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A UNIFIED APPROACH FOR COMPUTING PRESSURE DISTRIBUTION IN MULTI-OUTLET IRRIGATION PIPELINES*

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Abstract– A new analytical procedure taking into account the non-uniform outflow profile for hydraulic analysis and design of multiple outlets pipelines is presented. The method is developed based on presenting a new friction head loss distribution along the lateral. The proposed method simulates pressure and outflow profiles along the trickle or sprinkler irrigation laterals and manifolds, as well as gated pipes. The velocity head change was considered, whereas minor head losses were neglected. The presented technique was compared with the accurate step by step (SBS) method to justify its accuracy for lateral design. The comparison test for various design combinations indicated that the proposed method is sufficiently accurate. The suggested method could be applied in designing irrigation laterals.

Keywords– Pressure head distribution, lateral pipeline, head loss, non-uniform outflow

1. INTRODUCTION

Pressurized pipelines with multiple outlets are used extensively in irrigation systems (sprinkler, trickle or gated irrigation laterals and manifolds). A detailed hydraulic analysis of multiple outlets pipelines is very important for design and evaluation purposes. Nowadays, the increasing progress in computer technology has led to the development of numerical methods [1-4] which allow easy evaluation of head losses along a lateral. These models, however, have a high level of sophistication and require extensive programming and perhaps, long computer-execution time [5].

The previous hydraulic analyses are based on the assumption that outlet discharge along a lateral line is constant [6]. This case is represented by all types of solid-set, portable or mechanically movable linear sprinkler systems, as well as most trickle irrigation systems and gated pipes for surface irrigation [7]. This technique is simple, but is subject to certain errors as the energy grade line (EGL) is determined by the assumption of constant emitter outflow [8]. Based on this analysis of possible errors, for a turbulent flow emitter and an emitter flow variation equal to or less than 10%, the simple EGL approach is used for drip irrigation design. For non-turbulent flow emitters or when the emitter flow variation is larger than 10%, designs can be made by using an adjusted total discharge or revised energy gradient line (REGL) approach [8, 9]. The application of analytical EGL (or REGL) method is very easy; however, it has a limitation of application region due to constraints in emitter flow regimes and emitter flow variation along the lateral [10].

An alternative treatment in trickle laterals, presented by Yitayew and Warrick [11], uses a nonlinear, second-order, ordinary differential equation, which includes a spatially variable discharge function. It dismisses the assumption of uniform emitter flow along the lateral, as suggested by Keller and Bliesner [12] and Wu and Gitlin [13]. This method, however, requires numerical solution of a nonlinear, second-

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order differential equation. Vallesquino and Luque-Escamilla [14] and Vallesquino [15] proposed the successive-approximation method (SAM) for solving lateral hydraulic problems for laminar or turbulent flow. Yildirim [10] reported that the SAM method can be suitable for didactic applications since the solution requires long execution time and calculation effort.

A more practical analytical approach that takes into account the spatial variation of outlet discharge is presented by Valiantzas [5]. This method was then used in several researches for hydraulic design of irrigation pipelines [10, 16, 17].

In the present study, a new analytical form for the friction head loss function was presented. This is a new, simple, yet accurate analytical solution for designing multi-outlet irrigation laterals in different flow regimes and uniform slope cases.

2. THEORETICAL BACKGROUND

a) Traditional continuous approach- uniform outflows

Traditionally, flows in multiple outlet irrigation laterals and manifolds are considered to be steady and spatially varied flow. The discharge along the direction of flow decreases incrementally as the outlets, sprinklers, or emitters discharge water along the lateral. The hydraulics of these laterals and manifolds are usually evaluated by assuming a constant outflow q_{av} per unit length so that the flow rate at any position x is given by:

$$Q(x) = Q_{in}\left(\frac{x}{L}\right) \tag{1}$$

In Eq. 1, Q_{in} is the total flow rate into the inlet of the lateral and L is the lateral's total length.

The head loss gradient or slope of EGL at point x, $S_f(x)$, can be approximated by Darcy-Weisbach formula:

$$\frac{dH_f(x)}{dx} = S_f(x) = k\frac{Q^m(x)}{D^{m+3}} = \frac{f}{D}\times\frac{V^2}{2g} \tag{2}$$

where $H_f(x)$ is head loss at a distance x (m), k is roughness coefficient, f is friction head loss, D is lateral or manifold diameter (m), and m is an empirical constant.

For plastic pipes (PVC or PE),

$$k = \frac{a_1\upsilon^m\left(\frac{4}{\pi}\right)^m}{2g} \tag{3}$$

in which υ is kinematic viscosity and a_1 is a constant. For laminar flow (Reynolds Number R < 2000), m =1 and $a_1 = 0.64$; for turbulent flow in a smooth pipe ($3000 < R < 10^5$), $m = 1.75$ and $a_1 = 0.316$; for fully turbulent ($R > 10^5$), $m = 1.828$ and $a_1 = 0.13$.

For aluminum pipes, the appropriate formula such as well-known Churchill equation [18] can be used to estimate the friction coefficient f:

$$f = 8\left[\left(\frac{8}{R}\right)^{12} + \frac{1}{(P+Q)^{1.5}}\right]^{1/12} \tag{4}$$

where P and Q are empirical parameters for computing Darcy-Weisbach friction coefficient and are given as:

$$P=\left\{2.457\ln\left[\frac{1}{\left(\frac{7}{R}\right)^{0.9}+0.27\left(\frac{\varepsilon}{D}\right)}\right]\right\}^{16} \tag{5}$$

$$Q=\left(\frac{37530}{R}\right)^{16} \tag{6}$$

where parameter ε is absolute roughness height of the internal pipe surface.

Substituting the expression for $Q(x)$ in Eq. (1) into Eq. (2) and then integrating between the limits 0 and x yields:

$$H_f(x)=\frac{k}{m+1}\frac{Q^m}{D^{m+3}}\left(\frac{x}{L}\right)^{m+1}=\frac{H_{f0}}{m+1}\left(\frac{x}{L}\right)^{m+1} \tag{7}$$

where H_{fo} is total friction loss of a similar pipe transmitting the entire flow over its length. Thus, at the end of the lateral (x = 0), total head loss, H_f, is:

$$H_f=\frac{H_{f_0}}{m+1} \tag{8}$$

which allows Eq. (7) to be written as:

$$H_f(x)=H_f\left(\frac{x}{L}\right)^{m+1} \tag{9}$$

Christiansen [19] introduced the concept of a friction correction factor, F_c, to adjust H_{f_o} for multiple outlets pipes as follows:

$$H_f=F_C H_{f_0}=F_C\times k\frac{Q_{in}^m}{D^{m+3}}L \tag{10}$$

The value of F_c is determined by the following relation:

$$F_c=\frac{1}{m+1}+\frac{1}{2N}+\frac{\sqrt{m-1}}{6N^2} \tag{11}$$

where N is the number of outlets along the lateral assuming the first one is located one outlet spacing away from the lateral inlet.

b) Non-uniform outflow, ignoring outlet hydraulics

The traditional approach considers neither the hydraulic characteristics of the outlets themselves, nor the effect of lateral slope or conditions where flow regime- indicated by parameter m- might change along the lateral. Where pressure compensating emitters and sprinklers are used, the assumption of constant outlet flow is appropriate and the foregoing analysis can be applied (Walker, 2010, private communication). However, most systems employ emitters and sprinklers that do not have pressure compensating features and their discharge is related their hydraulic characteristics and the pressure distribution in the lateral pipe. Consequently, it is useful to develop a more general analytical concept.

Yidirim [10] as well as Valiantzas [20] approached this problem by assuming a non-linear distribution of outlet flows. However, an alternative that allows the inclusion of outlet hydraulic characteristics is to represent the friction head drop distribution along the lateral by a power function:

$$H_f(x) = H_f(\frac{x}{L})^{\varphi} \tag{12}$$

where φ is an empirical exponent. Following the same procedure used above to formulate Eq. (7):

$$\frac{d\lfloor H_f(x)\rfloor}{dx} = \frac{\varphi H_f}{L^{\varphi}} x^{\varphi-1} \tag{13}$$

Substitution of H_f from Eq. (10) into Eq. (13), equating with the first right hand side term of Eq. (2) and rearranging for $Q(x)$ yields:

$$Q(x) = (\varphi F_C)^{\frac{1}{m}} \times \left(\frac{x}{L}\right)^{\frac{\varphi-1}{m}} \times Q_{in} \tag{14}$$

It is worth noting that although H_f in Eq. (10) represents a constant outflow concept, its substitution into Eq. (13) is logical since a recent study by Sadeghi et al. [21] has demonstrated that the non-uniformity of outflow along the lateral does not significantly change the value of F_c given by Christiansen [19].

Considering Eq. (14), at the pipe inlet ($x = L$), $Q(x) = Q_{in}$ and ignoring the hydraulic characteristics of the outlets, it is seen that:

$$\varphi = \frac{1}{F_C} \tag{15}$$

Equation (15) demonstrates that the nonlinear distribution of flow along the lateral (i.e., Eq. (14)) depends not only on the flow regime, but also on the number of outlets. With the value of φ defined by Eq. (15), the resulting form of Eq. (14) is:

$$Q(x) = Q_{in}\left(\frac{x}{L}\right)^{\lambda} \tag{16}$$

and,

$$\lambda = \frac{\varphi - 1}{m} \tag{17}$$

As noted, λ allows Eq. (16) to simulate the lateral pipe flow when the individual outlets do not have a uniform discharge. However, when $\lambda = 1$, Eq. 16 is the same as Eq. (1). Thus, $\varphi = m+1$ when the outlets have a uniform discharge along the lateral and $\varphi = 1/F_c$ when the uniform discharge assumption is invalid.

Figure 1 demonstrates distribution of flow in a lateral pipe as a function of number of outlets, each discharging an equal flow (Eq. (1)). Also, in this figure, the non-uniform flow distribution described by Eq. (16), as well as the recent contributions by Valiantzas [20], are shown. It can be observed that the profile presented by Eq. (16) lays between Eq. (1) and that of Valiantzas [20]. But when the number of outlets reaches 30, the three models are nearly the same and are indistinguishable at N = 100.

The friction loss variation along a lateral for various numbers of outlets (N= 2, 5 and 10) is shown in Fig. 2 for four models: Eq. (9), Eq. (10), Eq. (12), and direct stepwise procedure (SBS) with a uniform outflow assumption. Again, it can be observed that there exists a slight difference between friction loss gradient assuming a non-uniform outlet flow and the stepwise method assuming a uniform outlet flow. These differences become negligible as the number of outlets increases.

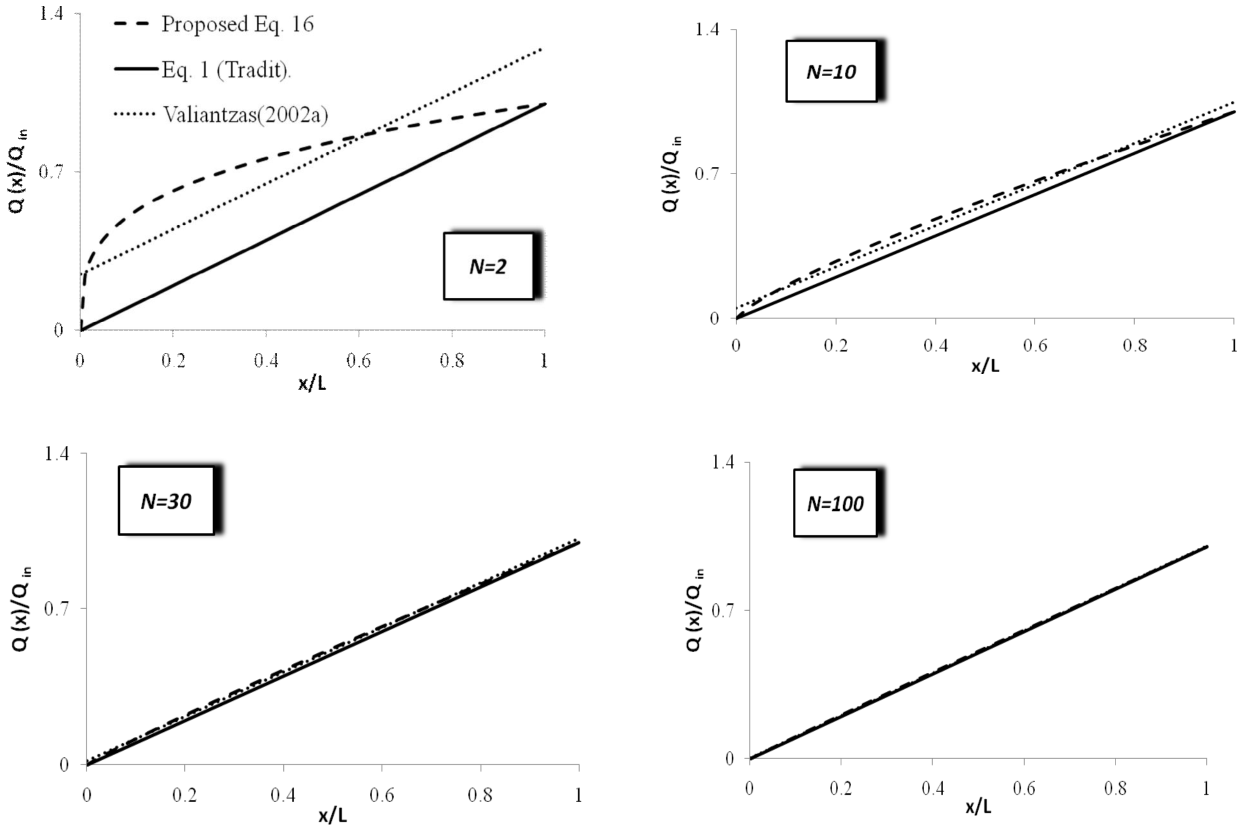

Fig. 1. Outflow profiles by the proposed function and traditional approach for N= 2, 10, 30 and 100 and m=2

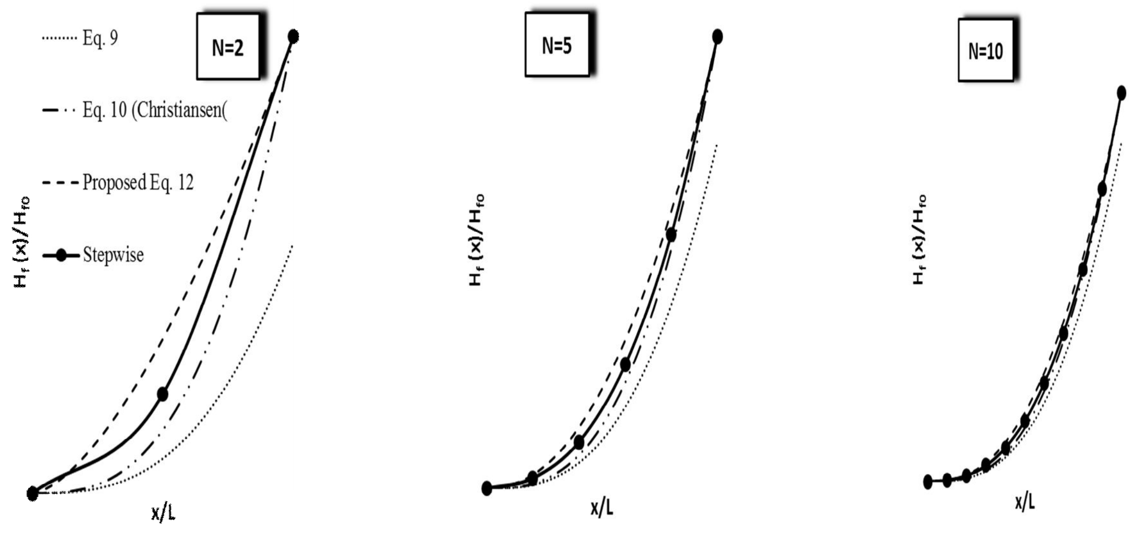

Fig. 2. Friction head loss function by the proposed equation and traditional approach (N=2, 5 and10 and m=1.852)

c) Non-uniform outflows, including outlet hydraulics

The pressure head H at any point x along the lateral is given by:

$$H(x) = H_d + H_f(x) - S_0 x - H_v(x) \tag{18}$$

where H_d is pressure head at the downstream end of the lateral pipe, $H_v(x)$ is velocity head gradient and S_0 is ground slope, which is assumed to be uniform along the lateral length. Laterals laid on downhill slope have positive values for S_0, since x is measured from the downstream end of the lateral. If it is initially assumed that $\varphi = 1/F_c$ and V_{in} represents the average flow velocity at the lateral inlet (m/sec), then from Eq. 16 the flow velocity distribution can be written as:

$$V(x) = \left(\frac{x}{L}\right)^{\lambda} \times V_{in} \tag{19}$$

The velocity head (m) in the lateral pipe, $H_v(x)$, can therefore be expressed as:

$$H_v(x) = \frac{V^2(x)}{2g} = \frac{V_{in}^2}{2g}\left(\frac{x}{L}\right)^{\frac{2\varphi-2}{m}} \tag{20}$$

The average pressure head along the lateral can be found by substituting Eqs. (12) and (20) into Eq. (18), integrating each term between 0 and L, and then dividing by L:

$$H_{av} = H_d + \frac{H_f}{1+\varphi} - \frac{V_{in}^2}{2g}\left(\frac{m}{2\varphi+m-2}\right) - S_0\frac{L}{2} \tag{21}$$

Subtracting Eq. (21) from Eq. (18) gives the distribution of pressure head in the lateral as a function of average pressure head:

$$H(x) = H_{av} + H_f\left[\left(\frac{x}{L}\right)^{\varphi} - \frac{1}{1+\varphi}\right] + S_0\left(\frac{L}{2} - x\right) + \frac{V_{in}^2}{2g}\left(\frac{m}{2\varphi+m-2} - \left(\frac{x}{L}\right)^{\frac{2\varphi-2}{m}}\right) \tag{22}$$

Equation (22) defines pressure head along an irrigation lateral and demonstrates the dependence of this head on the number of outlets ($\varphi = 1/F_c$) and flow regime, m. The question remains of what impact might outlet hydraulics have on distribution of pressure and flow along the lateral?

The actual value of λ can be determined directly from Eq. (16). Following the suggestion of Valiantzas [5], Eq. (16) can be written for the lateral midpoint and then solved for λ. The resulting expression is:

$$\lambda = LOG\left(\frac{Q_{0.5}}{Q_{in}}\right) \times \frac{1}{LOG0.5} \tag{23}$$

where, $Q_{0.5}$ is total discharge entering the lower half of the lateral (m^3/sec). The average value of pressure head over the lower half of lateral, $H_{0.5}$, can be derived by integrating Eq. (22) between 0 and $L/2$, dividing by $L/2$ and substituting $1/F_c$ for of φ:

$$H_{0.5} = H_{av} + \frac{F_C H_f}{1+F_C}\left(0.5^{\frac{1}{F_C}} - 1\right) + S_0\left(\frac{L}{4}\right) + \frac{mF_C V_{in}^2}{(4+2mF_C-4F_C)g}\left(1 - 0.5^{\frac{2-2F_C}{mF_C}}\right) \tag{24}$$

For simplicity, Eq. 24 can be rewritten as:

$$H_{0.5} = H_{av} + AH_f + S_0\left(\frac{L}{4}\right) + \frac{BV_{in}^2}{g} \tag{25}$$

where:

$$A = (1+\phi)(0.5^{\phi} - 1) \tag{26}$$

and,

$$B = \frac{m}{(4\phi + 2m - 4)}\left(1 - 0.5^{\frac{2\phi-2}{m}}\right) \tag{27}$$

The discharges through most pipe outlets that are not pressure-regulated are generally governed by an orifice relationship of the following type:

$$q = \frac{Q_{in}}{N} = cH^{y} \tag{28}$$

In Eq. (28), q is discharge through an outlet, H is opening pressure head of the outlet, and c and y are rating coefficients. The average outlet flow from the lower half of the lateral, $q_{av}|_{0.5}$, can be estimated by the average pressure head $H_{av}|_{0.5}$ as:

$$q_{av}\big|_{0.5} = cH^{y}_{av0.5} \tag{29}$$

On the other hand, total discharge, $Q_{0.5}$, passing through the lateral section located at distance $x_0=L/2$ represents the summation of discharge of all emitters located between x=0 and $x=L/2$. Consequently it can be calculated as:

$$Q_{0.5} = \left(\frac{N}{2}\right) q_{av}\big|_{0.5} \tag{30}$$

Substituting Eq. (25) in Eq. (29) and then substituting the result in Eq. (30) yields:

$$Q_{0.5} = \left(\frac{cN}{2}\right)\left(H_{av} + AH_f + S_0\left(\frac{L}{4}\right) + \frac{BV_{in}^2}{g}\right)^{y} \tag{31}$$

Total discharge at the inlet of the lateral is calculated as a function of average outlet flow along the entire lateral, q_{av}, and the total number of outlets:

$$Q_{in} = Nq_{av} = NcH^{y}_{av} \tag{32}$$

Finally, the actual or adjusted value of λ, λ_{adj}, is derived by substituting $Q_{0.5}$ and Q_{in} from Eqs. (31) and (33) into Eq. (23):

$$\lambda_{adj} = Log\left[0.5\left(\frac{H_{av} + AH_f + S_0\left(\frac{L}{4}\right) + \frac{BV_{in}^2}{g}}{H_{av}}\right)^{y}\right] \times \frac{1}{Log 0.5} \tag{33}$$

Similarly, an actual or adjusted value of φ can be determined by substituting λ_{adj} for λ of Eq. (33) into Eq. (17):

$$\varphi_{adj} = 1 - 3.22my\log\left\{0.5\left[\left(\frac{H_{av} + AH_f + \frac{S_0 L}{4} + \frac{BV_{in}^2}{2g}}{H_{av}}\right)^{y}\right]\right\} \tag{34}$$

Distribution of energy and discharge along the lateral can now be defined using the values of λ_{adj} and φ_{adj} by modifying Eqs. (12) and (16):

$$H_f(x) = H_f\left(\frac{x}{L}\right)^{\varphi_{adj}} = F_C H_{f_0}\left(\frac{x}{L}\right)^{\varphi_{adj}} \tag{35}$$

$$Q(x) = Q_{in}\left(\frac{x}{L}\right)^{\frac{\varphi_{adj}-1}{m}} \tag{36}$$

d) Modified EGL

From the above analysis it may be concluded that for a pipeline with multiple outlets the energy line can be determined by Eq. 22, if φ is replaced by φ_{adj} and the following data are known: diameter D, roughness coefficient in Eq. 2, slope S_0, number of outlets N, spacing s and the outlet discharge coefficients c and y.

e) Determination of uniformity coefficients (U_c and DU_{LQ})

The Christiansen coefficient of uniformity, U_C, is used here to express uniformity of outlet discharge. Using statistical analysis, Valiantzas [5] has shown that the coefficient of variation of outlet discharge is related to the variance of pressure head along the lateral as:

$$CV = \frac{y\sqrt{H_{\text{var}}}}{H_{av}} \tag{37}$$

where $CV = (q_{\text{var}})^{1/2}/q_{\text{av}}$ is coefficient of variation, and q_{var} is variance of outlet discharge along the lateral. The U_C is related to CV by:

$$U_c = 1 - 0.798CV \tag{38}$$

The variance of pressure head profile between the pipe segments $x = 0$ and $x = L$ can be expressed as [22]:

$$H_{\text{var}} = \frac{1}{L}\int_{x=0}^{x=L} DEV_x^2 dx = \frac{1}{L}\int_{x=0}^{x=L} \left(H(x) - H_{av}\right)^2 dx \tag{39}$$

where DEV_x is deviation between $H(x)$ and H_{av} at any distance x from the closed end of the pipeline. Substituting H_{av} from Eq. 22 into Eq. (39) and then solving the integration yields:

$$\begin{aligned} H_{\text{var}} = {} & H_f^2\left[\frac{1}{2\varphi_{adj}+1} - \frac{1}{(1+\varphi_{adj})^2}\right] + \frac{(S_0L)^2}{12} + \left(\frac{V_{in}^2}{2g}\right)\left[\frac{m}{4\varphi_{adj}+m-4} - \left(\frac{m}{2\varphi_{adj}+m-2}\right)^2\right] + \\ & H_f S_0 L\left[\frac{1}{\varphi_{adj}+1} - \frac{2}{\varphi_{adj}+2}\right] + 2m\left(\frac{V_{in}^2}{2g}\right)H_f\left[\frac{1}{2\varphi_{adj}^2 + m\varphi_{adj} + m - 2} - \frac{1}{2\varphi_{adj} + m\varphi_{adj} + m - 2}\right] + \\ & mLS_0\left(\frac{V_{in}^2}{2g}\right)\left[\frac{1}{\varphi_{adj}+m-1} - \frac{1}{2\varphi_{adj}+m-2}\right] \end{aligned} \tag{40}$$

Then, the coefficient of variation will be expressed as:

$$CV = y\frac{\left[\begin{array}{l} H_f^2\left[\dfrac{1}{2\varphi_{adj}+1}-\dfrac{1}{(1+\varphi_{adj})^2}\right]+\dfrac{(S_0L)^2}{12}+\left(\dfrac{V_{in}^2}{2g}\right)\left[\dfrac{m}{4\varphi_{adj}+m-4}-\left(\dfrac{m}{2\varphi_{adj}+m-2}\right)^2\right]+ \\ H_f S_0 L\left[\dfrac{1}{\varphi_{adj}+1}-\dfrac{2}{\varphi_{adj}+2}\right]+2m\left(\dfrac{V_{in}^2}{2g}\right)H_f\left[\dfrac{1}{2\varphi_{adj}^2+m\varphi_{adj}+m-2}-\dfrac{1}{2\varphi_{adj}+m\varphi_{adj}+m-2}\right]+ \\ mLS_0\left(\dfrac{V_{in}^2}{2g}\right)\left[\dfrac{1}{\varphi_{adj}+m-1}-\dfrac{1}{2\varphi_{adj}+m-2}\right] \end{array}\right]^{0.5}}{H_{av}} \tag{41}$$

For normal distribution, the relationship between U_C and lower-quarter distribution uniformity coefficient, DU_{LQ}, is given by the following relationship [11]:

$$U_c = 0.37 + 0.63DU_{LQ} \tag{42}$$

Substituting the expression for U_C given by Eq. (38) into Eq. (42) and rearranging for DU_{LQ}, the following simple transformation between DU_{LQ} and coefficient of variation of discharge (CV) can be deduced:

$$DU_{LQ} = 1 - 1.267CV \tag{43}$$

f) Determination of inlet pressure head

Consider a lateral line with all outlets being equally spaced except for the first one, which is some fraction, r, of the common outlet spacing s from the inlet:

$$s_{in} = rs \tag{44}$$

where s_{in} is distance from the inlet to the first outlet and r is the ratio ($0 < r < 1$), usually taken as 0.5 or 1 for typical installation [2, 12]. The inlet pressure head is obtained by Eq. 22 for $(x/L) = [(N-1)s+rs]/(Ns) = (N+r-1)/N$. Therefore:

$$\begin{aligned} H_{in} = H_{av} + H_f\left(\left(\frac{N+r-1}{N}\right)^{\varphi_{adj}} - \frac{1}{1+\varphi_{adj}}\right) - S_0L\left(\frac{N-2+2r}{2N}\right) \\ + \frac{V_{in}^2}{2g}\left(\frac{m}{2\varphi_{adj}+m-2} - \left(\frac{N+r-1}{N}\right)^{\frac{2\varphi_{adj}-2}{m}}\right) \end{aligned} \tag{45}$$

g) Maximum–minimum outlet pressure head

Considering Eqs. (12) and (20), Eq. (18) can be written as:

$$H(x) = H_d + H_f\left(\frac{x}{L}\right)^{\varphi_{adj}} - S_0L\left(\frac{x}{L}\right) + \frac{V_{in}^2}{2g}\left(\frac{x}{L}\right)^{\frac{2\varphi_{adj}-2}{m}} \tag{46}$$

For downhill slope, the point of minimum outlet discharge (and consequently of minimum pressure head) is found when the first derivative of $H(x)$ in Eq. (46) is set to zero. Neglecting the velocity head, the location x_{min}, where minimum pressure head occurs is:

$$x_{\min} = L\left(\frac{S_0 L}{\varphi_{adj} H_f}\right)^{\frac{1}{\varphi_{adj}-1}} \tag{47}$$

Because x=0 refers to the first outlet, the following equation should be used to obtain the number of outlet which acts with minimum pressure:

$$N_{\min} = [x_{\min}] + 2 \tag{48}$$

In special cases:
If $x_{\min} = 0$, then $N_{\min} = 1$
If $x_{\min} \geq L$, then $N_{\min} = N$

Maximum pressure head is located either in the first outlet (x=L) or at the end point (x=0). In the case of uphill and zero slope, the point of minimum pressure is always at the closed end of the pipeline (x=0).

h) Location of the average pressure head

If the location of the pressure head along the lateral is denoted by x_{av}, then Eq. (22) should give H_{av} for $x = x_{av}$. Simplifying H_{av} from both sides, we obtain:

$$H_f\left[\left(\frac{x_{av}}{L}\right)^{\varphi} - \frac{1}{1+\varphi}\right] - S_0 L\left[\left(\frac{x_{av}}{L}\right) - \frac{1}{2}\right] + \frac{V_{in}^2}{2g}\left[\frac{m}{2\varphi + m - 2} - \left(\frac{x_{av}}{L}\right)^{\frac{2\varphi-2}{m}}\right] = 0 \tag{49}$$

Note that Eq. (49) is implicit and requires a trial and error or some other iterative techniques, such as Newton-Raphson, for solution. However, most pocket programmable calculators can easily solve it. Considerable simplification results if the lateral is laid on level ground (S_0=0), φ is replaced by $1/F_C$ and the velocity term is neglected. In this case, there is a direct solution for each value of m and N and the location of average pressure head is given by the following equation:

$$x_{av} = L\left(\frac{F_c}{1+F_c}\right)^{F_c} \tag{50}$$

Table 1 shows a comparison between values of x_{av}/L calculated by Eq. (50) and proposed coefficients by Scaloppi and Allen (7) for three values of m. Note that the location of average pressure head is determined from the inlet for comparison. It is evident that considering the traditional form of friction head drop ($F=1/m+1$) for determining the location of average pressure head leads to a significant error when number of outlets along the lateral is small. As an example, when 1< N <10, for m=2, the relative error is 26% to 5%. However, errors decrease as total number of outlets increase since the traditional F and the Christiansen's F factor approach each other.

Conducting numerical analysis, Scaloppi and Allen [7] comprehended that their approach is incapable of determining the location x_{av} when N < 6. Anyhow, this conclusion can be explained by the values of Table 1. The other point here is that the average pressure head is always located at the middle of the lateral when N=1 without depending on the kind of the flow regime.

i) Pressure head variation

To maintain a coefficient of uniformity of about 97%, total pressure variation in the lateral with outlets is typically limited to 20% [12]. This criterion can be written as shown below:

$$\delta_H = \frac{\Delta H}{H_{av}} = \frac{H_{max} - H_{min}}{H_{av}} \tag{51}$$

Where δ_H is percentage of the maximum allowable difference in the outlet operating pressure head to the average outlet pressure head, ΔH *is* maximum allowable difference in the outlet operating pressure head along the lateral, and H_{max} and H_{min} are maximum and minimum values of the operating outlet pressure head, respectively.

Table 1. Location of the average pressure head for a horizontal lateral with uniform outflow (Scaloppi and Allen 1993) and non-uniform outflow conditions (Eq. (50))

m =1				*m* =1.852				*m* =2			
Number of Outlets	$\frac{x_{av}}{L}$ (Eq. 42)	$\frac{x_{av}}{L}$ (Scaloppi and Allen, 1993)	Relative error (%)	Number of Outlets	$\frac{x_{av}}{L}$ (Eq. 42)	$\frac{x_{av}}{L}$ (Scaloppi and Allen, 1993)	Relative error (%)	Number of Outlets	$\frac{x_{av}}{L}$ (Eq. 42)	$\frac{x_{av}}{L}$ (Scaloppi and Allen, 1993)	Relative error (%)
1	0.500	0.423	15.40	1	0.500	0.377	24.60	1	0.50	0.37	26.00
2	0.470	0.423	10.06	2	0.452	0.377	16.61	2	0.45	0.37	17.70
3	0.457	0.423	7.46	3	0.431	0.377	12.49	3	0.43	0.37	13.39
4	0.450	0.423	5.92	4	0.419	0.377	10.00	4	0.41	0.37	10.74
5	0.445	0.423	4.90	5	0.411	0.377	8.32	5	0.41	0.37	8.98
6	0.442	0.423	4.19	6	0.406	0.377	7.14	6	0.40	0.37	7.71
7	0.439	0.423	3.64	7	0.402	0.377	6.24	7	0.40	0.37	6.75
8	0.437	0.423	3.23	8	0.399	0.377	5.54	8	0.39	0.37	6.00
9	0.436	0.423	2.89	9	0.397	0.377	4.97	9	0.39	0.37	5.40
10	0.434	0.423	2.62	10	0.395	0.377	4.51	10	0.39	0.37	4.91
15	0.431	0.423	1.76	15	0.389	0.377	3.08	15	0.38	0.37	3.39
20	0.429	0.423	1.33	20	0.386	0.377	2.33	20	0.38	0.37	2.58
30	0.427	0.423	0.87	30	0.383	0.377	1.57	30	0.38	0.37	1.75
40	0.426	0.423	0.63	40	0.382	0.377	1.18	40	0.38	0.37	1.33
60	0.425	0.423	0.40	60	0.380	0.377	0.76	60	0.37	0.37	0.91
80	0.424	0.423	0.28	80	0.379	0.377	0.55	80	0.37	0.37	0.67
100	0.424	0.423	0.21	100	0.379	0.377	0.45	100	0.37	0.37	0.54
500	0.423	0.423	0.00	500	0.377	0.377	0.05	500	0.37	0.37	0.11
1000	0.423	0.423	0.00	1000	0.377	0.377	0.00	1000	0.37	0.37	0.05

Wu [8] proposed the following uniformity parameters, given by Eq. (52), to alternate Eq. (36), which can be specifically used as design criteria for trickle irrigation sub-main units. The uniformity criteria δ_H and δ_q (emitter flow variation) along a sub-main line in trickle irrigation system is given by:

$$\delta_H = \frac{\Delta H}{H_{max}} = 1 - \frac{H_{min}}{H_{max}}, \delta_q = \frac{\Delta q}{q_{max}} = 1 - \frac{q_{min}}{q_{max}}, \delta_q = 1 - (1 - \delta_H)^y \tag{52}$$

Where δ_q is emitter flow variation along the lateral line or in a sub-main unit in trickle irrigation system, q_{max} and q_{min} are maximum and minimum values of the emitter outflow, respectively, and $\Delta q = q_{max} - q_{min} =$ maximum allowable difference in the emitter outflow along a sub-main line.

Equation (52) shows that δ_q is essentially equivalent to δ_H for the laminar flow emitters (y =1). The criteria of hydraulic design are usually set as 10 and 20% emitter outflow variation, δ_q, which is equivalent to values of approximately 20 and 40% of δ_H for an emitter in turbulent flow or sprinkler irrigation system with y = 0.5 [8].

The maximum difference in pressure head is obtained by Eq. (22) as:

$$\Delta H = H_f\left[\left(\frac{x_{max}}{L}\right)^{\varphi_{adj}} - \left(\frac{x_{min}}{L}\right)^{\varphi_{adj}}\right] - S_0 L\left[\left(\frac{x_{max}}{L}\right) - \left(\frac{x_{min}}{L}\right)\right] + \frac{V_{in}^2}{2g}\left[\left(\frac{x_{max}}{L}\right) - \left(\frac{x_{min}}{L}\right)\right]^{\frac{2\varphi_{adj}-2}{m}} \tag{53}$$

3. APPLICATION AND VERIFICATION

An example is given here to illustrate the application of the proposed model in design of multiple outlet pipelines.

a) Design example

Determine the pressure head and outflow profiles along the pipeline for non-uniform outflow concept and the corresponding flow characteristics for a horizontal (S_0=0) polyethylene trickle irrigation lateral with turbulent emitters (x=0.54), using the following data. Total number of emitters N=151, which are equally spaced at s =1.0 m. The inlet spacing ratio from the main pipe r =1.0. The internal diameter of the lateral is D =14 mm. The required average emitter discharge and corresponding nozzle pressure head are q_{av} = 2.0 L h^{-1} (=5.555×10^{-7} m^3 s^{-1}) and H_{av} = 7.2 m, respectively. Irrigation water is at 20°C (kinematic viscosity υ =1.01×10^{-6} m^2 s^{-1}). Acceleration due to gravity g = 9.807 m s^{-2}.

b) Solution

Inlet flow rate $Q_{in}=Nq_{av}$ =8.389×10^{-5} m^3 s^{-1}; the kinetic head $V^2/2g=8(Nq_{av})^2/\pi^2 gD^4$ = 0.015 m; the inlet Reynolds number $R_{in}=(4\,Nq_{av}/\pi D\upsilon)$=7,554; the distance between first and last emitters L_0=150 m; total lateral length L=151 m. The turbulent flow (3,000 < R_{in} = 7,554 < 10^5) occurs at the inlet of trickle lateral. (a_1 =0.316, m=1.75, n=4.75, k=7.792×10^{-4}). So, $H_{f_0} = k\dfrac{Q_{in}^m}{D^{m+3}}L = 5.533m$. The Christiansen F factor for m =1.75 and N =151 is 0.3669 (Eq. (11)). Then, from Eqs. (15) and (34), we have $\varphi=1/F_C$ = 2.7251 and φ_{adj} = 2.8368.

Other calculation steps of the proposed analytical solution are summarized in Table 2. Note that $H_{f\,(L0)}$ corresponds to total friction drop between the first and last outlets and can easily be calculated by putting x =L-s =151-1 in Eq. (12).

Table 2. Calculation steps of the proposed analytical method for Design Example

Design parameter	Proposed analytical method		Calculated By	SBS
	φ=2.7251	φ_{adj}=2.8368		
H_{in} (m)	8.68	8.69	Eq. (45)	8.86
H_{max} (m)	8.64	8.65	-	8.86
H_{min} (m)	6.66	6.68	-	6.79
δ_H (%)	22.91	22.85	Eq. (52)	21.9
$H_{f\,(L0)}$ (m)	1.99	2.03	Eq. (12)	1.93
δ_q (%)	13.1	13.1	Eq. (52)	12.5
CV (%)	4.4	4.3	Eq. (41)	4.1
U_C (%)	96.5	96.5	Eq. (42)	96.7
DU_{LQ} (%)	94.5	94.5	Eq. (43)	94.8

It is observed that solving the example with the proposed method yields to an accurate estimation of hydraulic design parameters. Solving the example with the traditional φ ($\varphi=1/F_C$) also gives acceptable results, even though its accuracy is not permanent since it is independent of the slope, the average pressure head along the lateral and the kind of the emitters.

In order to show validity of the new approach, the variation of pressure head, outlets discharge, outflow along the lateral and also the Reynolds number are plotted against the relative length (x/L) in Fig. 3. It can be deducted that all profiles follow the SBS method accurately. However, in order to do a detailed comparative analysis, the solution of the Design Example is extended to cover various combinations of design parameters, varying emitter discharge exponents (x = 0.2, 0.5, 0.54, and 1.0) and different uniform pipeline slopes (S_0= 0.0, −0.02, −0.05, 0.02, and 0.05). The complete results are presented in Table 3. As shown in Table 3, for all design cases the results obtained from the analytical solution and those of the

SBS numerical method are in good correlation. For example, Table 3 shows that for S=0.02 and y=0.2, the pressure head at the inlet (H_{in}), pressure head at the first and the last outlets (H_{max} , H_{min}) and pressure head at the end of the lateral (H_d) are 7.17, 8.17, 6.82 and 8.17 m, whereas the SBS method yielded 7.14, 8.12, 6.75 and 8.12 m respectively.

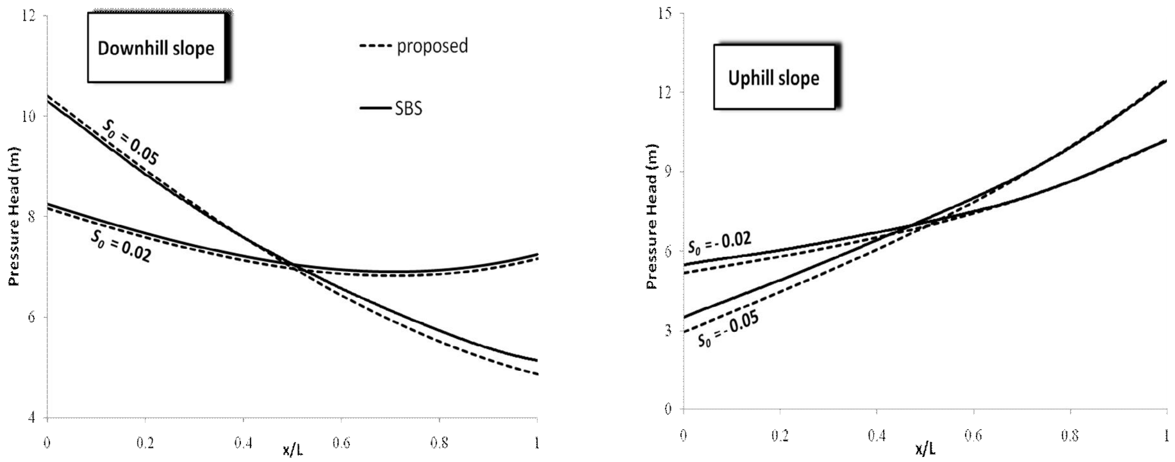

Fig. 3. Pressure head profile for Design Example with respect to the distance ratio from lateral end for downhill and uphill conditions (± 0.02 and ± 0.05), emitter discharge exponent y =0.54, obtained from the presented analytical solution and the numerical SBS solution

Table 3. Comparison of hydraulic design parameters based on the proposed analytical method and the numerical step-by-step (SBS) method (Hathoot et al., 1993) for various combinations of uniform slopes and different emitter parameters

Slope		Proposed analytical method									Numerical SBS method								
(S_0)	y	H_{in}	H_{max}	H_{min}	H_1	N_{min}	$H_{f(L_0)}$	δ_q	CV	U_C	H_{in}	H_{max}	H_{min}	H_1	N_{min}	$H_{f(L_0)}$	δ_q	CV	U_C
0.00	0.2	8.68	8.65	6.67	6.67	1	2.03	5.1	1.6	98.7	8.86	8.86	6.91	6.91	1	1.97	4.80	1.5	98.8
	0.5	8.69	8.65	6.67	6.67	1	2.03	12.2	4.0	96.8	8.7	6.7	6.78	6.78	1	1.94	11.7	3.9	96.9
	0.54	8.69	8.65	6.68	6.68	1	2.03	13.1	4.3	96.5	8.7	8.7	6.79	6.79	1	1.93	12.5	4.1	96.7
	1	8.70	8.66	6.69	6.69	1	2.03	22.8	8.0	93.6	8.57	8.57	6.69	6.69	1	1.89	21.9	7.9	93.7
0.02	0.2	7.17	8.17	6.82	8.17	108	2.03	3.6	1.1	99.1	7.14	8.12	6.75	8.12	108	2.04	3.60	1.1	99.1
	0.5	7.16	8.17	6.83	8.15	108	2.03	8.5	2.7	97.9	7.26	8.10	6.93	8.10	109	2.17	7.50	2.9	97.7
	0.54	7.16	8.17	6.82	8.17	108	2.03	9.3	2.9	97.6	7.25	8.23	6.87	8.23	109	2.03	9.00	2.9	97.7
	1	7.15	8.16	6.83	8.16	108	2.03	16.3	5.4	95.7	7.24	8.04	6.87	8.04	108	2.22	14.5	5.7	95.4
0.05	0.2	4.89	10.43	4.91	10.43	151	2.03	14.0	4.6	96.3	5.17	10.66	5.17	10.66	151	2.03	13.5	5.0	96.0
	0.5	4.87	10.40	4.89	10.40	151	2.03	31.5	11.4	90.9	5.11	10.37	5.11	10.37	151	2.25	29.8	12.9	89.7
	0.54	4.87	10.40	4.88	10.40	151	2.03	33.5	12.3	90.2	5.10	10.37	5.10	10.37	151	2.23	31.8	12.9	89.7
	1	4.83	10.36	4.85	10.36	151	2.02	53.2	22.6	82.0	5.02	10.54	5.02	10.54	151	1.99	52.4	23.8	81.0
-0.02	0.2	10.20	10.14	5.17	5.17	1	2.03	12.6	4.0	96.8	10.44	10.44	5.56	5.56	1	1.90	11.8	4.0	96.8
	0.5	10.22	10.16	5.18	5.18	1	2.03	28.6	9.8	92.2	10.21	10.21	5.39	5.39	1	1.83	27.3	10.0	92.0
	0.54	10.22	10.16	5.19	5.19	1	2.03	30.5	10.6	91.5	10.15	10.15	5.35	5.35	1	1.82	29.2	10.8	91.4
	1	10.25	10.18	5.21	5.21	1	2.03	48.8	19.5	84.4	10.62	10.62	5.33	5.33	1	2.31	49.8	19.7	84.3
-0.05	0.2	12.48	12.39	2.92	2.92	1	2.03	25.1	7.5	94.0	12.89	12.89	3.44	3.44	1	1.97	23.2	7.9	93.7
	0.5	12.52	12.42	2.95	2.95	1	2.03	51.3	18.7	85.0	12.51	12.51	3.36	3.36	1	1.66	48.2	19.7	84.3
	0.54	12.52	12.43	2.96	2.96	1	2.03	54.0	20.2	83.9	12.40	12.40	3.26	3.26	1	1.66	51.4	21.2	83.1
	1	12.57	12.47	3.00	3.00	1	2.03	75.9	37.1	70.4	12.94	12.94	3.08	3.08	1	2.38	76.2	38.1	69.6

For the friction head loss between the first and last emitters, $H_{f(L0)}$, the proposed method gave 2.03, whereas the SBS method yielded 2.04 m.

Position of the outlet which possesses minimum pressure head, N_{min}, in Table 3 is calculated from Eqs. (47) and (48), respectively. As an example, for S_0=0.02 and y=0.54, φ_{adj} is 2.692. Therefore using Eq. (47):

$$x_{\min} = 151 \times \left(\frac{0.02 \times 151}{2.692 \times 2.03} \right)^{\frac{1}{2.692-1}} = 106.354m$$

Then from Eq. (48) we have

$$N_{\min} = [106.354] + 2 = 108$$

Finally, Table 3 illustrates that parameters δ_q, CV and U_C are in excellent accordance with the SBS method in all cases.

The pressure head distribution for $S_0 = \pm 0.02$ and $S_0 = \pm 0.05$ (calculated from Eq. 22) is plotted against the SBS method in Fig. 3 to prove the validity of the model. This figure shows that the proposed method is accurate enough for different slopes of the lateral pipeline.

4. CONCLUSION

A new linear relationship describing the total flow rate variation along laterals (in sprinkler or trickle irrigation systems) is proposed. Simple analytical equations describing friction head loss and velocity head variation along laterals in which the number of outlets and the flow regime are taken into account are also developed. The various parameters characterizing the outflow profile along the lateral are calculated by simple analytical expressions and compared with the accurate SBS method. The comparison showed that the proposed method is sufficiently accurate in practical applications.

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