 1) \\ J_{ij} = 0 \quad \text{if } i \neq j \quad (i \geq 2 \text{ and } j \geq 1) \end{cases} \quad (30)$$

Using Theorem 3 the following equation can be written:

$$f_U(u) = \int_{\alpha_4}^{\beta_4}\int_{\alpha_3}^{\beta_3}\int_{\alpha_2}^{\beta_2}\int_{\alpha_1}^{\beta_1} f_{U,V,W,Z,P}(u,v,w,z,p)dvdwdzdp \tag{31}$$

In what follows, the integral bounds of the above equation are determined. As this integral has differentials dv, dw and dz and as v, w and z are the same functions k_2, k_3 and k_4, therefore, the bounds of the integral can also be obtained as functions k_2, k_3 and k_4. For this purpose, max is related to the bounds α_i and min is related to β_i.

$$\begin{cases} \alpha_1 = \max\left[k_{2\min}, \dfrac{k_{1\min} + z\left(ap - w\left(\chi - \chi_s.w\right)\right) - u\left(bp + \chi_c w^2\right)}{p\left(a.u + b.z\right)} \right] \\ \beta_1 = \min\left[k_{2\max}, \dfrac{k_{1\max} + z\left(ap - w\left(\chi - \chi_s w\right)\right) - u\left(bp + \chi_c w^2\right)}{p\left(a.u + b.z\right)} \right] \end{cases}$$

$$\begin{cases} \alpha_2 = \max\left[k_{3\min}, \min\left(F\left(u, k_{1\max}, k_{2\min}, z, p\right)\right)\right] \\ \beta_2 = \min\left[k_{3\max}, \max\left(F\left(u, k_{1\max}, k_{2\min}, z, p\right)\right)\right] \end{cases}$$

where:

$$F\left(u,k_1,k_2,z,p\right) = -\frac{\chi'}{2} \pm \frac{\sqrt{\Delta}}{2}$$

$$\chi' = \frac{\chi.z}{u.\chi_c + z.\chi_s}$$

$$\Delta' = \frac{u.p\left(a.k_2 + b\right) + z.p\left(b.k_2 - a\right) - k_1}{u.\chi_c + z.\chi_s}$$

$$\Delta = \chi'^2 - 4.\Delta'$$

$$\begin{cases} \alpha_3 = \max\left[k_{4\min}, \dfrac{u\left(p\left(b + a.k_{2\min}\right) + \chi_c.k_{3\min}^{\ 2}\right) - k_{1\max}}{p\left(a - b.k_{2\min}\right) - k_{3\min}\left(\chi - \chi_s.k_{3\min}\right)} \right] \\ \beta_3 = \min\left[k_{4\max}, \dfrac{u\left(p\left(b + a.k_{2\max}\right) + \chi_c.k_{3\max}^{\ 2}\right) - k_{1\min}}{p\left(a - b.k_{2\max}\right) - k_{3\max}\left(\chi - \chi_s.k_{3\max}\right)} \right] \end{cases}$$

$$\begin{cases} \alpha_4 = \max\left[k_{5\min}, \dfrac{k_{1\min} - k_{3\max}\left(k_{4\min}\left(\chi - \chi_s.k_{3\max}\right) - u.\chi_c.k_{3\max}\right)}{u\left(a.k_{2\max} + b\right) + k_{4\min}\left(b.k_{2\max} - a\right)} \right] \\ \alpha_4 = \min\left[k_{5\max}, \dfrac{k_{1\max} - k_{3\min}\left(k_{4\max}\left(\chi - \chi_s.k_{3\min}\right) - u.\chi_c.k_{3\min}\right)}{u\left(a.k_{2\min} + b\right) + k_{4\max}\left(b.k_{2\min} - a\right)} \right] \end{cases}$$

$$g_1(k_{1\min}, k_{2\max}, k_{3\max}, k_{4\min}, k_{5\max}) \le u = FS \le g_1(k_{1\max}, k_{2\min}, k_{3\min}, k_{4\max}, k_{5\min}) \tag{32}$$