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EVALUATION OF THE CHRISTIANSEN METHOD FOR CALCULATION OF FRICTION HEAD LOSS IN HORIZONTAL SPRINKLER LATERALS: EFFECT OF VARIABLE OUTFLOW IN OUTLETS*

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Abstract– The Christiansen's friction factor, F_c, is usually used to compute head loss due to friction in laterals of pressurized irrigation systems. In this research, this factor was evaluated considering variable discharge for outlets located along the laterals. For this purpose, two geometrical progressions were established between the outlets' discharges. The first progression was appointed between the discharge of the first outlet and the outlet which is assumed to possess the average pressure. The next progression was developed between the discharge of this outlet and the most downstream outlet which is assumed to operate at least discharge. The proposed F-factor is a function of number of outlets, the permitted variation of pressure between the first and the last outlet, and also the formula being used to calculate friction head loss along the lateral. The results showed that Christiansen's method accurately estimates total friction head loss along the laterals such that maximum relative error caused by assuming equal discharges in outlets does not exceed 3%.

Keywords– Friction factor, head loss, geometrical progression, lateral pipeline

1. INTRODUCTION

In a pressured irrigation system or any other hydraulic system including multiple outlets along the pipes, estimation of the total friction head loss along a lateral requires a stepwise analysis starting from the lowest outlet, working upward and computing the head loss caused by friction in each segment. Nowadays, the increasing progress in computer technology has led to the development of numerical methods [1- 4] which allow easy evaluation of head losses along the lateral. These models, however, have a high level of sophistication and require extensive programming and, perhaps, long computer-execution time [5]. In fact, although the stepwise computation of the friction loss has been greatly assisted by the use of spreadsheets, correction factors continue to be used in the literature [6, 7].

Several researchers have developed the concept of the reduction factor for closed-end multiple-outlets pipes, beginning with Dupuit [8], for continuous outflow water pipes, and followed by Christiansen [9] for discrete outflow irrigation lines considering the following assumptions:

1) No outflow exists at the downstream end of the pipeline (closed-end pipe).
2) All outlets are equally spaced (constant outlet spacing).
3) All outlets have equal discharge, and
4) The distance between the pipe inlet and the first outlet is equal to one outlet spacing.

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Other researchers have attempted to account for the effect of the first outlet's spacing length upon that factor since Jensen and Fratini [10] developed a solution for discrete outflow with the first spacing length equal to half of the full outlet spacing. Chu [11] developed a slight modification to the concepts of the previous researchers, while Scaloppi [12] generalized the formulation with a relative solution that uses the Christiansen [9] factor as a base for calculation. Cuenca [13] presented a solution to the Jensen and Fratini [10] condition while DeTar [14], Valiantzas [15], and Alazba [16] arrived at results similar to Christiansen [9].

An alternative treatment in the calculation of total friction loss considers the effect of outflow variation in calculation of the F factor [5, 15, 17-19]. These approaches, however, are based on introducing an equation for the F factor for uniform outflow along the lateral and then adjusting it by a more concrete occasion in which the outflow is considered to be non-uniform. Nevertheless, any of these studies have not specified the amount of error caused by assuming equal discharges for outlets in computation of the F factor. Therefore, the objective of this study was to evaluate the Christiansen's [9] method for calculation of total friction loss for a horizontal sprinkler lateral by considering different discharges in outlets and also to determine the maximum error caused by this procedure. In other words, we intend to dismiss the conventional assumption in which total flow at the inlet is equal to Nq (q is discharge of the outlet which possesses the average head, and N is total number of outlets). This goal is followed by assuming two logical geometrical progressions along the lateral.

It is to be noted that in a pipeline with multiple outlets, there will be energy losses caused by the couplings and the structure of the outlet usually called local losses. However, there is also a gradual reduction in the velocity head as flow passes the outlets and this will cause an increase in pressure, which will balance losses caused by turbulence at outlet couplings [12]. Hence, an exact procedure to calculate pressure losses in pipelines with multiple outlets cannot be justified [20]. These assumptions also underline the present work.

2. ANALYSIS

a) Determination of geometrical progressions

Consider a horizontal pipeline including N outlets (Fig. 1). On this pipe, the first and last outlets operate with maximum and minimum discharge, respectively. The outlet which works with an average head is located at approximately 40% of the lateral length from the inlet [7, 17, 21]. So, if the number of outlets (N) on the lateral pipeline is a multiple of 5, the outlet functioning with an average head will be the 0.4Nth. Introducing the average correction factor, F_{AVG}, Anwar [22] reports that $(1-F_{AVG})$ of the total friction head loss occurs before the outlet having an average pressure head. The value of $(1-F_{AVG})$ is shown in Fig. 2. It is evident that 75- 90 percent of total friction head loss (N=2 to N=500, respectively) occurs before the outlet having an average pressure head. So more discharge variation is expected along the pipe ahead of the outlet which works with the average head than after it (i.e., the remaining 60%). In other words, the rate of the outlets' discharge variation before the outlet with the average head will not be equal to the rate of discharge variation for the rest of the lateral. Heuristically, in order to consider variable discharges for outlets and also offer more flexibility to the model, we decided to consider two geometrical progressions. The first is appointed between the first outlet's discharge, located at the upper part of the pipe, and the discharge of the outlet which operates with average head (the 0.4Nth outlet). The second progression is developed between the 0.4Nth outlet and the last outlet on the lateral. It is assumed that if the first outlet were positioned at the inlet of the lateral, its discharge would be q but this discharge changes to qc_1 for the outlet at a distance l from the inlet, where c_1 is the first progression value ($c_1<1$). Mathematically, the discharge of the second outlet reduces to $qc_1 \times c_1 = qc_1^2$ and the 0.4Nth outlet's discharge must be qc_1^2 , where α = 0.4*N*. Considering the parameter c_2 for the second progression (c_1<1), the last outlet's discharge will be $qc_1^{\alpha}c_2^{\beta}$, where β=0.6*N*. The variation of discharge of the outlets is shown schematically in Fig. 1.

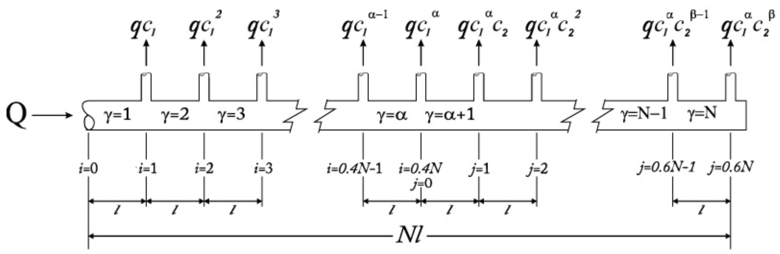

Fig. 1. Variable discharge for *N* outlets along a lateral. The first outlet is one full- spacing from the inlet

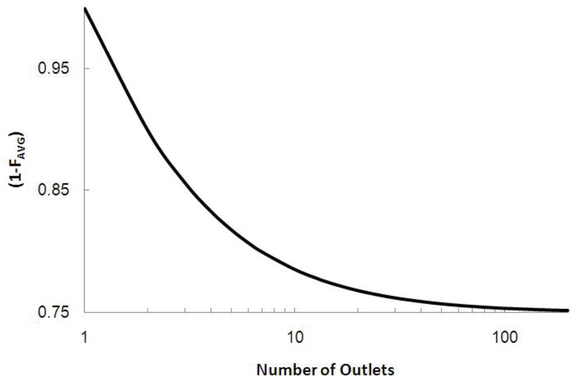

Fig. 2. The value of (1-F_{AVG}) for different values of N and *m*=2 [22]

To calculate c_1 and c_2, three logical assumptions have been made:

1- To maintain a uniformity coefficient of about 97%, total pressure variation (head loss) in a lateral pipeline with numerous outlets is typically limited to 20% [7]. This quantity was denoted as ΔP. Hence we have:

$$\Delta q = \sqrt{1+\Delta P} - 1 \tag{1}$$

where Δq and ΔP are total discharge variation and total pressure variation along the lateral, respectively. For instance, if the total head loss permitted along the lateral is 20%, then ΔP = 0.2 and Δq= 0.0954.

2- For a horizontal lateral, the outlet which operates with average pressure head is located approximately in 0.4L, and therefore its discharge is qc_1^{α} .

3- According to Fig. 3, it is assumed that $\lambda \times H_f$ occurs between the first outlet and the outlet which operates with average pressure head, and therefore $(1-\lambda)\times H_f$ occurs after this outlet. Note that the value of λ will be computed in section (c).

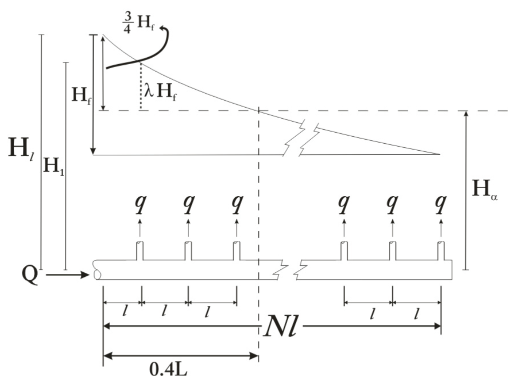

Fig. 3. Determination of parameter λ

An equivalent term to Eq. (1) is that the ratio of the first outlet's discharge to the last one can be $\sqrt{1+\Delta P}$. Hence we have:

$$\frac{qc_1}{qc_1^{\alpha}c_2^{\beta}} = \sqrt{1+\Delta P} \tag{2}$$

Considering $\delta = \sqrt{1+\Delta P}$, Eq. (2) can be written as shown below:

$$qc_1^{\alpha}c_2^{\beta} = \frac{qc_1}{\delta} \tag{3}$$

In a pressurized sprinkler irrigation system, discharge of an outlet on a lateral is a function of pressure head, and can be written as:

$$q = kH^{0.5} \tag{4}$$

where q is the outlet's discharge, H is head at the outlet, and k is a constant coefficient which depends on the outlet characteristics. Applying Eq. (4) for the first outlet, the outlet which operates with average pressure, and the last outlet, we will have:

$$H_1 = \left(\frac{qc_1}{k}\right)^2 \tag{5}$$

$$H_{\alpha} = \left(\frac{qc_1^{\alpha}}{k}\right)^2 \tag{6}$$

$$H_N = \left(\frac{qc_1^{\alpha}c_2^{\beta}}{k}\right)^2 \tag{7}$$

where H_1 is head at the first outlet, H_{α} is head at the outlet working with average pressure, and H_N is head at the last outlet.

Considering the third above-mentioned assumption and also Eqs. (5) to (7), we can write:

$$\left(\frac{qc_1}{k}\right)^2 - \left(\frac{qc_1^{\alpha}}{k}\right)^2 = \lambda \times H_f \tag{8}$$

$$\left(\frac{qc_1^{\alpha}}{k}\right)^2 - \left(\frac{qc_1^{\alpha}c_2^{\beta}}{k}\right)^2 = (1-\lambda) \times H_f \tag{9}$$

where λ is a constant value describing the total friction head loss before the outlet which operates with average pressure head.

Substituting Eq. (3) into Eq. (9) and rearranging gives:

$$\left(\frac{q}{k}\right)^2 \left(c_1^{2\alpha} - \frac{c_1^2}{\delta^2}\right) = (1-\lambda) \times H_f \tag{10}$$

Also, simplifying Eq. (8) yields:

$$\left(\frac{q}{k}\right)^2 \left(c_1^2 - c_1^{2\alpha}\right) = \lambda \times H_f \tag{11}$$

Parameter c_1 is found by dividing Eq. (11) by (10):

$$c_1 = \left(1-\lambda+\frac{\lambda}{\delta^2}\right)^{\frac{1}{2\alpha-2}} \tag{12}$$

A better form for the first progression value could be found by substituting δ by $\sqrt{1+\Delta P}$ in Eq. (12) and rearranging, which yields:

$$c_1 = \left(\frac{1+\Delta P(1-\lambda)}{1+\Delta P}\right)^{\frac{1}{2\alpha-2}} \tag{13}$$

On the other hand, simplifying Eq. (3) gives the parameter c_2:

$$c_2 = \left(\frac{c_1^{\,1-\alpha}}{\sqrt{1+\Delta P}}\right)^{\frac{1}{\beta}} \tag{14}$$

b) A new approach to develop F factor

Christiansen [9] proposed the F_c factor for calculating total friction head loss in a lateral pipeline with multiple outlets. We tried to relate our analysis with this factor and find an analytic relation for it. The general equation for computing friction loss in a pipe can be shown as:

$$H_f = TD^n LQ^m \tag{15}$$

where H_f is total friction head loss, T is a coefficient based on the friction formula used, D is pipe diameter, L is length of the lateral, Q is total discharge, and n and m are exponents. Considering $\rho = TD^n$ to be a constant term, the friction head loss for each segment ($h_{f\gamma}$) illustrated in Fig. 1 could be written as (γ represents the segment number):

$$h_{f_1} = \rho l Q^m$$

$$h_{f_2} = \rho l (Q - qc_1)^m$$

$$h_{f_3} = \rho l (Q - qc_1 - qc_1^{\,2})^m$$

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$$h_{f_{\alpha-1}} = \rho l (Q - qc_1 - qc_1^{\,2} - \ldots\ldots - qc_1^{\,\alpha-2})^m$$

$$h_{f_\alpha} = \rho l (Q - qc_1 - qc_1^{\,2} - \ldots\ldots - qc_1^{\,\alpha-2} - qc_1^{\,\alpha-1})^m$$

$$h_{f_{\alpha+1}} = \rho l (Q - qc_1 - qc_1^{\,2} - \ldots\ldots - qc_1^{\,\alpha-2} - qc_1^{\,\alpha-1} - qc_1^{\,\alpha})^m$$

$$h_{f\alpha+2} = \rho l \left(Q - \frac{qc_1(1-c_1^{\,\alpha})}{1-c_1} - qc_1^{\,\alpha} c_2\right)^m$$

$$h_{f_N} = \rho l \left(Q - \frac{qc_1(1-c_1^{\,\alpha})}{1-c_1} - qc_1^{\,\alpha} c_2 - qc_1^{\,\alpha} c_2^{\,2} - \ldots\ldots - qc_1^{\,\alpha} c_2^{\,\beta-1}\right)^m \tag{16}$$

From Eq. (16) we can write:

$$H_{f_{40\%}} = h_{f_1} + h_{f_2} + h_{f_3} + ... + h_{f_{\alpha-1}} + h_{f_\alpha} = \rho l \left\{ \sum_{i=0}^{\alpha-1} \left(Q - \frac{qc_1(1-c_1{}^i)}{1-c_1} \right)^m \right\} \quad (17)$$

$$H_{f_{60\%}} = h_{f_{\alpha+1}} + h_{f_{\alpha+2}} + ... + h_{f_{N-1}} + h_{f_N} = \rho l \left\{ \sum_{j=0}^{\beta-1} \left(Q - \frac{qc_1(1-c_1{}^\alpha)}{1-c_1} - \frac{qc_1{}^\alpha c_2(1-c_2{}^j)}{1-c_2} \right)^m \right\} \quad (18)$$

where $H_{f_{40\%}}$ is the total friction head loss in 40% of the upper section of the lateral and $H_{f_{60\%}}$ is the total friction head loss in 60% of the lower part of the lateral. Hence:

$$H_f = H_{f_{40\%}} + H_{f_{60\%}} \quad (19)$$

Substituting Eq.(17) and (18) into Eq.(19) leads to:

$$H_f = \rho\, l \left\{ \sum_{i=0}^{\alpha-1} \left(Q - \frac{qc_1(1-c_1{}^i)}{1-c_1} \right)^m + \sum_{j=0}^{\beta-1} \left(Q - \frac{qc_1(1-c_1{}^\alpha)}{1-c_1} - \frac{qc_1{}^\alpha c_2(1-c_2{}^j)}{1-c_2} \right)^m \right\} \quad (20)$$

Total discharge of the lateral is the sum of all outlets' discharge. Therefore:

$$Q = \frac{qc_1(1-c_1{}^\alpha)}{1-c_1} + \frac{qc_1{}^\alpha c_2(1-c_2{}^\beta)}{1-c_2} \quad (21)$$

Substituting Eq. (21) for Q in Eq. (20) gives:

$$H_f = \rho\, lq^m \left\{ \sum_{i=0}^{\alpha-1} \left(\frac{c_1{}^{1+i} - c_1{}^{1+\alpha}}{1-c_1} + \frac{c_1{}^\alpha c_2(1-c_2{}^\beta)}{1-c_2} \right)^m + \sum_{j=0}^{\beta-1} \left(\frac{c_1{}^\alpha c_2(c_2{}^j - c_2{}^\beta)}{1-c_2} \right)^m \right\} \quad (22)$$

Considering Eq. (15) and L=N*l* we have:

$$H_f = FTNlD^nQ^m \quad (23)$$

By substituting Eq. (21) in Eq. (23) we have:

$$H_f = F\rho Nlq^m \left(\frac{c_1(1-c_1{}^\alpha)}{1-c_1} + \frac{c_1{}^\alpha c_2(1-c_2{}^\beta)}{1-c_2} \right)^m \quad (24)$$

Finally, if Eqs. (22) and (24) are equated, then:

$$F = \frac{\sum_{i=0}^{\alpha-1} \left(\frac{c_1{}^{1+i} - c_1{}^{1+\alpha}}{1-c_1} + \frac{c_1{}^\alpha c_2(1-c_2{}^\beta)}{1-c_2} \right)^m + \sum_{j=0}^{\beta-1} \left(\frac{c_1{}^\alpha c_2(c_2{}^j - c_2{}^\beta)}{1-c_2} \right)^m}{N \left(\frac{c_1(1-c_1{}^\alpha)}{1-c_1} + \frac{c_1{}^\alpha c_2(1-c_2{}^\beta)}{1-c_2} \right)^m} \quad (25)$$

Note that the proposed F factor here is indirectly related to the pressure head variation along the lateral because c_1 and c_2 are related to this parameter.

Table 1 shows a comparison between the F values calculated from Eq. (25) and Christiansen's F factor for m=1.852 and different values of ΔP. It is seen from this table that, as ΔP increases, the value of the proposed F factor decreases. Also the Christiansen’s F factor is very close to F of Eq. (25) when ΔP=0.05 since, for this case, the values of the geometrical progressions are close to 1.0 and the assumption of equal discharges in outlets is more logical. Finally, it is concluded that a maximum relative error of 2.2% may occur for N=65 and ΔP=0.2.

Table 1. F-factor for m=1.852 and different values of ΔP

Number of outlets	F (calculated by Eq. (25))				F_C
	$\Delta P = 0.2$	$\Delta P = 0.15$	$\Delta P = 0.1$	$\Delta P = 0.05$	
5	0.4476	0.4497	0.4520	0.4543	0.4568
10	0.3938	0.3957	0.3978	0.3999	0.4022
15	0.3765	0.3784	0.3804	0.3825	0.3846
20	0.3680	0.3699	0.3718	0.3739	0.3760
25	0.3629	0.3648	0.3667	0.3687	0.3709
30	0.3595	0.3614	0.3633	0.3654	0.3675
35	0.3571	0.3590	0.3609	0.3629	0.3650
40	0.3554	0.3572	0.3591	0.3611	0.3632
45	0.3540	0.3558	0.3577	0.3597	0.3618
50	0.3528	0.3547	0.3566	0.3586	0.3607
55	0.3519	0.3538	0.3557	0.3577	0.3598
60	0.3512	0.3530	0.3549	0.3569	0.3590
65	0.3505	0.3524	0.3543	0.3563	0.3584
70	0.3500	0.3518	0.3537	0.3557	0.3578
75	0.3495	0.3513	0.3532	0.3552	0.3573
80	0.3491	0.3509	0.3528	0.3548	0.3569
85	0.3487	0.3506	0.3525	0.3545	0.3565
90	0.3484	0.3502	0.3521	0.3541	0.3562
95	0.3481	0.3499	0.3518	0.3538	0.3559
100	0.3479	0.3497	0.3516	0.3536	0.3556

c) Determination of λ

In order to calculate the value of both geometrical progressions (i.e., Eqs. (13) and (14)), the value of λ must be derived. For this purpose, consider a horizontal pipeline as illustrated in Fig. 2. Parameters H_l , H_1 , H_f and H_α are shown on the figure. The following relationship can be written for the pressure at the pipe inlet, H_l, and the pressure head of the first outflow, H_1:

$$H_l - \rho l Q^m = H_1 \tag{26}$$

On the other hand, the relationship between the inlet pressure and the outlet, which operates at average pressure for zero ground slopes is [7]:

$$H_l = H_\alpha + \frac{3}{4} H_f \tag{27}$$

Considering Eqs. (23), (26) and (27), Eq. (27) can be transformed to:

$$H_1 - H_\alpha = \left(\frac{3}{4} - \frac{1}{NF_c} \right) H_f \tag{28}$$

where F_c is the Christiansen's F factor. Comparison of Eq. (28) and our third assumption shows that the item $(\frac{3}{4}-\frac{1}{NF_C})$ in Eq. (28) is equal to λ. The equation for directly calculating the F_c factor is given by DeTar [14] as:

$$F_c=\frac{1}{m+1}+\frac{1}{2N}+\frac{m}{12N^2} \tag{29}$$

Therefore, by substituting Eq. (29) in (28), λ is found as:

$$\lambda=\left(\frac{3}{4}-\frac{1}{\frac{N}{m+1}+0.5+\frac{m}{12N}}\right) \tag{30}$$

Table 2 shows the value of λ for different number of outlets and m values. As the number of outlets increases, λ approaches 0.75, which is proposed by Keller and Bliesner [7]. However, the observed difference between these two coefficients is because λ represents the friction head loss between the first outlet and the outlet which operates under average pressure, whereas the coefficient 0.75 considers the friction loss between the inlet of the lateral and the outlet with average pressure head.

Table 2. Values of λ for different values of m and N

Number of outlets	λ			
	m=1.75	m=1.852	m=1.9	m=2
5	0.3240	0.3122	0.3067	0.2955
10	0.5091	0.5014	0.4977	0.4903
15	0.5823	0.5767	0.5740	0.5685
20	0.6215	0.6170	0.6149	0.6106
25	0.6458	0.6421	0.6404	0.6369
30	0.6624	0.6593	0.6578	0.6548
35	0.6744	0.6717	0.6705	0.6678
40	0.6836	0.6812	0.6801	0.6777
45	0.6907	0.6886	0.6876	0.6855
50	0.6965	0.6946	0.6936	0.6918
55	0.7012	0.6995	0.6986	0.6969
60	0.7052	0.7036	0.7028	0.7012
65	0.7086	0.7071	0.7064	0.7049
70	0.7115	0.7101	0.7094	0.7080
75	0.7140	0.7127	0.7121	0.7108
80	0.7162	0.7150	0.7144	0.7132
85	0.7182	0.7170	0.7165	0.7153
90	0.7199	0.7188	0.7183	0.7172
95	0.7215	0.7204	0.7199	0.7189
100	0.7229	0.7219	0.7214	0.7204

The calculated λ here offers a new form of calculating c_1 which could be derived by substituting Eq. (30) in Eq. (13):

$$c_1 = \left(\frac{1 + 0.25\Delta P + \dfrac{\Delta P}{\dfrac{N}{m+1} + 0.5 + \dfrac{m}{12N}}}{1 + \Delta P} \right)^{\frac{1}{2\alpha - 2}} \tag{31}$$

c_1 and c_2 are calculated from Eqs. (31) and (14) and are summarized in Table 3 for different number of outlets along the lateral, four values of ΔP, and m=1.852. As it is seen in the table, both geometrical progressions approach 1.0 when the number of outlets increases. Also, it is evident that as ΔP increases, the value of c_1 and c_2 decreases.

Table 3. Variation of $c_1$and c_2 for m=1.852 and different values of N and ΔPs

Number of outlets	$\Delta P = 0.2$		$\Delta P = 0.15$		$\Delta P = 0.1$		$\Delta P = 0.05$	
	c_1	c_2	c_1	c_2	c_1	c_2	c_1	c_2
5	0.97364	0.97875	0.97943	0.98377	0.98571	0.98897	0.99254	0.99438
10	0.98556	0.99211	0.98879	0.99401	0.99226	0.99595	0.99598	0.99795
15	0.98995	0.99550	0.99221	0.99659	0.99463	0.99770	0.99722	0.99884
20	0.99228	0.99693	0.99402	0.99768	0.99589	0.99844	0.99787	0.99921
25	0.99373	0.99770	0.99515	0.99826	0.99666	0.99883	0.99828	0.99941
30	0.99472	0.99817	0.99592	0.99862	0.99719	0.99907	0.99855	0.99953
35	0.99544	0.99849	0.99648	0.99886	0.99758	0.99923	0.99875	0.99961
40	0.99599	0.99871	0.99690	0.99903	0.99787	0.99935	0.99890	0.99967
45	0.99642	0.99888	0.99724	0.99915	0.99810	0.99943	0.99902	0.99971
50	0.99677	0.99901	0.99750	0.99925	0.99829	0.99950	0.99912	0.99975
55	0.99705	0.99912	0.99772	0.99933	0.99844	0.99955	0.99919	0.99977
60	0.99729	0.99920	0.99791	0.99940	0.99856	0.99959	0.99926	0.99980
65	0.99750	0.99927	0.99807	0.99945	0.99867	0.99963	0.99932	0.99981
70	0.99767	0.99933	0.99820	0.99949	0.99877	0.99966	0.99936	0.99983
75	0.99782	0.99938	0.99832	0.99953	0.99885	0.99969	0.9994	0.99984
80	0.99796	0.99942	0.99842	0.99956	0.99892	0.99971	0.99944	0.99985
85	0.99807	0.99946	0.99851	0.99959	0.99898	0.99973	0.99947	0.99986
90	0.99818	0.99949	0.99859	0.99962	0.99904	0.99974	0.99950	0.99987
95	0.99827	0.99952	0.99867	0.99964	0.99909	0.99976	0.99953	0.99988
100	0.99836	0.99955	0.99873	0.99966	0.99913	0.99977	0.99955	0.99988

The other point here is that according to Table 3, c_1 is always less than c_2. Logically this means that for a horizontal lateral, the rate of the outlets' discharge variation is more severe before the outlet possesses the average pressure head.

d) Total discharge of the lateral

The consideration of two geometrical progressions not only affects the F factor, but will also influence the total flow entering the lateral inlet since Q is equal to the summation of all outlets discharge. In order to approve this effect, Eq. (21) is rewritten as:

$$Q = qc_1^{\alpha} \left(\frac{1 - c_1^{\alpha}}{(1 - c_1)(c_1^{\alpha - 1})} + \frac{c_2(1 - c_2^{\beta})}{1 - c_2} \right) \tag{32}$$

On the other hand, based on our second assumption, qc_1^{α} is approximately equal to $\overline{q}$. Therefore, Eq. (32) takes the following form:

$$Q = \overline{q} \times \left(\frac{1 - c_1^{\alpha}}{(1 - c_1)(c_1^{\alpha - 1})} + \frac{c_2(1 - c_2^{\beta})}{1 - c_2} \right) \tag{33}$$

In order to compare the computed total inflow by the suggested model and the conventional approach by Christiansen (1942) in which $Q = N\overline{q}$, a series of numerical computations were carried out. For this purpose, a combination of input parameters N, *m* and Δ*P* were considered. The value of N was varied between 5 to 100 by an increment of 5; *m* was chosen as 1.75, 1.852, 1.9 and 2; and four values were considered for Δ*P* as Δ*P*= 0.2, 0.15, 0.1 and 0.05. Using these data, the values of c_1 and c_2 were calculated using Eqs. (31) and (14). The value of $Q/\overline{q}$ was obtained using Eq. (33). The relative error between this quantity and the value of total inflow at the inlet ($Q = N\overline{q}$) was computed and plotted against the total number of outlets in Fig. 4. This figure demonstrates that relative errors are negligible and are always between -0.5% and 2%.

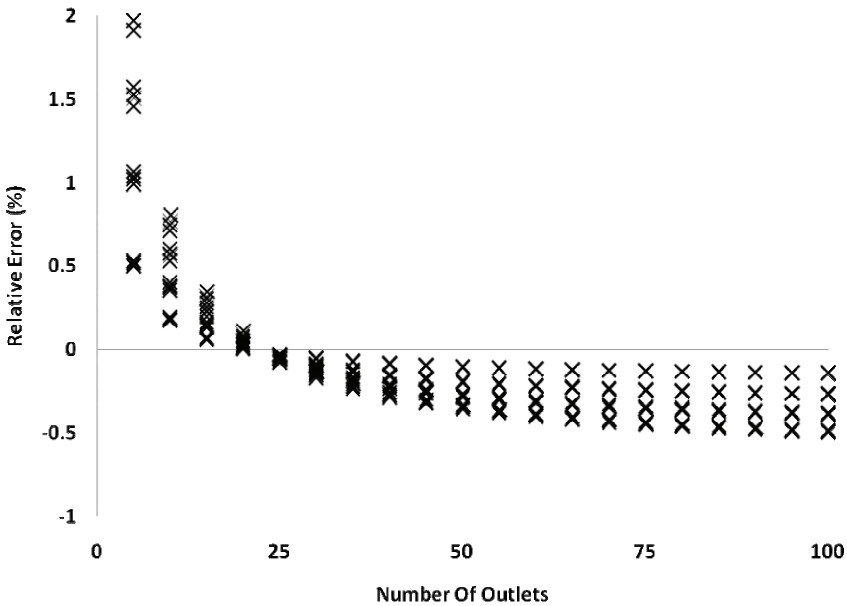

Fig. 4. Relative errors in calculation of $Q/\overline{q}$ using Eq. (33) and the conventional approach ($Q = N\overline{q}$)

e) Determination of ΔP

To determine Δ*P*, we consider a horizontal lateral pipeline on which *N* outlets, with a spacing of *l* and equal discharge of *q*, are located. Neglecting the velocity head along the lateral, which is acceptable for drip and sprinkler irrigation systems [21], we can write:

$$H_l - H_N = H_f \tag{34}$$

On the basis of our first assumption, we have:

$$H_l - H_N = \Delta P \times H_{\alpha} \tag{35}$$

Equating Eqs. (34) and (35) gives:

$$\Delta P = \frac{H_f}{H_a} \tag{36}$$

Considering Eqs. (4) and (23):

$$\Delta P = \frac{\rho k^2 N^2 L Q^m F_c}{Q^2} \tag{37}$$

Finally, using Eq. (29) yields:

$$\Delta P = \rho k^2 L Q^{m-2} \left(\frac{N^2}{m+1} + \frac{N}{2} + \frac{m}{12}\right) \tag{38}$$

It is observed that ΔP depends on the number of outlets, pipe length and type, total discharge of the pipe, coefficient m, and pipe diameter.

3. COMPARISON TO THE CHRISTIANSEN'S METHOD

An example is given here to illustrate the application of the adjusted friction coefficient (Eq. 25) and calculation of total friction head loss along a lateral. Suppose that a wheel-move sprinkle irrigation system is designed such that 15 outlets are fixed on an aluminum pipeline. The spacing between outlets and also between the first outlet and the lateral inlet is 10 m. Calculate the total friction head loss for this pipe by using both Christiansen's F-factor and the method proposed in this paper. Suppose that the diameter of the pipe is 3", the design discharge of the outlets is 0.9 liter/sec with an average pressure head equal to 35m. The friction loss is assumed to be described by the Hazen–Williams formula [C_{HW} =130, for aluminum pipes with couplers approximately every 10 m [7]].

Solution

1. Christiansen method: For m=1.852 and 15 outlets, F_C = 0.3846 (Table 2). The total flow at the inlet of the lateral is calculated as $Q=Nq=15\times0.9 lps$ = 13.5 *lps*.

The Hazen-Williams formula for calculating friction head loss in the SI system is:

$$H_f = 10.672 \times F_C L \left(\frac{Q}{C_{HW}}\right)^{1.852} D^{-4.871} \tag{39}$$

where C_{HW} is the Hazen-Williams coefficient.

$$H_f = 10.672 \times FL\left(\frac{Q}{C_{HW}}\right)^{1.852} D^{-4.871} = 10.672 \times 0.3846 \times 150 \times \left(\frac{13.5\times10^{-3}}{130}\right)^{1.852} (0.0762)^{-4.871}$$

$$H_f = 7.206\, m$$

2. The method of this paper:

1- Using Eq. (38) , ΔP= 0.2059 ≈ 0.2.

2- From Table 3, for ΔP=0.2 and N=15, c_1= 0.9899 and c_2= 0.9955.

3- From Table 1, the adjusted F-factor for 15 outlets (for m=1.852 and ΔP=0.2) is 0.3763.

4- α = 0.4×15=6; β=0.6×15=9.

$$qc_1^{\alpha} = 0.9 \rightarrow q = \frac{0.9}{c_1^{\alpha}} = 0.9563\, lps$$

5- Total discharge of the lateral (from Eq. (21)) is:

$$Q = \frac{qc_1(1-c_1^{\alpha})}{1-c_1} + \frac{qc_1^{\alpha} c_2(1-c_2^{\beta})}{1-c_2} = 13.496\, lps$$

$$H_f = 10.672 \times FL(\frac{Q}{C_{HW}})^{1.852} D^{-4.871}$$

$$= 10.672 \times 0.3763 \times 150 \times (\frac{13.496 \times 10^{-3}}{130})^{1.852} (0.0762)^{-4.871} \cong 7.047\ m$$

This example demonstrates that the Christiansen [9] method slightly overestimates the total friction head loss along the lateral (the relative error here is 2.26%). Fabricating a test rig in order to evaluate the Christiansen's F factor in manifolds, Mohammed et al. [23] also reported that the value of F_C was relatively greater than the experimental F factor. However, Scaloppi and Allen [21] considered errors of such an order of magnitude to be within the range of accuracy of pressure gauges commonly used in sprinkler irrigation systems. It is worth noting that these errors will significantly decrease if the number of outlets along the pipe increases. A recent study by Yildirim [18] also testifies to this statement. Solving an example for the hydraulic design of a sprinkler irrigation system, Yildirim [18] reports that the Christiansen coefficient for unequal discharges of the outlets and m= 1.852, l= 12 m, q= 0.315 L/s and N= 33 was calculated as 0.3627, while Christiansen [9] gave F = 0.3689 for N= 33. Our method also gives F = 0.36, which shows that the assumption of considering equal discharge in the outlets becomes logical when the total number of outlets is large.

4. CONCLUSION

In most sprinkler irrigation systems, the outlets are uniformly spaced along the lateral pipeline. Because of the importance of estimating the total friction head loss along the lateral, Christiansen [9] proposed the friction coefficient factor F and submitted a table for his results. In the present research, this method was evaluated by considering variable discharge in outlets in order to quantify the relative error caused by assuming equal discharge in outlets. For this purpose, two geometrical progressions were assumed to exist between the outlets' discharges. Results showed that 1) the value of the F factor changes as the total pressure head variation along the lateral changes, and 2) although Christiansen's method and our proposed equations give approximately similar values for F factor, Christiansen [9] slightly overestimates the total friction head loss along the lateral, particularly when $N \leq 10$ and $\Delta P \geq 0.1$. However, this overestimation can be neglected since the maximum relative error between the two methods does not exceed 3%.

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