

## ACCURACY EVALUATION OF THE MODAL PUSHOVER ANALYSIS METHOD IN THE PREDICTION OF SEISMIC RESPONSE OF VERTICALLY IRREGULAR FRAMES\*

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**Abstract**– The traditional pushover analysis procedure does not represent adequately, the effects of varying dynamic characteristics caused by structural yielding and the contributions of higher modes to the structural responses. The recently developed modal pushover analysis (MPA) procedure significantly improves the traditional pushover analysis by including the effects of a sufficient number of modes in the analysis.

The present study evaluates the accuracy of the modal pushover analysis in estimating the seismic demands of vertically irregular planar moment-resisting frames in comparison with the exact results from nonlinear time-history analysis (NLTHA). In this research, seven irregular 12-story frames representing three types of height-wise irregularities: mass irregularity, vertical geometric irregularity (i.e., setback) and difference in floor levels, and also one regular 12-story frame as a reference are analyzed due to an ensemble of seven ground acceleration records. The earthquake records are chosen such that a wide range of frequency contents, as well as the effects of both near and far fault distances, be taken into account in the analysis. Findings from this investigation indicate that in most of the irregular models considered, the method of MPA can reasonably estimate the values of critical parameters such as peak floor displacement and story-drift ratios. Furthermore, it is observed that the inclusion of the first two modes improves the accuracy of predictions relative to using only the fundamental mode in the analysis. Such degree of improvement in accuracy is not reached by including the third mode effects in the MPA procedure.

**Keywords**– Modal pushover analysis, nonlinear time history analysis, vertically irregular buildings

### 1. INTRODUCTION

Current seismic design practice is still governed by force-based design principles. However, the emergence of performance-based seismic engineering has resulted in increasing use of nonlinear methods to estimate expected seismic demands in a building structure. While nonlinear time history analysis (NLTHA) is the most rigorous procedure available to estimate seismic demands for buildings, it remains impractical for widespread use by the professionals due to the high analysis-cost.

Due to the above reasons, in recent years, structural engineering professions prefer to use the nonlinear static procedure or pushover analysis described in FEMA-273, FEMA-356 and ATC-40 [1-3].

Krawinkler and Seneviratna [4] have shown that the pushover analysis provides good predictions of seismic demands for low-rise structures having uniform distribution of inelastic behavior over height. In this procedure, none of the invariant force distributions account for the contributions of higher modes to response or redistribution of inertia forces because of structural yielding and associated changes in the vibration properties of a structure. To overcome these limitations, several researchers have proposed adaptive force distributions that attempt to follow more closely the time-variant distributions of inertia forces.

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Fajfar and Fischinger [5] suggested the use of story forces proportional to the deflected shape of the structure. Eberhard and Sozen [6] proposed the use of load patterns based on mode shapes derived from secant stiffness at each load step; and Bracci et al. [7] proposed the use of stiffness-dependent lateral force distributions in which story forces are proportional to story shear resistances at the previous step.

Saski et al. [8] proposed Multi- mode pushover (MMP) procedure to identify failure mechanisms due to higher modes. The procedure uses independent load patterns based on higher modes instead of only the fundamental mode.

Moghadam [9] proposed a procedure to quantify the effects of higher modes in tall building responses.

Gupta [10] proposed adaptive modal pushover procedure, which accounts for the effects of higher modes and limitations of traditional pushover analysis.

Chopra and Goel [11] developed an improved pushover analysis procedure named modal pushover analysis (MPA) which is based on structural dynamics. Firstly, the procedure was applied to linearly elastic buildings and it was shown that the procedure is equivalent to the well-known response spectrum analysis. The procedure was then extended to estimate the seismic demands of inelastic systems. Earthquake induced demands for a 9-story building were determined by MPA and nonlinear time history analysis. It was concluded that by including the contributions of a sufficient number of modes (two to three), the height-wise distribution of responses estimated by MPA is generally similar to the exact results obtained from nonlinear dynamic time history analysis [11].

Chintanapakdee and Chopra [12] evaluated the accuracy of the MPA procedure for a wide range of buildings. Generic one-bay frames of 3, 6, 9, 12, 15 and 18-stories with five strength levels corresponding to SDOF-system ductility factors of 1, 1.5, 2, 4 and 6 were examined. It was shown that the MPA is able to accurately predict the building's earthquake demands for its different height and different SDOF-system ductility factor.

Moreover, the MPA procedure is used to study forty-eight 12-story irregular frames with strong-columns and weak-beams designed with three types of irregularity; stiffness; strength; and combined stiffness and strength, introduced in eight different locations along the height [13]. In this case it was found that the MPA can estimate the seismic demands of irregular frames, provided the irregularity is in the top story or mid-height story.

Behavior of vertically irregular structures can be significantly different compared to the regular structures. Since 1998 the uniform Building code started to distinguish vertically irregular structures from regular ones based on certain limits on the ratio of strength, stiffness, mass, setbacks or offsets of one story with respect to an adjacent story [14]. Another type of irregularity has been mentioned in SIAC, the irregularity due to the difference in floor levels, which is usually used in Iran [15].

The objective of this paper is to evaluate the capability of modal pushover analysis (MPA) to estimate the critical seismic demands in building frames involving three types of vertical irregularities. The irregularity cases include mass irregularity, geometric irregularity and difference in floor levels. In order to consider the effects of frequency contents and fault distance, an ensemble of seven ground acceleration records are used for the analysis of frames. From the numerical results of the study, it was found that the MPA procedure is able to estimate the floor displacement and inter-story drift ratio with acceptable accuracy compared to the results obtained from nonlinear time history analysis.

## 2. MODAL PUSHOVER ANALYSIS (MPA) PROCEDURE

The MPA procedure involves several steps that can be summarized as follows [12]:

1. Calculation of the natural periods,  $T_n$ , and mode shapes,  $\varphi_n$ , corresponding to the linearly-elastic vibration of the structure.
2. Developing the base-shear roof-displacement pushover curve ( $V_{bn} - u$ ) based on the distribution of force as  $\mathbf{S}_n^* = \mathbf{m}\varphi_n$  for the  $n$ th-mode, where  $\mathbf{m}$  is the mass matrix.
3. Idealization of the pushover curve as a bilinear curve (Fig. 1a).  
(This step is according to FEMA- 273).
4. Conversion of the idealized pushover curve to the force-deformation relation ( $\frac{F_{sn}}{L_n} - D_n$ ) corresponding to the “ $n$ th-mode” inelastic SDOF system by applying the following relationships:

$$\frac{F_{sny}}{L_n} = \frac{V_{bny}}{M_n^*} \tag{1}$$

$$D_{ny} = \frac{u_{rny}}{\Gamma_n \varphi_{rn}} \tag{2}$$

where  $M_n^*$  is the effective modal mass,  $\varphi_{rn}$  is the  $n$ th-mode shape value at the roof,  $\Gamma_n$  is the modal participation factor and  $L_n = \varphi_n^T \mathbf{m} \mathbf{r}$  in which  $\mathbf{r}$  is the influence vector.

5. Calculation of the peak deformation,  $D_n$ , of the “ $n$ th-mode” inelastic SDOF system with a force-deformation relation of Fig.1b and damping ratio  $\xi_n$ .  $D_n$  for a given ground motion can be computed by nonlinear time history analysis.

- 6- Applying Eq. (3) for computing the average value of  $D_n$  associated with each mode. In this equation  $m$  shows the number of earthquake records considered in the analysis. This method is proposed in reference [11] to decrease the number of pushover analysis procedures needed for MPA.

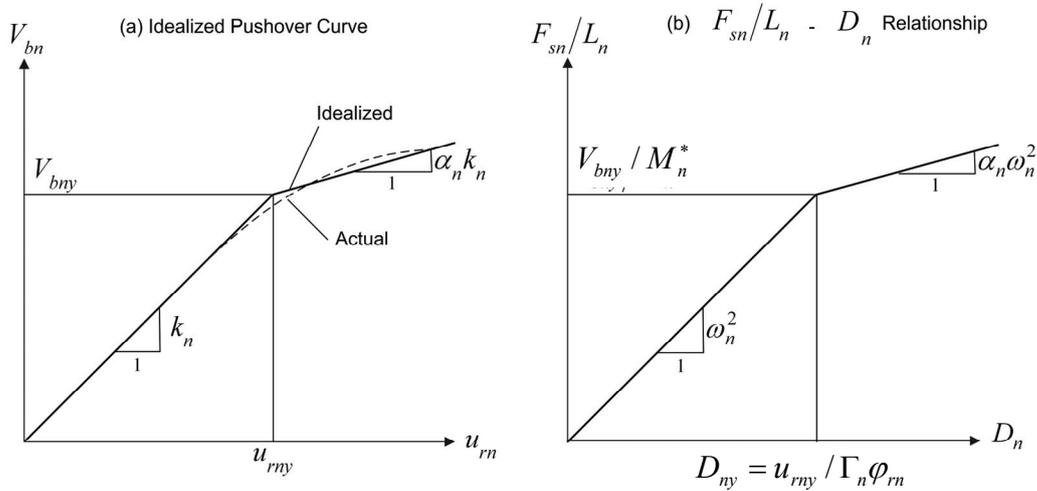


Fig. 1. Properties of the “ $n$ th-mode” inelastic SDOF system from the pushover curve [12]

$$\hat{D} = \exp \left[ \frac{\sum_{i=1}^m Ln(D_i)}{m} \right] \tag{3}$$

- 7- Using Eq. (4) to compute the target displacement of the “ $n$ th-mode” inelastic SDOF system.

$$U_{rno} = \Gamma_n \varphi_{rn} \hat{D}_n \tag{4}$$

8. From the pushover database, corresponding to roof displacement  $U_{rno}$ , the values of the desired responses such as floor displacements, story drifts, plastic hinge rotations, etc. can be extracted.
9. Repeat steps 3 to 8 for a sufficient number of modes. Generally two to three first modes will provide sufficient accuracy [12].
10. Applying an appropriate modal combination rule, e.g. the SRSS combination rule, and combining the peak modal responses to calculate the total response (demand)  $r_{MPA}$ :

$$r_{MPA} = \left( \sum_{n=1}^q r_n^2 \right)^{\frac{1}{2}} \quad (5)$$

where  $q$  is the number of modes participated in the analysis.

### 3. DESCRIPTION OF BUILDING MODELS CONSIDERED FOR ANALYSIS

In order to study the capability of the MPA procedure in the analysis of vertically irregular buildings, seven 12-story irregular frames and one regular frame are chosen. In the irregular frames, the irregularities considered in the analysis are mass irregularity; vertical geometric irregularity (i.e., setback) and difference in floor levels (Fig. 2).

In mass irregular models, the effective mass of the irregular story is more than 150 percent of the mass of any other stories. In Fig. 2 the irregular story floor is shown with a thick line in the first, middle and the last floors. All frames are steel moment resisting frames with four bays of 5.5 m, and story height of 3.0 m.

The frames are designed according to the Iranian code of practice [16] in a high seismicity zone with soil type II. The frames are designed to satisfy the strong-column weak-beam requirement of the UBC-97 [14] as well. For the design of the frames, the software ETABS2000 and for NLTHA and MPA the software SAP2000 are utilized. For the frame systems, the plastic hinge models are chosen according to the FEMA- 273 [1, 14].

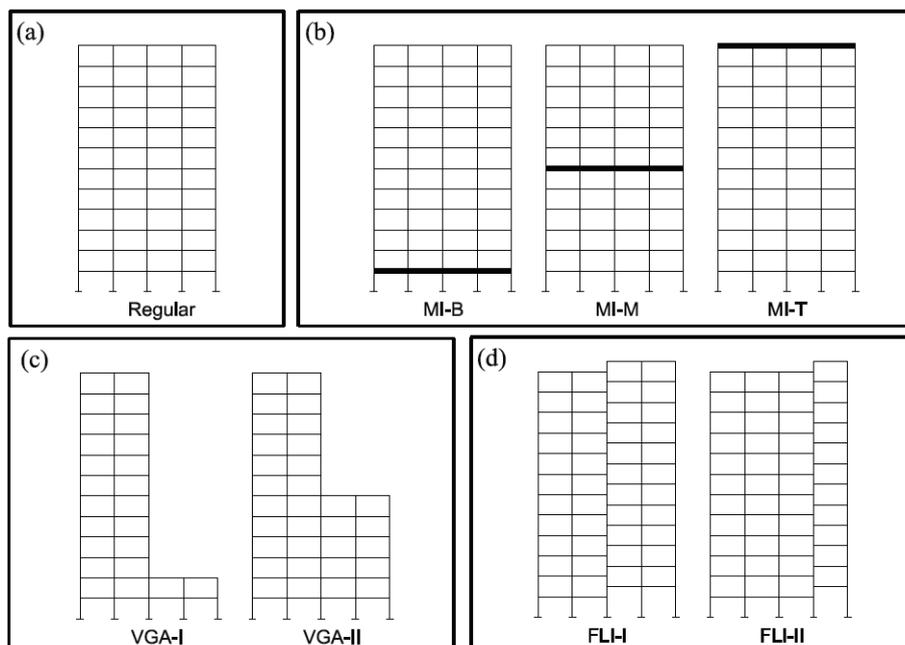


Fig. 2. Frames considered in this study: (a) Regular, (b) Mass Irregularity, (c) Vertical geometric irregularity, and (d) difference in floor levels

#### 4. GROUND MOTION DATA

Each structural frame model is subjected to an ensemble of seven ground motion acceleration records, shown in Table 1. All the input ground motion acceleration records are corrected for baseline errors, and filtered using a band-path filter to eliminate the unwanted noises. Also, the earthquake inputs are scaled according to the IBC 2006 [17] provisions such that the PGA of all acceleration inputs becomes about 0.35g. The earthquake inputs are chosen such that a wide range of frequency contents, as well as the effects of both near and far fault distances are taken into account in the analysis. Table 1 shows the characteristics of the ground motions considered in this study and their pseudo-acceleration spectra are shown in Fig. 3.

Table 1. Ground motion data set considered in this study

No.	Year	Earthquake	$M_w$	Recording station	Fault Dis. (Km)	Comp.	PGA (cm/sec <sup>2</sup> )
1	1979	Imperial Valley	6.5	El Centro Array	12.6	140	357.1
2	1995	Kobe	6.9	Kobe	69.4	090	51.4
3	1989	Loma Prieta	6.9	Diamond Heights	77	000	95.74
4	1971	San Fernando	6.6	La Hollywood Stor Lot	21.2	090	206
5	1966	Parkfield	6.1	Cholame	0.1	065	466.95
6	1978	Tabas	7.4	Tabas	3	000	820.11
7	1994	Northridge	6.7	Hollywood Villoughby	25.7	090	133.41

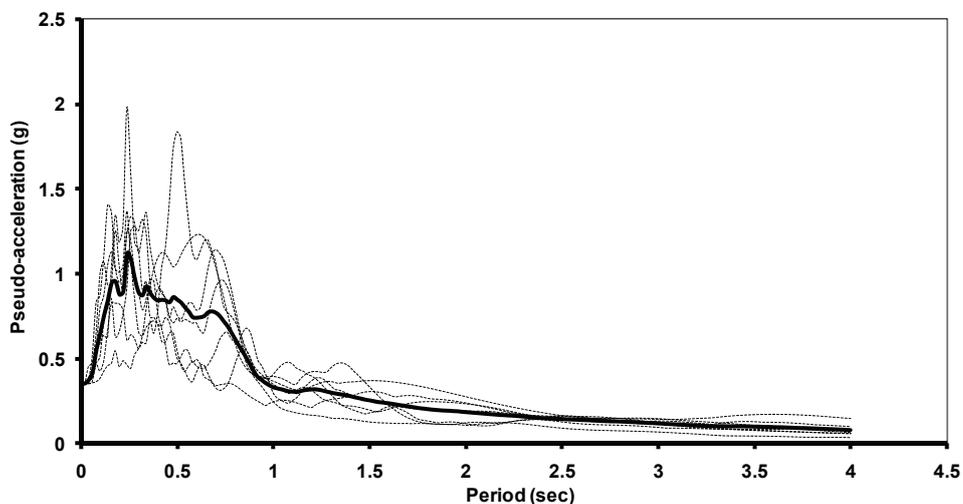


Fig. 3. Pseudo-acceleration spectra of scaled ground motions, damping ratio=5% (solid line shows the average spectrum)

#### 5. NUMERICAL STUDIES

In order to evaluate the modal pushover analysis method, the irregular frames given in Fig. 2 are analyzed under the different earthquake acceleration records described in the previous section. To study the effect of height-wise irregularity, the frames height was fixed on 12 stories. To evaluate the effect of the considered irregularities on the accuracy of the MPA procedure, a 12 story regular frame, given in Fig. 2a, is also chosen as the reference for comparison purposes. The periods of the first three modes of the reference regular frame are 2.24, 0.78, and 0.46 seconds, respectively.

The modal characteristics of the irregular frames are given in Table 2. In the table,  $T$  is the modal period;  $\xi_n$  is the modal damping ratio and the other parameters have already been introduced in the previous sections. It should be noted that in the nonlinear time-history analysis, the damping matrix is developed based on the Rayleigh damping. For defining the Rayleigh damping matrix, the modal damping ratios are used based on the considerations presented in [12, 18].

The dynamic response of each structural frame is determined using both the NLTHA and MPA procedures under the selected ground motion ensembles given in Table 1. The peak story displacement and story drift ratio of each floor for the reference regular frame and the irregular frames are obtained by applying the MPA and NLTHA methods. It is noted that in the MPA method, three cases are considered in the analysis: 1 "mode" case, in which only the fundamental mode of the frames is considered in the modal pushover analysis; and the 2 "modes" and 3 "modes" cases, in which the first two and three vibration modes are considered in the analysis procedure, respectively. The accuracy of the results is discussed in the next sections.

Table 2. Modal properties of the considered frames

Irregular frame	Mode	$T$ (sec)	$\xi_n$	$L_n$	$\Gamma_n$	$M_n^*$
MI-B	1	2.19	0.05	2116.902	216.116	457497.890
	2	0.78	0.0271	879.116	89.798	78943.645
	3	0.45	0.0279	543.049	55.442	30108.169
MI-M	1	2.19	0.05	2175.775	222.127	483299.241
	2	0.80	0.0272	834.875	85.269	71189.616
	3	0.44	0.0279	498.307	50.874	25351.042
MI-T	1	2.27	0.05	2167.822	221.296	479731.072
	2	0.78	0.0263	855.338	87.368	74729.217
	3	0.45	0.0269	514.510	52.534	27029.644
VGI-I	1	2.30	0.05	1505.823	153.725	231483.215
	2	0.79	0.0271	677.705	69.265	46941.604
	3	0.46	0.0284	558.533	57.025	31850.557
VGI-II	1	2.04	0.05	1757.729	179.444	315414.890
	2	0.89	0.03	867.659	88.668	76934.408
	3	0.46	0.0295	427.527	43.685	18676.743
FLI-I	1	2.27	0.05	2128.399	219.382	466932.932
	2	0.78	0.0232	817.385	84.382	68972.753
	3	0.46	0.0212	507.035	52.347	26542.041
FLI-II	1	2.23	0.05	2110.523	216.928	457832.435
	2	0.78	0.0239	837.692	86.213	72220.293
	3	0.46	0.0220	511.268	52.618	26902.280
Regular Frame	1	2.24	0.05	2108.579	215.266	453906.423
	2	0.78	0.0268	847.001	86.515	73278.778
	3	0.46	0.0276	539.902	55.126	29763.045

## 6. COMPARISON OF THE PEAK STORY DISPLACEMENTS DETERMINED BY MPA AND NLTHA

In Figs. 4 to 7 the peak displacements of different stories determined by the MPA and NLTHA procedures are compared for the regular and irregular structural models. As can be seen in these figures, the results of MPA are presented corresponding to three cases (1 mode, 2 modes, and 3 modes), as explained in the previous section. In Fig. 4 the height-wise variation of the peak floor displacements corresponding to the regular reference frame has been shown. Figure 5 shows the peak story displacements of the different mass irregular frames (for models see Fig. 2). Figures 6 and 7 also show the peak story displacements of the models VGA and FLI, respectively.

The height-wise variation of peak floor displacements shown in Fig. 4 and those shown in Figs. 5 to 7 of the irregular frames indicate that the vertical irregularities considered have not noticeable effects on the capability of MPA in predicting the peak story displacements

It can also be seen in Figs. 4 to 7 that by including only the fundamental mode in the MPA procedure, the method can reasonably predict the responses, especially for the lower stories. Including the higher modes slightly improves the accuracy of the MPA predictions for the lower stories, but there are no noticeable improvements in accuracy for the upper stories.

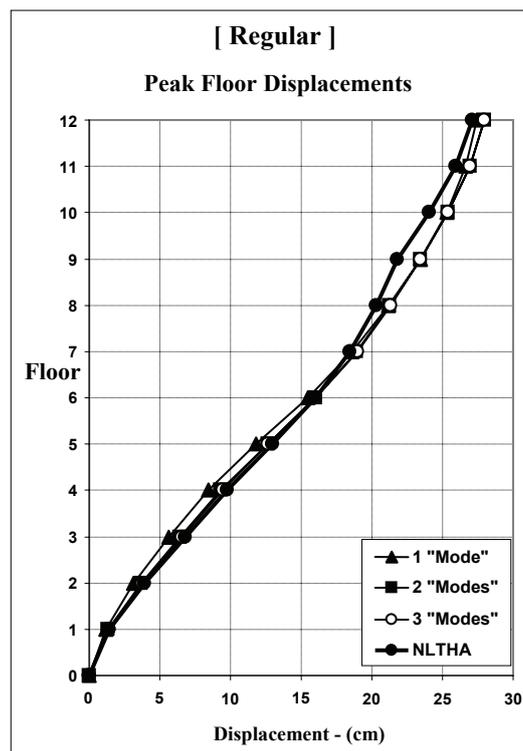
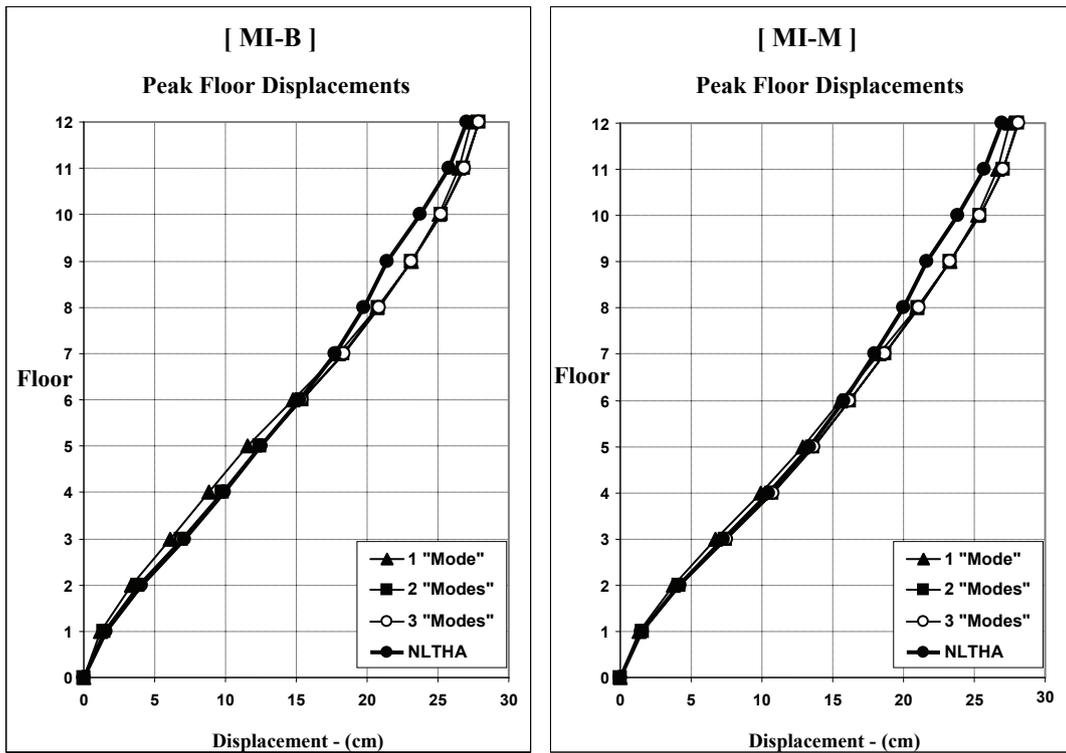
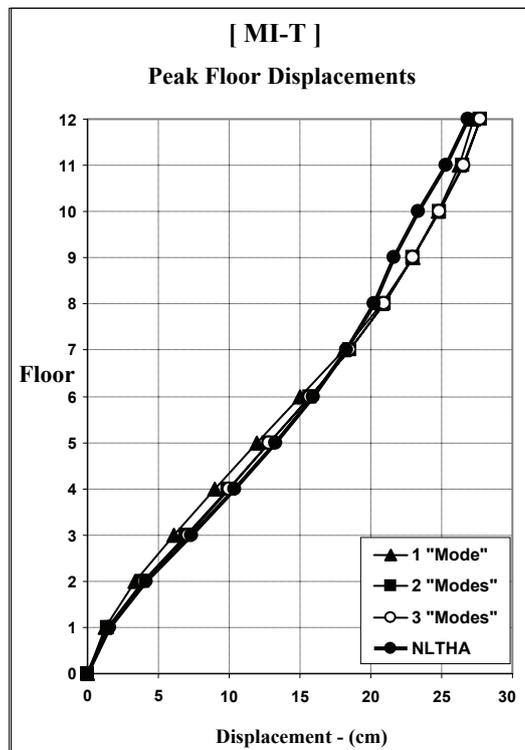


Fig. 4. Comparison of the peak story displacements of the reference regular frame computed by MPA and NLTHA procedures



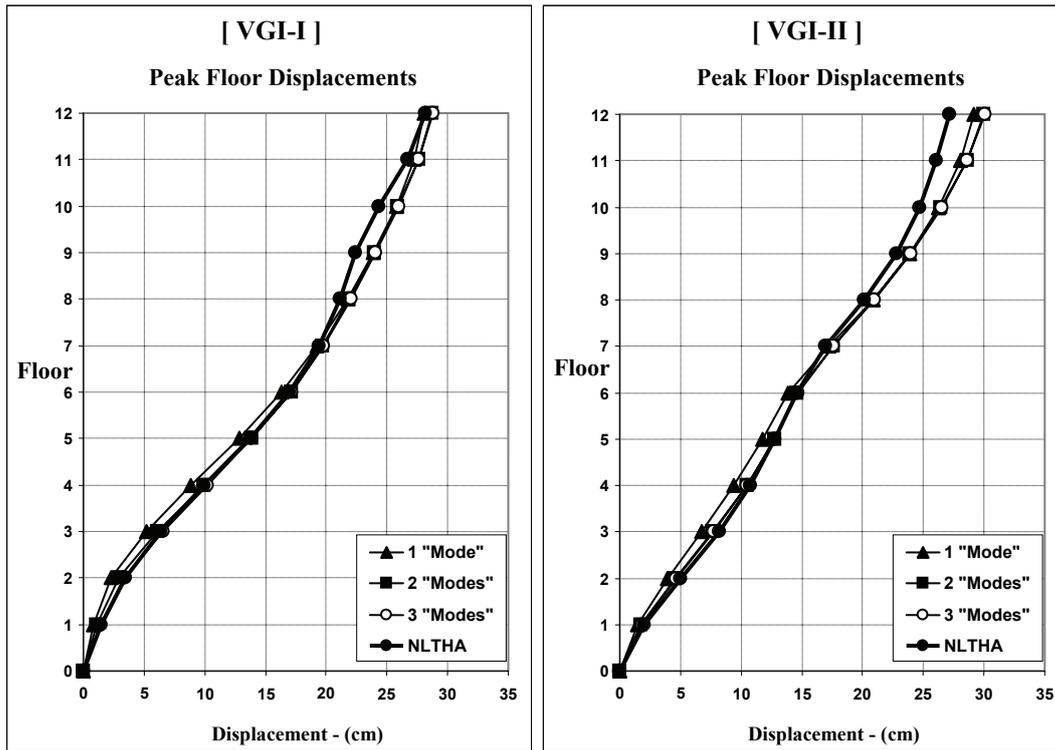
(a)

(b)

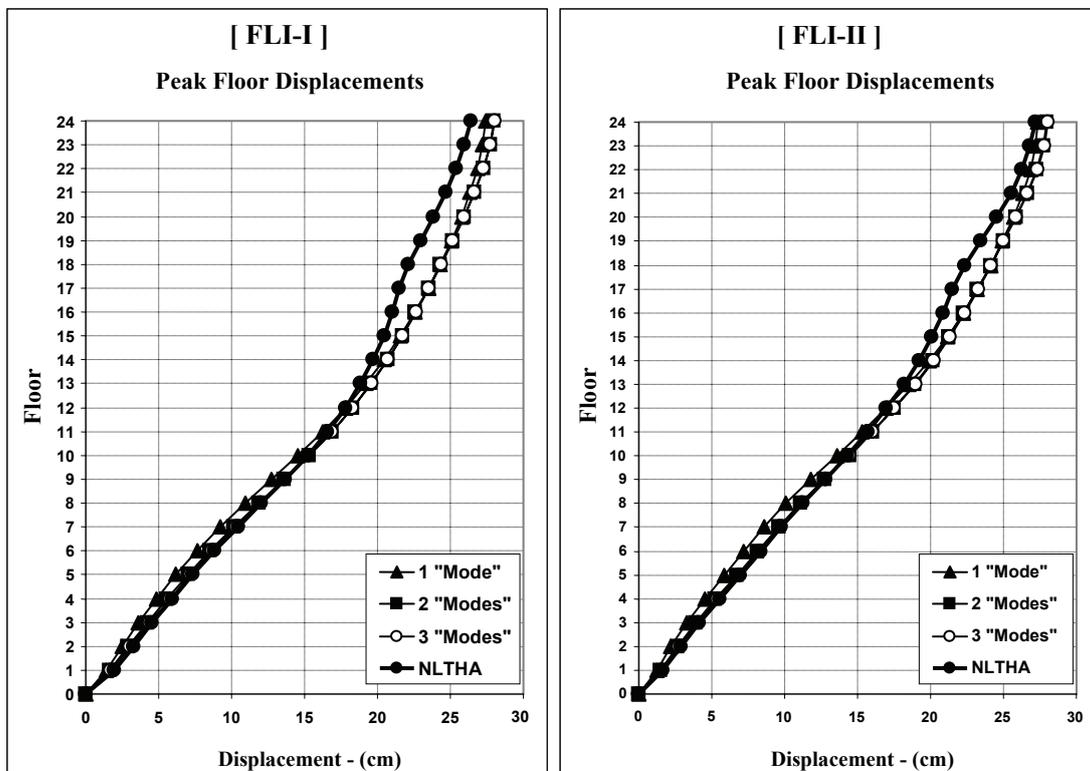


(c)

Fig. 5. Comparison of the peak story displacements of the mass irregular models computed by MPA and NLTHA procedures



(a) (b)  
 Fig. 6. Comparison of the peak story displacements of the geometrically irregular models computed by MPA and NLTHA procedures



(a) (b)  
 Fig. 7. Comparison of the peak story displacements of the floor-levels irregular models computed by MPA and NLTHA procedures

### 7. COMPARISON OF THE STORY-DRIFT RATIOS DETERMINED BY MPA AND NLTHA

The story-drift ratios of the selected frames determined by the MPA and NLTHA methods are shown in Figs. 8 to 11.

In Fig. 8 the story-drift ratios of the reference regular frame predicted by the MPA and NLTHA methods are compared. This figure illustrates the capability of the MPA in the estimation of the story-drift ratios for the reference regular frame. It can be observed that by including the second mode of vibration in the MPA procedure, the accuracy improvements in the MPA predictions is much more than the inclusion of both the second and third modes along with the first mode. This indicates the importance of the contributions of the two first modes in the responses and also reveals the significant contributions of the second mode relative to the higher mode contributions.

Figure 9 shows the story-drift ratios corresponding to the irregular models, MI-B, MI-M, and MI-T (see Fig. 2) predicted by the MPA and NLTHA methods. The figure shows that the MPA method is able to estimate the response of irregular frames as accurate as the regular frame. Moreover, for the above irregular frames the inclusion of the first three modes in the MPA procedure has effects on the accuracy of the predictions with similarities to what was observed in the analysis of the regular reference frame.

Figures 10 and 11 show the comparison between the story-drift ratios obtained by the MPA and NLTHA methods for the irregular models of VGA-I, VGA-II; and FLI-I, FLI-II, respectively. These figures also reveal the capability of the MPA procedure in predicting the story-drift ratios of the above irregular frames. The observed capability is consistent with the trends seen in the predictions for the mass irregular cases.

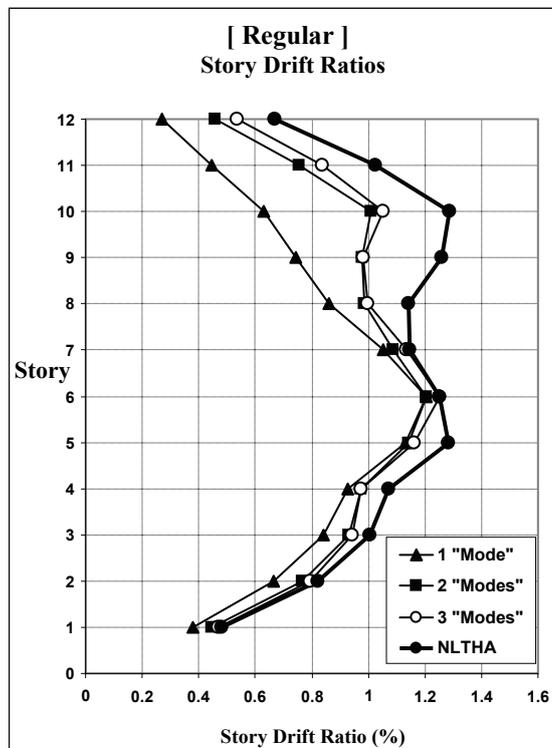
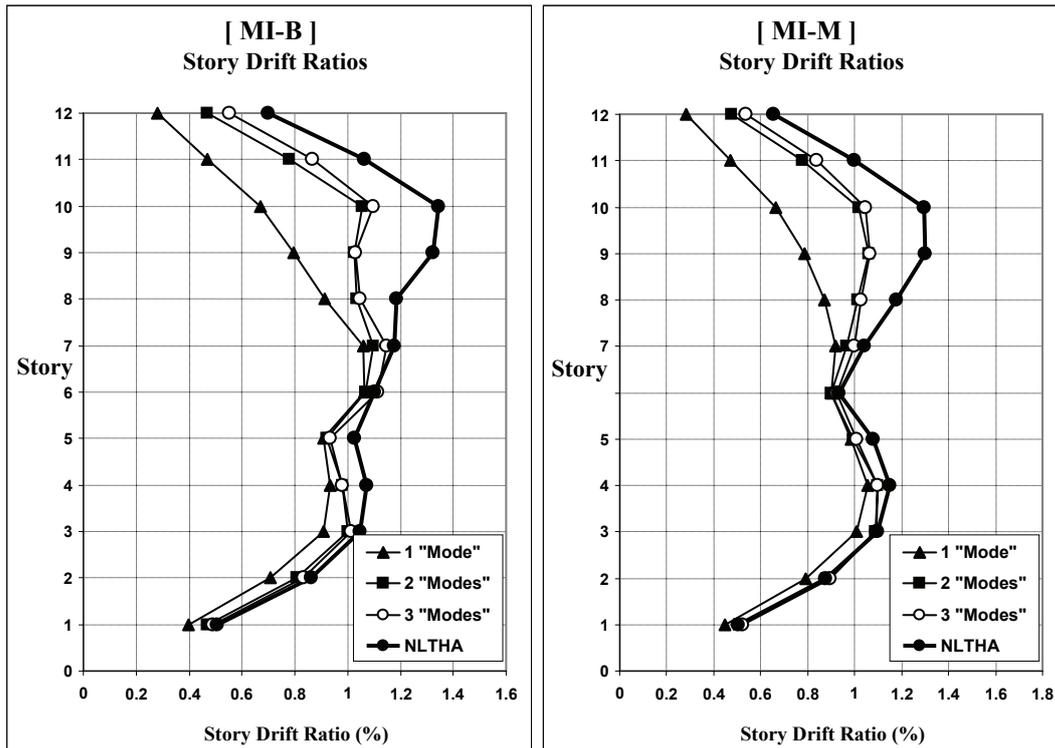
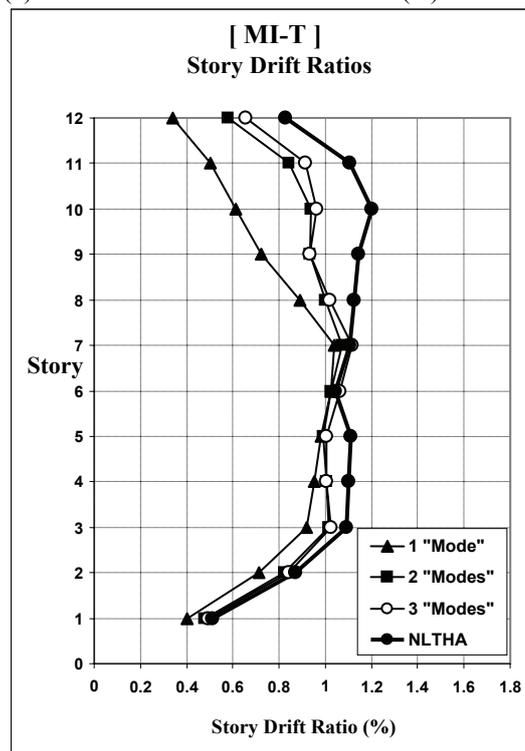


Fig. 8. Comparison of the story-drift ratios of the reference regular frame computed by MPA and NLTHA procedures



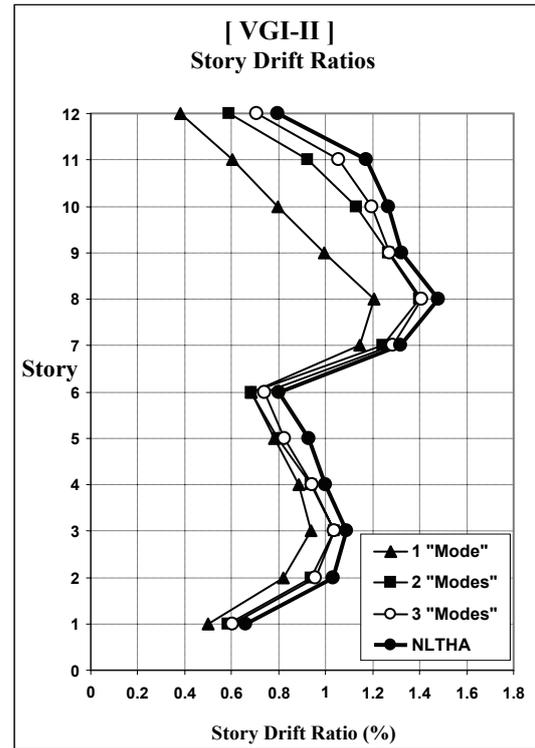
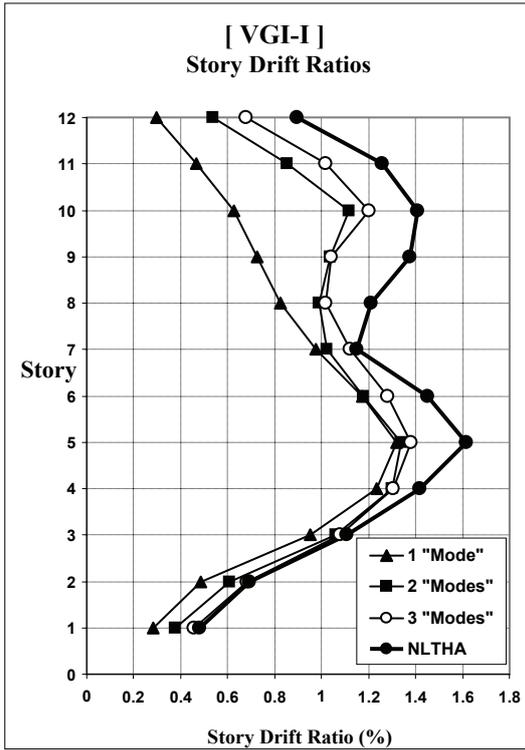
(a)

(b)



(c)

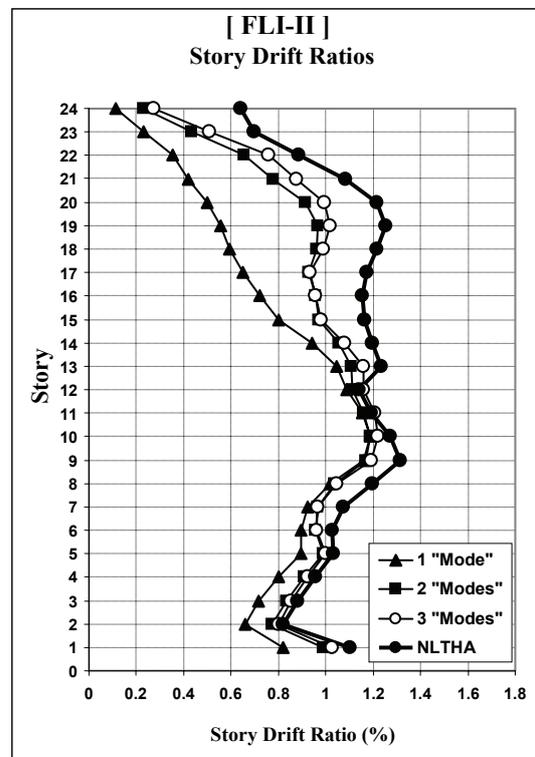
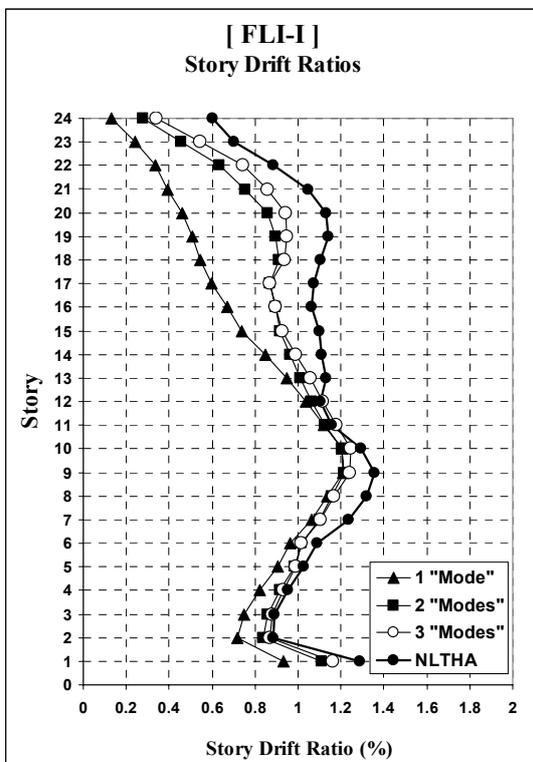
Fig. 9. Comparison of the story-drift ratios of the mass irregular frames computed by MPA and NLTHA procedures



(a)

(b)

Fig. 10. Comparison of the story-drift ratios of the geometrically irregular frames computed by MPA and NLTHA procedures



(a)

(b)

Fig. 11. Comparison of the story-drift ratios of the floor-levels irregular frames computed by MPA and NLTHA procedures

## 8. CONCLUSION

The objective of this research was to evaluate the MPA accuracy in predicting the seismic demands for vertically irregular planar moment-resisting frames. Three types of height-wise irregularities including: mass irregularity, vertical geometric irregularity (i.e., setback) and difference in floor levels were considered. The study has led to the following conclusions:

1. The findings indicate that for the considered irregular frames, MPA can predict the maximum floor displacements of the lower stories with high accuracy; however, it overestimates that response for the upper stories. Similar capability of MPA is observed in the analysis of reference regular frame.
2. The accuracy of the MPA in predicting the height-wise variation of peak floor displacement and story-drift ratio is improved by including the first two modes in the procedure relative to using only the fundamental mode. However, using the three first modes does not improve the accuracy of the peak floor displacement noticeably, but there is little improvement in the estimation of story-drift ratios. This observation emphasizes the significant contributions of the second mode compared to the higher modes contributions in the responses. This trend is seen in the analysis of regular and irregular frames similarly.
3. The capability of the MPA in the prediction of story-drift ratio of the regular and irregular frames is sensitive to the number of modes used in the procedure. The observed sensitivity is more viable in the predictions for the top stories relative to the lower stories. Nevertheless, this sensitivity is not observed to such extent in the prediction of peak floor displacements.
4. In an overall sense, it is observed that the accuracy of the MPA procedure does not deteriorate by the vertical irregularities considered.

The conclusions made above are based on the performance of MPA for the irregular frames considered herein. Further investigations need to be done to examine the accuracy of MPA in the analysis of frames with different stories in which different pattern of height-wise irregularities exist.

## REFERENCES

1. FEMA-273 (Federal Emergency Management Agency), (1997). *NEHRP guidelines for the seismic rehabilitation of buildings*. Building Seismic Safety Council. Washington DC.
2. FEMA-356 (Federal Emergency Management Agency). (2000). *Prestandard and commentary for the seismic rehabilitation of buildings*. American Society of Civil Engineers, Washington DC.
3. ATC-40 (Applied Technology Council), (1996). *Seismic Evaluation and Retrofit of Concrete Buildings*.
4. Krawinkler, H. & Seneviratna, G. D. P. K. (1998). Pros and Cons of a pushover analysis of seismic performance evaluation. *Engineering Structures*, Vol. 20, pp. 452-64.
5. Fajfar, P. & Fischinger, M. (1987). Nonlinear seismic analysis of R/C buildings: Implications of a Case Study. *European Earthquake Engineering*, pp. 31-43.
6. Eberhard, M. O. & Sözen, M. A. (1993). Behavior-based method to determine design shear in earthquake resistant walls. *Journal of the Structural Engineering ASCE*, Vol. 119 No. 2, pp. 619-40.
7. Bracci, J. M., Kunnath, S. K. & Reinhorn, A. M. (1997). Seismic performance and retrofit evaluation of reinforced concrete structures. *Journal of Structural Engineering ASCE*, Vol. 123, pp. 3-10.
8. Sasaki, F., Freeman, S. & Paret, T. (1998). Multi-mode pushover procedure (MMP), a method to identify the effect of higher modes in a pushover analysis. *Proc 6th US National Conference on Earthquake Engineering, Seattle, CD-ROM, EERI, Oakland*.

9. Moghadam, A. S. (2002). A pushover procedure for tall buildings. *12th European Conference on Earthquake Engineering*, 395, Elsevier Science Ltd.
10. Gupta, B. (1999). Enhanced pushover procedure and inelastic demand estimation for performance-based seismic evaluation of buildings. PhD Dissertation, University of Central Florida, Orlando, FL.
11. Chopra, A. K. & Goel, R. K. (2001). *A modal pushover analysis procedure to estimating seismic demands for buildings: theory and preliminary evaluation*. Pacific Earthquake Engineering Research Center, University of California, Berkeley. PERR Report.
12. Chintanapakdee, C. & Chopra, A. K. (2003). Evaluation of modal pushover analysis using generic frames. *Earthquake Engineering and Structural Dynamics*, Vol. 32, pp. 417-42.
13. Chintanapakdee, C. & Chopra, A. K. (2003). *Evaluation of the modal pushover analysis procedure using vertically regular and irregular generic frames*. Earthquake Engineering Research Center, University of California, Berkeley. Report EERC.
14. Uniform Building Code, (1997). International Conference of Building Officials, Whittier, California.
15. Farzad, N. (2005). *The seismic design handbook*. New York: Van Nostrand Reinhold.
16. Iranian code of practice for seismic resistant design of buildings, (2005). *Standard 2800*. Building and Housing Research Center, 3<sup>rd</sup> Edition.
17. International Building Code IBC, (2006). *International building code council, ICC*.
18. Esteki, S. & Pourzeynali, S. (2006). Evaluation of the damping matrix and its effect on the dynamic response of suspension bridges. *Proceedings of the Eighth International Conference on Computational Structures Technology*, B.H.V. Topping, G. Montero and R. Montenegro, (Editors), Civil-Comp Press, Scotland.