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VIBRATION ANALYSIS OF ASYMMETRIC SHEAR WALL STRUCTURES USING THE TRANSFER MATRIX METHOD*

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Abstract– In this paper a method for the vibration analysis of proportional asymmetric shear wall structures is presented. The whole structure is assumed as an equivalent bending-warping torsion beam in this method. The governing differential equations of equivalent bending-warping torsion beam are formulated using the continuum approach and are posed in the form of a simple storey transfer matrix. By using the storey transfer matrices and point transfer matrices, which take into account the inertial forces, the system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. The structural properties of the building may change in the proposed method. A numerical example has been solved and presented at the end of the study by means of a program written in MATLAB to verify the method that is being proposed. The results of this example display the agreement between the proposed method and the other valid method given in the literature.

Keywords– Vibration, asymmetric, wall, transfer matrix

1. INTRODUCTION

A number of methods, such as finite element method, have been developed for the analyses of buildings. The continuum model is a very simple and efficient method used in the static and dynamic analysis of shear wall-frame buildings.

There are numerous studies [1-43] in the literature regarding the continuum method.

Rosman [1] proposed a continuum medium method for a pair of high rise coupled shear walls. Heidebrecht and Stafford Smith [2] derived the differential equations of a system for buildings with uniform stiffness along their height and then obtained closed-form solutions for uniform and triangular static lateral load distributions.

Zalka [18] derived simplified expressions for the circular frequency of wall-frame buildings. Kuang and Ng [14] considered the problem of doubly asymmetric structures, in which the motion is dominated by shear walls. For the analysis, the structure was replaced by an equivalent uniform cantilever whose deformation was coupled in flexure and warping torsion. An approximation method for estimating floor acceleration demands in multistory buildings subjected to earthquake ground motions has been developed in a recent study by Miranda and Taghavi [30]. The dynamic properties of multistory buildings were approximated using the equivalent continuum model consisting of a flexural cantilever beam and a shear cantilever beam that were assumed to be connected by an infinite number of axially rigid members in the proposed method. The dimensionless parameter, which controls the degree of overall flexural and overall shear deformations, was presented in the simplified model of buildings. In a companion paper [32], the

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accuracy of the methodology was evaluated by comparing the results of the approximation method with the computed response by using detailed finite element analyses for two generic buildings; and then the results were compared to the recorded accelerations for the case of the four instrumented buildings.

Rafezy et al. [35] proposed a global approach to the calculation of natural frequencies of doubly asymmetric, three dimensional, multi-bay, and multi-storey frame structures. It was assumed that the primary frames of the original structure ran in two original directions and that their properties may have varied in a step-wise fashion at one or more storey levels. The structure was therefore divided naturally into uniform segments according to changes in section properties.

A typical segment was then replaced by an equivalent shear-flexure-torsion coupled beam, whose governing differential equations were formulated using the continuum approach and being posed in the form of a dynamic member stiffness matrix.

Kuang and Ng [42] derived the governing equation and the corresponding eigenvalue problem of asymmetric frame structures using the continuum assumption. A method for a theoretical solution was proposed and a general solution to the eigenvalue equation of the problem was presented for determining the coupled natural frequencies and associated mode shapes based on the theory of differential equations.

Bozdogan [44] proposed the Transfer Matrix method for lateral static and dynamic analyses of wall-frame buildings. Step changes of properties along the height of the structure were not allowed in any of the studies with the exception of Rafezy and Howson’s and Bozdogan’s work.

A method for the vibration analysis of proportional asymmetric shear wall structures is suggested in this study. The following assumptions are made in this study; the behavior of the material is linear elastic, small displacement theory is valid, P-delta effects are negligible, the flexural rigidity and geometric center at each floor is assumed to lie on a vertical line through the height of the structures, the shear deformations of walls are negligible and the floor system is rigid in its plane.

2. ANALYSIS

a) Transfer matrix method

As the number of constants to be determined by the use of boundary conditions increases in various engineering problems, the computations become more tedious and the possibility of making errors increases. Therefore, ways of reducing the number of constants to a minimum number is required and the transfer matrix method makes this possible. The main principle of this theory, which is applied to problems with one variable, is to convert all the boundary value problems into problems of initial values. Thus new constants that may result from the use of intermediate condition are eliminated. Therefore, it can be stated as a method of expressing the equations in terms of the initial constants and that this method makes no distinction between the so-called determinate and indeterminate problems of elastomechanics [45]. The transfer matrix method is an efficient and easily computerized method and it also provides a fast and practical solution since the dimensions of the matrix for elements and the system never change [46].

b) Physical model

Figures 1 and 2 show a typical floor plan of asymmetric, three dimensional shear wall structures and deformations of flexural center [14]. If shear deformations are ignored, shear wall structures demonstrate the bending-warping torsional beam behavior. The differential equation of this equivalent bending – warping torsional beam can be initially written.

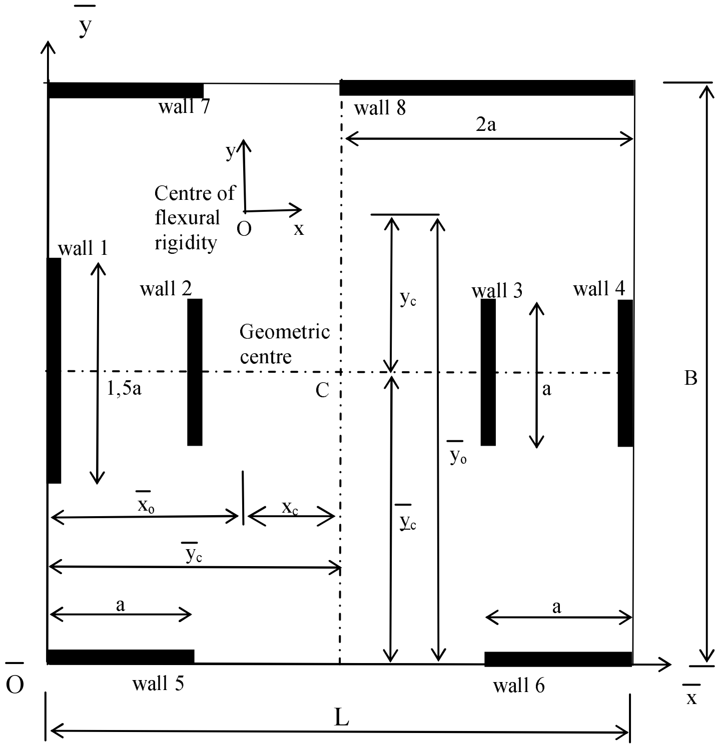

Fig. 1. Typical floor plan of asymmetric frame structures [14]

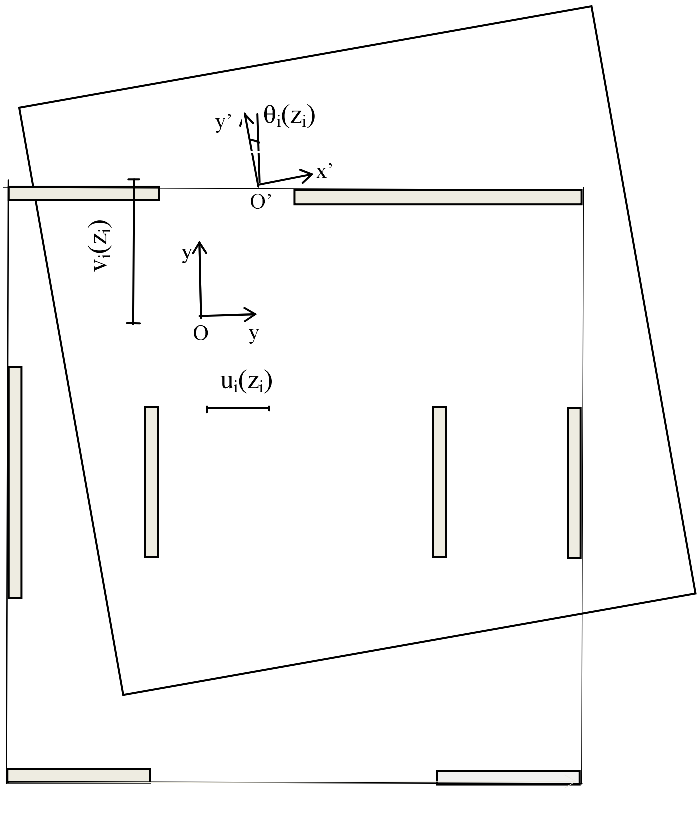

Fig. 2. Coupled translational-torsional vibration of the structure

c) Storey transfer Matrices

Under the horizontal loads governing equations of the ith storey can be written as:

$$(EI)_{xi}\frac{d^4u_i}{dz_i^4}=0 \tag{1}$$

$$(EI)_{yi}\frac{d^4v_i}{dz_i^4}=0 \tag{2}$$

$$(EI)_{wi}\frac{d^4\theta_i}{dz_i^4}=0 \tag{3}$$

where u_i and v_i are the lateral deflections of the flexural center, respectively, θ_i is the torsional rotation of the floor plan about flexural rigidity at the given height, and z_i is the vertical axis of each storey (Fig. 2). $(EI)_{xi}$ and $(EI)_{yi}$ are the equivalent flexural rigidity of the storey for wall structures in x and y directions and can be calculated as follows [14, 41]:

$$EI_{yi}=\sum_j EI_{yi,j} \qquad EI_{xi}=\sum_j EI_{xi,j} \tag{4}$$

Where j is the number of the bent .

$(EI)_{wi}$ are the warping stiffness of ith storey and can be calculated as follows [14];

$$(EI)_{wi}=\sum_j[(\bar{y}_j-\bar{y}_0)^2(EI)_{xi,j}+(\bar{x}_j-\bar{x}_0)^2(EI)_{yi,j}] \tag{5}$$

where $\bar{y}_o$ and $\bar{x}_o$ are the coordinates at the location of the center of flexural rigidity of the jth bent at ith storey in a coordinate system ($\bar{y}_j$, $\bar{x}_j$).

$\bar{y}_o$ and $\bar{x}_o$ are the coordinate of the flexural center and can be calculated as follows [14]:

$$\bar{y}_o=\frac{\sum_j \bar{y}_j(EI)_{xj}}{\sum_j (EI)_{xj}} \tag{6}$$

$$\bar{x}_0=\frac{\sum_j \bar{x}_j(EI)_{yj}}{\sum_j (EI)_{yj}} \tag{7}$$

When Eqs. (1), (2) and (3) are solved with respect to the z_i, $u_i(z_i)$ and $v_i(z_i)$ and $\theta_i(z_i)$ can be obtained as follows:

$$u_i(z_i)=c_4z_i^3+c_3z_i^2+c_2z_i+c_1 \tag{8}$$

$$v_i(z_i)=c_8z_i^3+c_7z_i^2+c_6z_i+c_5 \tag{9}$$

$$\theta_i(z_i)=c_{12}z_i^3+c_{11}z_i^2+c_{10}z_i+c_9 \tag{10}$$

where $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}$ are integral constants. By using Eqs. (8), (9) and (10), the rotation angles in x and y direction (u_i',v_i') , the rate of twist (θ_i'), bending moments in x and y directions (M_{xi}, M_{yi}) and bi-moment (M_{wi}), shear forces in x and y directions (V_{xi},V_{yi}) and torque (T_i) for ith storey can be obtained as follows:

$$u_i'(z_i) = 3c_4 z_i^2 + 2c_3 z_i + c_2 \tag{11}$$

$$v_i'(z_i) = 3c_8 z_i^2 + 2c_7 z_i + c_3 \tag{12}$$

$$\theta_i'(z_i) = 3c_{12} z_i^2 + 2c_{11} z_i + c_{10} \tag{13}$$

$$M_{xi}(z_i) = (EI)_{xi} \frac{d^2 u_i}{dz_i^2} = (EI)_{xi}(2c_3 + 6c_4 z_i) \tag{14}$$

$$M_{yi}(z_i) = (EI)_{yi} \frac{d^2 v_i}{dz_i^2} = (EI)_{yi}(2c_7 + 6c_8 z_i) \tag{15}$$

$$M_{wi}(z_i) = (EI)_{wi} \frac{d^2 \theta_i}{dz_i^2} = (EI)_{wi}(2c_{11} + 6c_{12} z_i) \tag{16}$$

$$V_{xi}(z_i) = (EI)_{xi} \frac{d^3 u_i}{dz_i^3} = (EI)_{xi} 6c_4 \tag{17}$$

$$V_{yi}(z_i) = (EI)_{yi} \frac{d^3 v_i}{dz_i^3} = (EI)_{yi} 6c_8 \tag{18}$$

$$T_i(z_i) = (EI)_{wi} \frac{d^3 \theta_i}{dz_i^3} = (EI)_{wi} 6c_{12} \tag{19}$$

Equation (20) shows the matrix form of the Eqs. (8)-(19):

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ u_i'(z_i) \\ v_i'(z_i) \\ \theta_i'(z_i) \\ M_{xi}(z_i) \\ M_{yi}(z_i) \\ M_{wi}(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ T_i(z_i) \end{bmatrix} = \begin{bmatrix} 1 & z_i & z_i^2 & z_i^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & z_i & z_i^2 & z_i^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & z_i & z_i^2 & z_i^3 \\ 0 & 1 & 2z_i & 3z_i^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2z_i & 3z_i^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2z_i & 3z_i^2 \\ 0 & 0 & 2EI_x & 6z_iEI_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2EI_y & 6z_iEI_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2EI_w & 6z_iEI_w \\ 0 & 0 & 0 & 6EI_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6EI_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6EI_w \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} = A(z_i) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{20}$$

At the starting point of the storey for z_i=0, Eq. (20) can be written as:

$$\begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = A(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \quad (21)$$

The vector in the right-hand side of Eq. (21) can be shown as follows:

$$\mathrm{c} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} \end{bmatrix}^t \quad (22)$$

When vector c is solved by implementing Eq. (21) and then substituted in Eq. (20), Eq. (23) is obtained.

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ u_i'(z_i) \\ v_i'(z_i) \\ \theta_i'(z_i) \\ M_{xi}(z_i) \\ M_{yi}(z_i) \\ M_{wi}(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ T_i(z_i) \end{bmatrix} = A(z_i)A(0)^{-1} \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = T_i \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} \quad (23)$$

T_i represents the storey transfer matrix for z=h_i in Eq. (23).

The storey transfer matrices obtained from Eq. (23) can be used for the dynamic analysis of the proportional asymmetric wall structures.

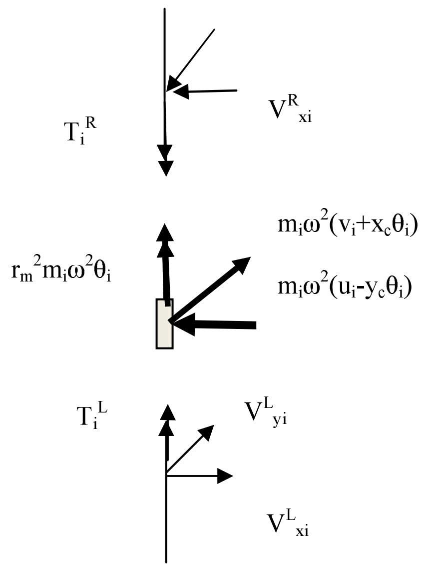

Fig. 3. Free body diagram for a typical floor

When considering the inertial forces in the storey levels (Fig. 3), the relationship between the shear forces of under the floor and above the floor can be written as;

$$V_{xi}^{R} = V_{xi}^{L} + m_i\omega^2(u_i - y_c\theta_i) \tag{24}$$

$$V_{yi}^{R} = V_{yi}^{L} + m_i\omega^2(v_i + x_c\theta_i) \tag{25}$$

$$T_i^{R} = T_i^{L} + r_m^2 m_i\omega^2\theta_i \tag{26}$$

Bending moments in x and y directions (M_{xi}, M_{yi}), bi-moment (M_{wi}) are the same in both sides of the floor.

By continuity, the displacement and the slope on both sides of the floor must be the same:

$$u_i^R = u_i^L \tag{27}$$

$$v_i^R = v_i^L \tag{28}$$

$$\theta_i^R = \theta_i^L \tag{29}$$

$$u_i^{,R} = u_i^{,L} \tag{30}$$

$$v_i^{,R} = v_i^{,L} \tag{31}$$

$$\theta_i^{,R} = \theta_i^{,L} \tag{32}$$

Therefore, when considering the inertial forces in the storey levels, the relationship between the ith and the (i+1)th stories can be shown by the following matrix equation;

$$\begin{bmatrix} u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \\ M_{xi}(h_i) \\ M_{yi}(h_i) \\ M_{wi}(h_i) \\ V_{xi}(h_i) \\ V_{yi}(h_i) \\ T_i(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega^2 m_i & 0 & -\omega^2 m_i y_c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \omega^2 m_i & \omega^2 m_i x_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega^2 m_i y_c & \omega^2 m_i x_c & \omega^2 m_i r_m^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} T_i \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} \tag{33}$$

where m_i is the mass of the ith storey and ω are the natural frequencies of the system and r_m^2 is the inertial radius of gyration, and can be calculated as [14, 41];

$$r_m^2 = \frac{L^2 + B^2}{12} + y_c^2 + x_c^2 \tag{34}$$

y_c and x_c are the dimensions of the location of the geometric center and can be calculated as follows:

$$y_c = \bar{y}_C - \bar{y}_O \tag{35}$$

$$x_c = \bar{x}_C - \bar{x}_O \tag{36}$$

where the coordinate ($\bar{y}_C$, $\bar{x}_c$) is the location of the geometric center C in the coordinate system ($\bar{y}$, $\bar{x}$). $\bar{y}_O$ and $\bar{x}_O$ is the location of the flexural center O in the coordinate system ($\bar{y}$, $\bar{x}$).

Dynamic transfer matrix can be shown as T_{di}.

$$T_{di} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega^2 m_i & 0 & -\omega^2 m_i y_c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \omega^2 m_i & \omega^2 m_i x_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega^2 m_i y_c & \omega^2 m_i x_c & \omega^2 m_i r_m^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} T_i \tag{37}$$

The displacements - internal forces relationship between the base and the top of the structure can be found as follows:

$$\begin{bmatrix} u_{top} \\ v_{top} \\ \theta_{top} \\ u'_{top} \\ v'_{top} \\ \theta'_{top} \\ M_{xtop} \\ M_{ytop} \\ M_{wtop} \\ V_{xtop} \\ V_{ytop} \\ T_{top} \end{bmatrix} = T_{dn} * T_{d(n-1)} * .....T_{di} * ....T_{d2} * T_{d1} \begin{bmatrix} u_{base} \\ v_{base} \\ \theta_{base} \\ u'_{base} \\ v'_{base} \\ \theta'_{base} \\ M_{xbase} \\ M_{ybase} \\ M_{wbase} \\ V_{xbase} \\ V_{ybase} \\ T_{base} \end{bmatrix} \quad (38)$$

The boundary conditions of the bending- warping beam are:

1) $u_{base}=0$ 2) $v_{base}=0$ 3) $\theta_{base}=0$ 4) $u^{I}_{base}=0$ 5) $v^{I}_{base}=0$ 6) $\theta^{I}_{base}=0$
7) $M_{xtop}=0$ 8) $M_{ytop}=0$ 9) $M_{wtop}=0$ 10) $V_{xtop}=0$ 11) $V_{ytop}=0$ 12) $T_{top}=0$

When boundary conditions are considered for equation (38) for the nontrivial solution of $\mathbf{t_d} = \mathbf{T_{dn}} \mathbf{T_{dn-1}} \mathbf{T_{dn-2}} \mathbf{......T_{d1}}$, equation (39) can be attained:

$$\text{f} = \begin{bmatrix} t(7,7) & t(7,8) & t(7,9) & t(7,10) & t(7,11) & t(7,12) \\ t(8,7) & t(8,8) & t(8,9) & t(8,10) & t(8,11) & t(8,12) \\ t(9,7) & t(9,8) & t(9,9) & t(9,10) & t(9,11) & t(9,12) \\ t(10,7) & t(10,8) & t(10,9) & t(10,10) & t(10,11) & t(10,12) \\ t(11,7) & t(11,8) & t(11,9) & t(11,10) & t(11,11) & t(11,12) \\ t(12,7) & t(12,8) & t(12,9) & t(12,10) & t(12,11) & t(12,12) \end{bmatrix} \quad (39)$$

The values of ω, which set the determinant to zero, are natural frequencies of the asymmetric wall building.

3. PROCESS OF COMPUTATION

The process of the computation for the transfer matrix method is presented below:

1. The equivalent rigidities of each storey are calculated using the geometric and the material properties of the structure.
2. Storey transfer matrices are calculated for each storey by using the equivalent rigidities.
3. System transfer matrix (equation 38) is obtained with the help of storey transfer matrices and inertia forces effecting the storey levels with the procedure specified in section 3.

4) The nontrivial equation is obtained using Eq. (39) as a result of the application of the boundary conditions.

5) The angular frequencies and relevant periods are found with the help of a method obtained from numerical analysis (Newton-Raphson, Regula Falsi etc.).
6) The modes are found with the help of the angular frequency and the equation (33).
7) The effective mass ratio (M) and participation factor (Γ) can be found as,

$$M_{xn} = \frac{\left[\sum_{i=1}^{N} (m_i \Phi_{ixn})\right]^2}{\sum_{i=1}^{N} (m_i \Phi_{ixn}^2 + m_i \Phi_{ixn}^2 + m_i r_m^2 \Phi_{i\theta n})} \quad (40) \qquad M_{xn} = \frac{\left[\sum_{i=1}^{N} (m_i \Phi_{ixn})\right]^2}{\sum_{i=1}^{N} (m_i \Phi_{ixn}^2 + m_i \Phi_{ixn}^2 + m_i r_m^2 \Phi_{i\theta n})} \quad (41)$$

$$\Gamma_{xn} = \frac{\sum_{i=1}^{N} (m_i \Phi_{ixn})}{\sum_{i=1}^{N} (m_i \Phi_{ixn}^2 + m_i \Phi_{ixn}^2 + m_i r_m^2 \Phi_{i\theta n})} \quad (42) \qquad \Gamma_{yn} = \frac{\sum_{i=1}^{N} (m_i \Phi_{iyn})}{\sum_{i=1}^{N} (m_i \Phi_{ixn}^2 + m_i \Phi_{ixn}^2 + m_i r_m^2 \Phi_{i\theta n})} \quad (43)$$

Where N is the number of storey Φ is the mod shape.
8) With the help of the acceleration and the displacement spectrums, obtained from an earthquake record or design spectrum from codes, the displacement and internal forces are found using the effective mass and the participation factor.

4. A NUMERICAL EXAMPLE

A numerical example has been solved by a program written in MATLAB to verify the proposed method in this part of the study. The results are then compared with those given in the literature.

Example 1. A typical asymmetric shear-wall system (Fig 1) is analyzed as an example. The general multi-bent is considered as an asymmetric reinforced concrete shear-wall building (Fig.1). The structure has 25 storeys with total height H=75 m, and floor dimensions L=24 m and B=24 m. The structure consists of eight walls 0.25-m thick and 6 m long (a=6 m). An elastic modulus is $E=20*10^6$ kN/m^2 and the density of the floor slabs is $\rho=2.350$ kg/m^3. The structural properties are given in Table 1. The natural frequencies calculated by this method are compared with the results in the reference [14]. The results are presented in Table 2.

Table 1. Structural properties of asymmetric shear wall structures

Structural properties of asymmetric frame structures	
$(EI)_x$	$990.70*10^6$ kNm^2
$(EI)_y$	$574.53*10^6$ kNm^2
$(EI)_w$	$136.66*10^9$ kNm^4
x_c	4.463 m
y_c	-7.631 m
m	202.980 kNsn^2/m
r_m	13.1966 m

Table 2. Comparison of natural frequencies in Example 1

Natural frequencies of the first three modes (s^{-1})									
	Proposed Method			Kuang and Ng			ETABS (Kuang and Ng)		
Mode	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1	1.560	1.927	3.696	1.622	2.004	3.843	1.592	1.963	3.753
2	9.782	12.085	23.180	10.163	12.556	24.083	9.794	12.207	22.906
3	27.411	33.864	53.757	28.457	35.156	67.432	27.292	33.395	63.724

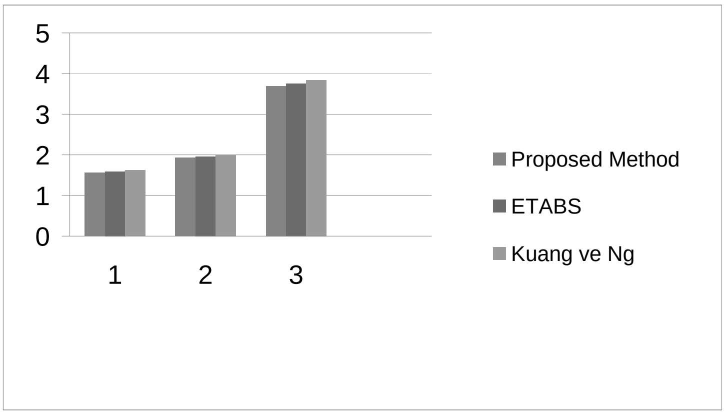

Fig. 4. Comparison of natural frequencies of the first mode

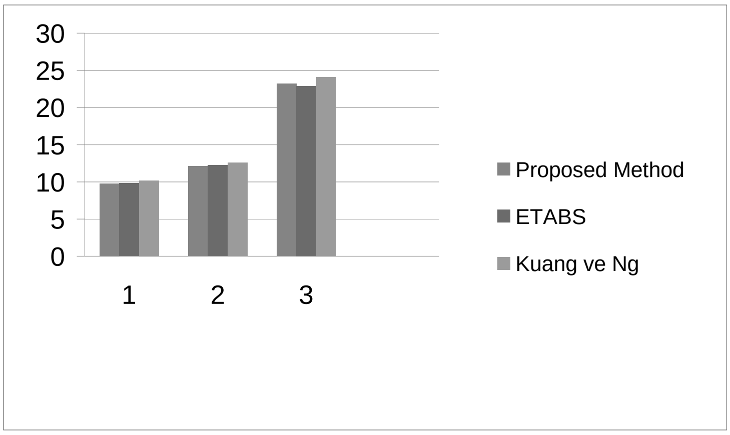

Fig. 5. Comparison of natural frequencies of the second mode

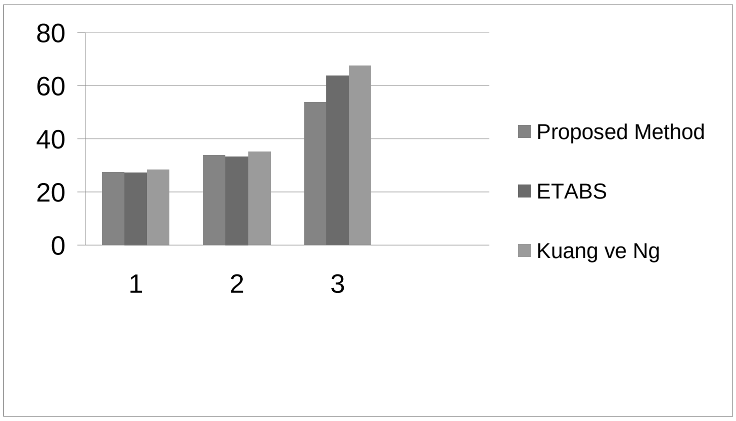

Fig. 6. Comparison of natural frequencies of the third mode

5. CONCLUSION

This paper presents a method for the vibration analysis of proportional asymmetric shear wall structures. The whole structure is assumed as an equivalent bending –warping torsion beam in this method. The governing differential equations of equivalent bending-warping torsion beam are formulated using the continuum approach and are posed in the form of the simple storey transfer matrix. By using the storey transfer matrices and the point transfer matrices which take into account the inertial forces, the system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. The example solved in this study shows that the results obtained from the proposed method are in close agreement with the solution that was developed in the literature. In the proposed method the structural properties of a building may change and different numerical examples can also be solved. The proposed method is simple and accurate enough to be used both at the concept design stage and for final analyses.

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