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# MODIFIED PREDICTIVE OPTIMAL LINEAR CONTROL OF STRUCTURES IN SEISMIC REGION*

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**Abstract**– A modified predictive optimal linear control (MPOLC) algorithm is proposed for controlling the seismic response of elastic structures. This algorithm compensates for the time delay that occurs in real control applications by predicting the structural response in the modified optimal linear control equation. Since the environmental loads and disturbances are not measured during real-time control, they are not involved in the derivation of the control algorithm. Therefore the predictive optimal linear controller (POLC) is a proportional feedback of the only predicted current state. In the modified control algorithm (MPOLC), using a logical assumption, the immeasurable disturbances are considered in the state space equation and also in the derivation of the control algorithm, so that the controller is a combination of the control force in the last step and the proportional feedback of the predicted states in the last two steps. Hence, the control performance of the modified control algorithm is superior to that of the original one. The feasibility and effectiveness of the proposed control algorithm is verified through frequency-domain and time-domain analyses, and compared with the original one. The tendon control system of a three-degree-of-freedom structure is illustrated to demonstrate the control effectiveness of the modified predictive control algorithm.

**Keywords**– Time delay, response prediction, active structural control, structural dynamics, earthquake

## 1. INTRODUCTION

Structural control has been widely studied since the early 1970s as a new method of reducing structural damages and preventing collapses during earthquake excitations. Many analytical and experimental studies have been performed and have shown that significant response reduction during earthquakes can be achieved by adding an active control system to the structure [1-5]. However, some of these experimental results differ from those computed analytically, even in a well-controlled laboratory environment. One of the most important reasons is that time delay exists in all structural control systems. An important assumption is generally made in most numerical studies that all operation in the control system can be performed instantaneously. In reality, however, time has to be consumed in processing measured information, performing on-line computation, and executing the required control forces. Therefore, as stated clearly by Agrawal and Yang [6], time delay causes an unsynchronized application of the control forces, and as a result, it degrades the control performance and even renders the controlled structures unstable [8, 9].

Research efforts in active control have been focused on a variety of control algorithms based on several control design criteria. Some are considered classical as they are applications of modern control theory. In classical linear optimal control, the control forces are to be chosen in such a way that a

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quadratic performance functional is minimized subject to the constraining dynamic equilibrium equation. Since the environmental loads and disturbances are not measured during real-time control, they are not involved in the derivation of the control algorithm, making the controller a proportional feedback of current state [4 -11, 16].

Wong [1] developed a simple analysis method using predictive optimal linear control (POLC) to reduce the degradation of control systems due to time delay $\tau$. In this predictive algorithm, the structural response at time $t$ is predicted at time $t - \tau$ based on the structural response and earthquake ground motion at time $t - \tau$. Using this predicted response, the control force at time $t$ can be computed at time $t - \tau$ and be applied when the time $t$ arrives.

The objective of this paper is to develop a modified predictive linear optimal control (MPOLC) to reduce the degradation of control systems due to time delay $\tau$ and the immeasurable disturbances which can be ground motions, wind excitations, modeling errors or parametric uncertainties. In this modified control algorithm, the immeasurable disturbances are considered in the state space equation and also in the derivation of the control algorithm. A new variable is defined and the disturbances are assumed to have a constant value for two successive time steps. With this definition and using this logical assumption, a new state space form of the dynamic equation is obtained so that the immeasurable disturbances are eliminated. Then using a new prediction model of structural response and earthquake ground motion, the control force at time $t$ can be computed at time $t - \tau$ and be applied when time $t$ arrives. The feasibility and effectiveness of the proposed control algorithm is verified through frequency-domain and time-domain analyses, and compared with the original one.

## 2. DERIVATION OF CONTROL ALGORITHMS

The response of an elastic structure with $n$ degrees of freedom, installed with an active control system consisting of $n_c$ control actuators, is described by the following dynamic equilibrium equation:

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = E_1\ddot{X}_g(t) + B_1u(t) \tag{1}$$

Where $M$, $C$, and $K$ are structural mass, damping, and stiffness matrices, respectively; $X(t)$, $\dot{X}(t)$, and $\ddot{X}(t)$ are structural displacement, velocity, and acceleration vectors, respectively; $\ddot{X}_g(t)$ is earthquake acceleration corresponding to each degree of freedom (DOF); $B_1$ is $n \times n_c$ location matrix of active control forces; $E_1$ is the $n \times 1$ allocation matrix of external loads; and u(t) is $n_c$-dimensional control force vector.

The state space form of Eq. (1) can be written as

$$\dot{z}(t) = A_c z(t) + B_c u(t) + E_c d(t) \tag{2}$$

Where $z(t)$ is $2n$ state vector; $A_c$ is $2n \times 2n$ continuous time state transition matrix; $B_c$ is $2n \times n_c$ control force transition matrix; $E_c$ is $2n \times 1$ ground acceleration transition matrix; and $d(t)$ is generally environmental load or disturbance, which can be wind excitations, modeling errors or parametric uncertainties. But in this paper, $d(t)$ is the earthquake acceleration time history ($\ddot{X}_g(t)$ in Eq.(1)). Hence, in other equations, $\ddot{X}_g(t)$ is replaced to $d(t)$, which emphasizes the general concept of disturbances.

These matrices and vectors are given by

$$z(t) = \begin{pmatrix} X(t) \\ \dot{X}(t) \end{pmatrix}, \quad A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ M^{-1}B_1 \end{bmatrix}, \quad E_c = \begin{bmatrix} 0 \\ M^{-1}E_1 \end{bmatrix} \tag{3}$$

The incremental solution of Eq. (2) can be obtained as follows:

$$z(t+\Delta t) = e^{A_c \Delta t} z(t) + e^{A_c (t+\Delta t)} \int_t^{t+\Delta t} e^{-A_c s} [B_c u(s) + E_c d(s)] ds \tag{4}$$

Where $\Delta t$ is integration time step and "s" is a time variable that in integral Eq. (4) is varied from $t$ to $t+\Delta t$. The reason for using "s" instead of "t" is only for distinguishing the integral limitation. Let $t_{k+1} = t + \Delta t$ and $t_k = t$, then integrating Eq. (4) by assuming that the control force $u(s)$ is uniform over the duration $t_k$ to $t_{k+1}$ and that the ground motion $d(s)$ as pulses gives

$$z_{k+1} = Az_k + Bu_k + Ed_k \tag{5}$$

Where

$$A = e^{A_c \Delta t} \qquad , B = A_c^{-1}(A - I)B_c \qquad , E = AE_c \Delta t \tag{6}$$

and $z_k$, $u_k$, and $d_k$ are discretized forms of $z(t)$, $u(t)$, and $d(t)$, respectively. Eq.(5) is a recursive equation of performing the entire time history analysis.

In fact, the discrete state space form of a linear system must be expressed as

$$z_{k+1} = Az_k + Bu_k \tag{7}$$

Since the environmental loads and disturbances are not available and so not measurable during real-time control, they are not involved in the state space equation and also in the derivation of the control algorithm. In other words, since there is a time delay in the real-time control process as well as the environmental load, e.g. the earthquake acceleration is unknown and not predictable, (in fact, the disturbance $d_k$ is not effective in the calculation of control force $u_k$) we can say that the discrete state space equilibrium equation as shown in Eq.(7).

Therefore, the control force at time step *k* (i.e., $u_k$) in Eq.(7) depends on the response of the structure. To determine this control force that gives an optimal linear control, define the cost function $J$ to be

$$J = \tfrac{1}{2} \sum_{k=0}^{N} (z_k^T Q z_k + u_k^T R u_k) \tag{8}$$

Where Q is a $2n \times 2n$ positive semi-definite weighing matrix and R is $p \times p$ positive-definite weighing matrix. Minimizing the cost function $J$, generally, according to Eq. (5), the control force equation is obtained as [1, 12]

$$u_k = -(B^T PB + R)^{-1} B^T PAz_k - (B^T PB + R)^{-1} B^T PEd_k \tag{9}$$

Where P is steady state Riccati matrix and is given by

$$P = A^T P(I + BR^{-1} B^T P)^{-1} A + Q \tag{10}$$

but due to the elimination of the immeasurable disturbances in Eq.(7), Kalman proposed just the first term on the right-hand side of Eq. (9), which is incorporated to the control force equation as [13]

$$u_k = -K_{ss} z_k \tag{11}$$

Where $K_{ss}$ is proportional state feedback gain matrix and is given by

$$K_{ss} = \left[R + B^T PB\right]^{-1} B^T PA \tag{12}$$

It is observed from Eq. (11) that disturbance $d_k$ is not incorporated into the minimizing process of cost function $J$ and so to the control force equation (Eq. (11)). Therefore, this method of calculating the control force without considering disturbance is not suitable enough.

## 3. MODIFIED OPTIMAL LINEAR CONTROL

So far in the optimal linear control method, the control force at instant $k$ ($u_k$) is only dependent and affected by the response of the structure at the same instant $k$, i.e. $z_k$. In the definition of cost function $J$, only the control force and the state of system $z_k$ are considered (regardless of effect of acceleration $d_k$), and by minimizing $J$ and solving the Riccati equation, coefficient $K_{ss}$ is achieved without considering $d_k$. So it appears that the acceleration $d_k$ does not exist in the state space equation, i.e. Eq. (7) is valid. It should be noted that due to the validity of Eq. (5), proportional coefficient $K_{ss}$ can be multiplied by $Ed_k$, as shown in Eq. (9). However, the effect of acceleration $d_k$ is not incorporated yet in calculating $K_{ss}$. But the manner which is discussed below overcomes this problem.

The discrete state space form of a linear system with immeasurable disturbances is expressed as

$$z_{k+1} = Az_k + Bu_k + Ed_k \tag{13}$$

Where $d_k$ are immeasurable disturbances if we define $y_k$ as

$$y_k = Bu_k + Ed_k \tag{14}$$

With this definition of the vector $y_k$, the original equation can be expressed as

$$z_{k+1} = Az_k + y_k \tag{15}$$

The disturbances are assumed to have a constant value for two successive time steps, so we have

$$y_{k+1} - y_k = \Delta y_k = B\Delta u_k \tag{16}$$

and consequently,

$$y_{k+1} = y_k + B\Delta u_k \tag{17}$$

The reason for this essential assumption is that, with developments in technology, advances are made in control devices such as systems for measuring data (such as disturbances, internal noises to a system, earthquake acceleration, a state of the system, etc.), so that the possibility of measuring and sampling this data is provided for an infinitesimal fraction of time by sampler devices. Hence, if the interval of the time-step for data sampling is very small, the variation of the sampled data in two successive time steps will be small too. On the other hand, by using the zero order holder (ZOH) technique in the sampler device, the function of the sampled data will be the same as a piecewise continuous function. Hence, for small time steps (in this paper $\Delta t = 0.01s$), the variation of sampled data in two successive time steps is too small and can be assumed equal to zero.

Combining Eqs. (15) and (17) yields

$$\begin{pmatrix} z_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{bmatrix} A & I \\ 0 & I \end{bmatrix} \begin{pmatrix} z_k \\ y_k \end{pmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} \Delta u_k \tag{18}$$

Now, the state variable, the control variable and the parameters of the system are defined as follows:

$$\tilde{z}_k = \begin{pmatrix} z_k \\ y_k \end{pmatrix} \quad , \quad \tilde{u}_k = \Delta u_k \tag{19}$$

$$\tilde{A} = \begin{bmatrix} A & I \\ 0 & I \end{bmatrix} \quad , \quad \tilde{B} = \begin{bmatrix} 0 \\ B \end{bmatrix} \tag{20}$$

Then the original problem is transformed to the equation set

$$\tilde{z}_{k+1} = \tilde{A}\tilde{z}_k + \tilde{B}\tilde{u}_k \tag{21}$$

We want to develop the optimal control for the quadratic performance functional

$$J = \tfrac{1}{2}\sum_{k=1}^{N-1}(\tilde{z}_k^T\tilde{Q}\tilde{z}_k^T + \tilde{u}_k^T\tilde{R}\tilde{u}_k) \tag{22}$$

Where

$$\tilde{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix} \quad , \quad \tilde{R} = R \tag{23}$$

Performance parameters defined above result in the same performance index as Eq. (8)
Equations (21) and (22) show that we have transformed the problem into an equivalent LQR problem so that the optimal control is

$$\tilde{u}_k = -\tilde{K}_k\tilde{z}_k \tag{24}$$

Where

$$\tilde{K}_k = \left[\tilde{R} + \tilde{B}^T P_{k+1}\tilde{B}\right]^{-1}\tilde{B}^T P_{k+1}\tilde{A} \tag{25}$$

And $P_k$ can be computed from the appropriate discrete Riccati equation

$$P_k = \tilde{A}^T P_{k+1}\tilde{A} - \tilde{A}^T P_{k+1}\tilde{B}\left[\tilde{B}^T P_{k+1}\tilde{B} + \tilde{R}\right]^{-1}\tilde{B}^T P_{k+1}\tilde{A} + \tilde{Q} \tag{26}$$

This is a recursive relation that can be solved with one final condition. The steady-state solution to the Riccati equation leads to

$$\tilde{u}_k = -\tilde{K}_{ss}\tilde{z}_k \tag{27}$$

Finally, in terms of the original problem variables, we have

$$\Delta u_k = -\left[K_1 \;\vdots\; K_2\right]\begin{pmatrix} z_k \\ y_k \end{pmatrix} \tag{28}$$

By rearranging Eq. (15), then

$$y_k = z_{k+1} - Az_k \tag{29}$$

Using Eq. (29) in (28) gives

$$u_{k+1} - u_k = -K_1 z_k - K_2(z_{k+1} - Az_k) \tag{30}$$

This equation expresses that the control is a unique function of the state variable. By rearranging Eq. (30) and transition instant *k* to *k*-1, the linear control law is obtained as

$$u_k = u_{k-1} - K_2 z_k - \left[K_1 - K_2 A\right]z_{k-1} \tag{31}$$

This shows that the modified optimal linear control for a linear system with immeasurable disturbances is the proportional feedback of states in the two last steps, added to the control force in the last step.

So far, based on the conventional optimal linear control theory, the disturbances and environmental loads are not incorporated to minimize the process of cost function, and so to the control force calculation.

But now, for solving this problem, a modified optimal linear control (MOLC) algorithm for the active control of structures is proposed, in which the disturbances are effective in minimizing the process of the cost function (Eq. (22)) and the control force calculation (Eq. (31)).

## 4. OPTIMAL LINEAR CONTROL WITH TIME DELAY

If time delay $\tau$ exists in the control system, the control force computed at time $t - \tau$ will be applied to the structure at time $t$ [14]. Let $j$ be the number of time steps of the time delay, i.e., $\tau = j\Delta t$ , then the control force computed in Eqs. (11) and (31) will be applied at time step $k+j$ , i.e.,

$$\mathrm{u}_{k+j} = -\mathrm{K}_{ss}\mathrm{z}_k \tag{32}$$

$$\mathrm{u}_{k+j} = \mathrm{u}_{k+j-1} - \mathrm{K}_2\mathrm{z}_k - [\mathrm{K}_1 - \mathrm{K}_2\mathrm{A}]\mathrm{z}_{k-1} \tag{33}$$

The state space equation as given in Eq. (5) for the optimal linear control or a modified one with time delay $\tau$ becomes

$$\mathrm{z}_{k+1} = \mathrm{A}\mathrm{z}_k - \mathrm{B}\mathrm{K}_{ss}\mathrm{z}_{k-j} + \mathrm{E}\mathrm{d}_k \tag{34}$$

$$\mathrm{z}_{k+1} = \mathrm{A}\mathrm{z}_k - \mathrm{B}(\mathrm{u}_{k-1} - \mathrm{K}_2\mathrm{z}_{k-j} - [\mathrm{K}_1 - \mathrm{K}_2\mathrm{A}]\mathrm{z}_{k-j-1}) + \mathrm{E}\mathrm{d}_k \tag{35}$$

In these OLC and MOLC algorithms with time delay, the control force $\mathrm{u}_k$ depends only on the states at time steps $k$-$j$ and $k$-$j$-$1$. Therefore, the actual implementation of this algorithm is physically possible, however the control result may be jeopardized due to the time delay.

## 5. PROPOSED PREDICTIVE OPTIMAL LINEAR CONTROL FOR COMPENSATING FOR TIME DELAY

According to the control force equations presented in Eqs. (11) and (31), the control force at time step $k$ should be regulated by the structural states and the earthquake ground motion at the same time step. Consider a time delay $\tau = j\Delta t$ and let $t_k = t$ and $t_{k-j} = t - \tau$ , the optimal control force, computed at time step $k$-$j$ (i.e., $\mathrm{u}_{k-j}$ ), will only be applied when time step $k$ arrives if Eqs. (11) and (31) are used directly, and this unsynchronized application of the control force with structural vibration may cause detrimental effects to the stability of the controlled structure. In order to avoid this problem, the structural response at time $t$, i.e., $\mathrm{z}_k$ , must be calculated at time step $k$-$j$, then the control force at time step $k$, i.e., $\mathrm{u}_k$ , can be applied at the right time when time step $k$-$j$ arrives. However, the precise calculation of $\mathrm{z}_k$ at time step $k$-$j$ is practically impossible because the ground motion within the time steps $k$-$j$ to $k$ (i.e., $\mathrm{d}_{k-j}, \mathrm{d}_{k-j+1}, \ldots, \mathrm{d}_k$ ) are unknown. Hence predicting $\mathrm{z}_k$ becomes necessary in order to compensate for the negative effects of the time delay [1].

Consider again Eq. (2), where the solution can be written in the form

$$\hat{\mathrm{z}}_k = e^{\mathrm{A}_c j\Delta t}\mathrm{z}_{k-j} + e^{\mathrm{A}_c t_k}\int_{t_{k-j}}^{t_k} e^{-\mathrm{A}_c s}[\mathrm{B}_c\mathrm{u}(s) + \mathrm{E}_c\mathrm{d}(s)]\mathrm{d}s \tag{36}$$

Where $\hat{\mathrm{z}}_k$ represents the predicted value of $\mathrm{z}_k$ . Eq. (36) is similar to Eq. (4) except that $\Delta t$ in Eq. (4) is replaced by $j\Delta t$ in Eq. (36). Performing integration in Eq. (36) first requires numerical discretization of both the control force u(s) and the earthquake ground acceleration d(s) between times $t_{k-j}$ and $t_k$ .

First consider the control force u(s), where the objective is to determine the control force at time step *k* (i.e., $u_k$). At time step *k-j*, the control forces within time steps *k-j* and *k*-1 will be known since they are determined at the previous time steps. For example, the control force at time step *k*-1 (i.e., $u_{k-1}$) should have been computed at time step *k-j*-1, and similarly the control force at time step *k-j* (i.e., $u_{k-j}$) should have been computed at time step *k-2j* [1]. Therefore integration of the control force term in Eq. (36), by using the trapezoidal rule, can be performed as follows:

$$\int_{t_{k-j}}^{t_k} e^{-A_c s} B_c u(s)ds = \int_{t_{k-j}}^{t_{k-j+1}} e^{-A_c s} B_c u(s)ds + \int_{t_{k-j+1}}^{t_{k-j+2}} e^{-A_c s} B_c u(s)ds + \cdots + \int_{t_{k-1}}^{t_k} e^{-A_c s} B_c u(s)ds$$
$$= \Delta t\left[\tfrac{1}{2} e^{-A_c t_{k-j}} B_c u_{k-j} + e^{-A_c t_{k-j+1}} B_c u_{k-j+1} + \cdots + e^{-A_c t_{k-1}} B_c u_{k-1} + \tfrac{1}{2} e^{-A_c t_k} B_c u_k\right] \tag{37}$$

Now, substituting this result back into Eq. (36) gives

$$\hat{z}_k = e^{A_c j\Delta t} z_{k-j} + \Delta t\left[\tfrac{1}{2} e^{A_c j\Delta t} B_c u_{k-j} + e^{A_c (j-1)\Delta t} B_c u_{k-j+1} + \cdots + e^{A_c \Delta t} B_c u_{k-1} + \tfrac{1}{2} B_c u_k\right]$$
$$+ e^{A_c t_k} \int_{t_{k-j}}^{t_k} e^{-A_c s}(E_c d(s))ds \tag{38}$$

In order to integrate the last term on the right-hand side of Eq. (38), approximation to the earthquake ground motion is necessary. In this paper, it is assumed to be a white noise process with a mean equal to zero. Hence the expectations of the acceleration values at time $t_{k-j+1}$ to $t_k$ are

$$E(d_{k-j+1}) = E(d_{k-j+2}) = \cdots = E(d_k) = 0 \tag{39}$$

Based on this assumption, the integral in Eq. (38) becomes

$$e^{A_c t_k} \int_{t_{k-j}}^{t_k} e^{-A_c s}(E_c d(s))ds = e^{A_c j\Delta t} E_c \Delta t d_{k-j} \tag{40}$$

After evaluating the integrals, Eq. (38) becomes

$$\hat{z}_k = e^{A_c j\Delta t} z_{k-j} + \Delta t\left[\tfrac{1}{2} e^{A_c j\Delta t} B_c u_{k-j} + e^{A_c (j-1)\Delta t} B_c u_{k-j+1} + \cdots + \tfrac{1}{2} B_c u_k\right] + e^{A_c j\Delta t} E_c \Delta t d_{k-j} \tag{41}$$

Where $\hat{z}_k$ is the predictive state at time step $t_k$. Let the control force $u_k$ follow the optimal linear control or modified one, i.e.

$$u_k = -K_{ss}\hat{z}_k \tag{42}$$

Or

$$u_k = u_{k-1} - G_1 \hat{z}_{k-1} - G_2 \hat{z}_k \tag{43}$$

According to Eq. (31), $G_1$ and $G_2$ are $K_1 - K_2 A$ and $K_2$, respectively.

Then substituting the control force in Eqs. (42) or (43) into Eq.(41), and solving for the predictive state give, respectively,

$$\hat{z}_k = [I + \frac{\Delta t}{2} B_c K_{ss}]^{-1}\left\{e^{A_c j\Delta t}(z_{k-j} + E_c \Delta t d_{k-j}) + \Delta t\left[\tfrac{1}{2} e^{A_c j\Delta t} B_c u_{k-j} + \cdots + e^{A_c \Delta t} B_c u_{k-1}\right]\right\} \tag{44}$$

$$\hat{z}_k = [I + \frac{\Delta t}{2} B_c G_2]^{-1}\left\{e^{A_c j\Delta t}(z_{k-j} + E_c \Delta t d_{k-j}) + \Delta t\left[\tfrac{1}{2} e^{A_c j\Delta t} B_c u_{k-j} + \cdots + \tfrac{1}{2} B_c (u_{k-1} - G_1 \hat{z}_{k-1})\right]\right\} \tag{45}$$

As shown in the above equations, the control forces at time step $k$ can be totally determined by the state and earthquake acceleration at time step $k$-$j$, and the control forces within time steps $k$-$j$ and $k$-1.

Finally, the real state at time step k+1 is calculated when the time actually arrives, and it is computed based on Eq. (5). In this equation, the optimal linear control force is calculated by using Eqs. (42) and (44) and the modified one is obtained by using Eqs. (43) and (45).

The real state $z_k$ determined in Eq. (5), the control force $u_k$ determined in Eqs. (42) or (43), and the measured earthquake ground acceleration $d_k$ will then be used to predict the states $\hat{z}_{k+j}$ using Eqs. (44) and (45). This recursive calculation process continues until the strong motion phase of the earthquake has diminished.

## 6. NUMERICAL VERIFICATION

The tendon control system of a multiple-degree-of-freedom (MDOF) structure is studied to verify the feasibility of the modified predictive optimal linear control algorithm numerically. The control effectiveness is evaluated through frequency-domain and time-domain analyses.

In time-domain analysis, the reduction of the structural responses and the applied control forces are demonstrated by subjecting the structure to real earthquakes, 1940 El Centro, 1995 Kobe and 1994 Northridge. (Fig. 1)

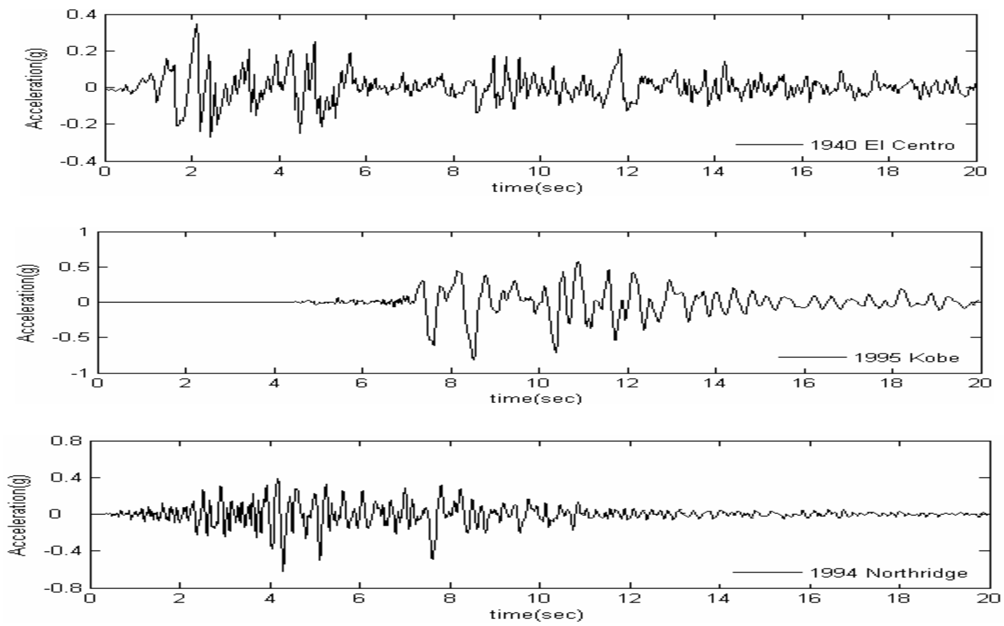


Fig. 1. Magnified El Centro, Kobe and Northridge earthquakes time history

A three-story structure (Fig. 2) is subjected to earthquake excitation and counteracted by the tendon control device implementation on the first floor [15]. Relevant parameters of the control system are listed in Table 1.

In frequency-domain analysis, the degree of vibration suppression is shown by the magnitude of the frequency response functions of the control systems.

The natural frequencies of the uncontrolled structure are 2.24 Hz, 6.80 Hz and 11.49 Hz, respectively, and the damping ratios are 1.61%, 0.39%, and 0.36%, respectively. The structural states are sampled with period $\Delta t = 0.01s$.

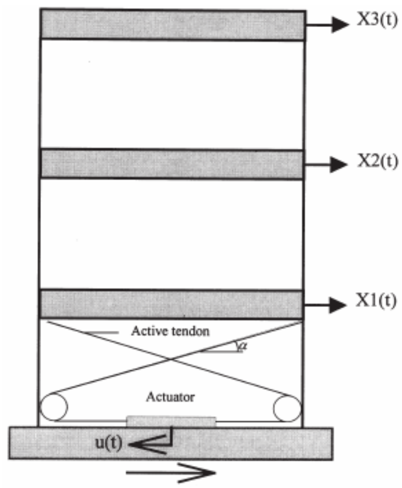


Fig. 2. MDOF model structure with control device

Table 1. Relevant parameters of MDOF control system

| System parameter | Parameter value |
|---|---|
| Mass matrix, **M** (kg) d(t) | $\begin{bmatrix} 981 & 0 & 0 \\ 0 & 981 & 0 \\ 0 & 0 & 981 \end{bmatrix}$ |
| Stiffness matrix, **K** (N/cm) | $\begin{bmatrix} 27417 & -16416 & 3691 \\ -16416 & 30222 & -16248 \\ 3691 & -16248 & 13336 \end{bmatrix}$ |
| Damping matrix, **C** (N-s/cm) | $\begin{bmatrix} 3.828 & -0.573 & 0.617 \\ -0.573 & 4.569 & -0.026 \\ 0.617 & -0.026 & 4.375 \end{bmatrix}$ |
| Tendon stiffness, $k_c$ (N/cm) | 3721 |
| Tendon inclination, **α** (°) | 36 |
| Control forces location matrix, $\mathbf{B}_1$ | $\begin{bmatrix} 4k_c \cos\alpha \\ 0 \\ 0 \end{bmatrix}$ |
| Loads allocation matrix, $\mathbf{E}_1$ | $\begin{bmatrix} -981 \\ -981 \\ -981 \end{bmatrix}$ |
| Response weighing matrix, **Q** | $\begin{bmatrix} \mathbf{K} & 0 \\ 0 & \mathbf{M} \end{bmatrix}$ |
| Control weighing matrix, **R** | $4k_c$ |

Top-floor displacements of a three-story structure for ideal control (without time delay) and predictive control with time delay steps L=10, 20, 30 are studied in time-domain analyses, as shown in Figs. 3, 5 and 7. The same results are shown in Fig. 9 for frequency-domain analyses. L is identical to $j$ index in Eqs. (44) and (45).

Also, the displacement response of this structure for control with time delay is represented for the same time delay steps in time-domain analyses. It should be noted that time delay is not compensated in this control algorithm. Hence, time delay has negative effect on the control performance (according to Eqs. (34) and (35)

In Figs. 4, 6 and 8, the control forces required for ideal control (without time delay) and predictive control with time delay steps L=10, 20, 30 are studied in time-domain analyses.

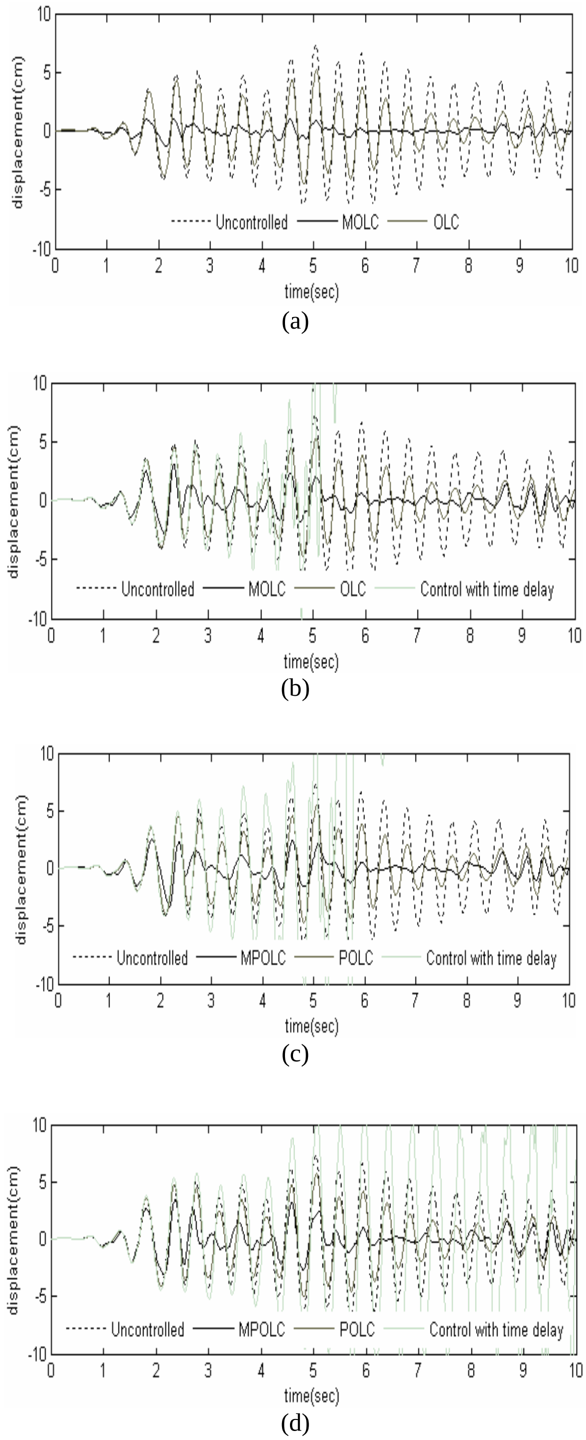


Fig. 3. Top-floor displacement of three-story structure under El Centro earthquake for a) without time delay, b) L=10, c) L=20, d) L=30

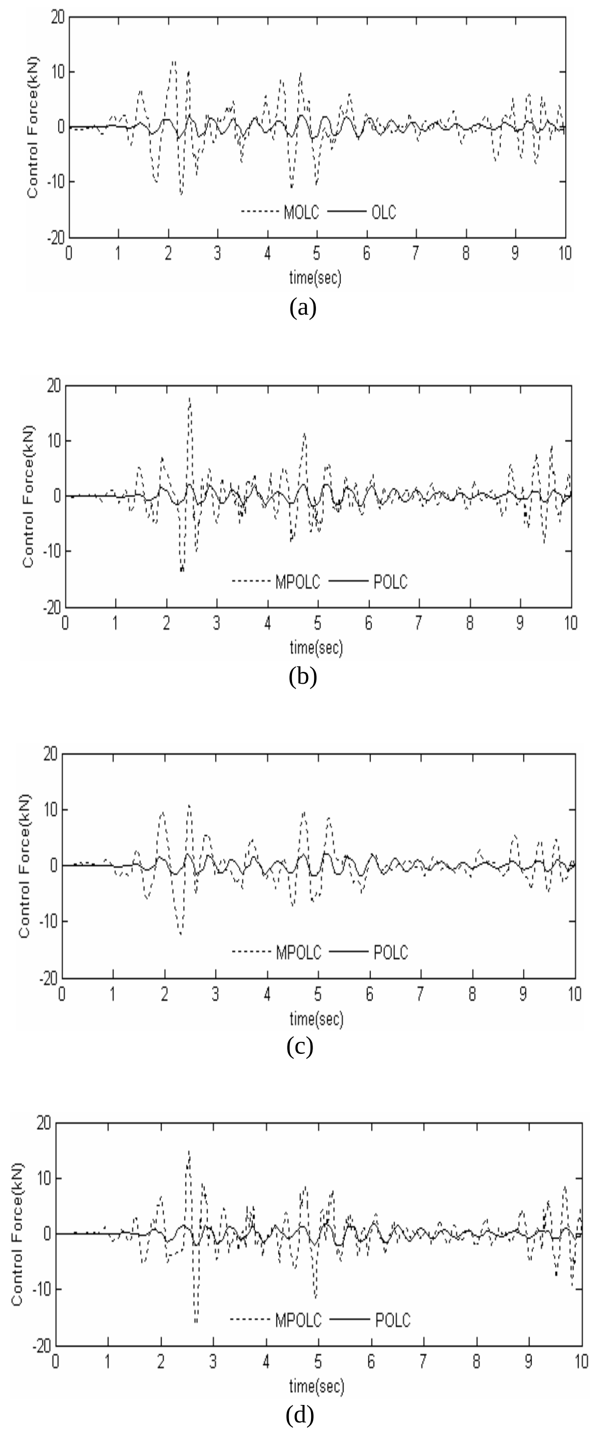


Fig. 4. Control force of three-story structure under El Centro earthquake for a) without time delay, b) L=10, c) L=20, d) L=30

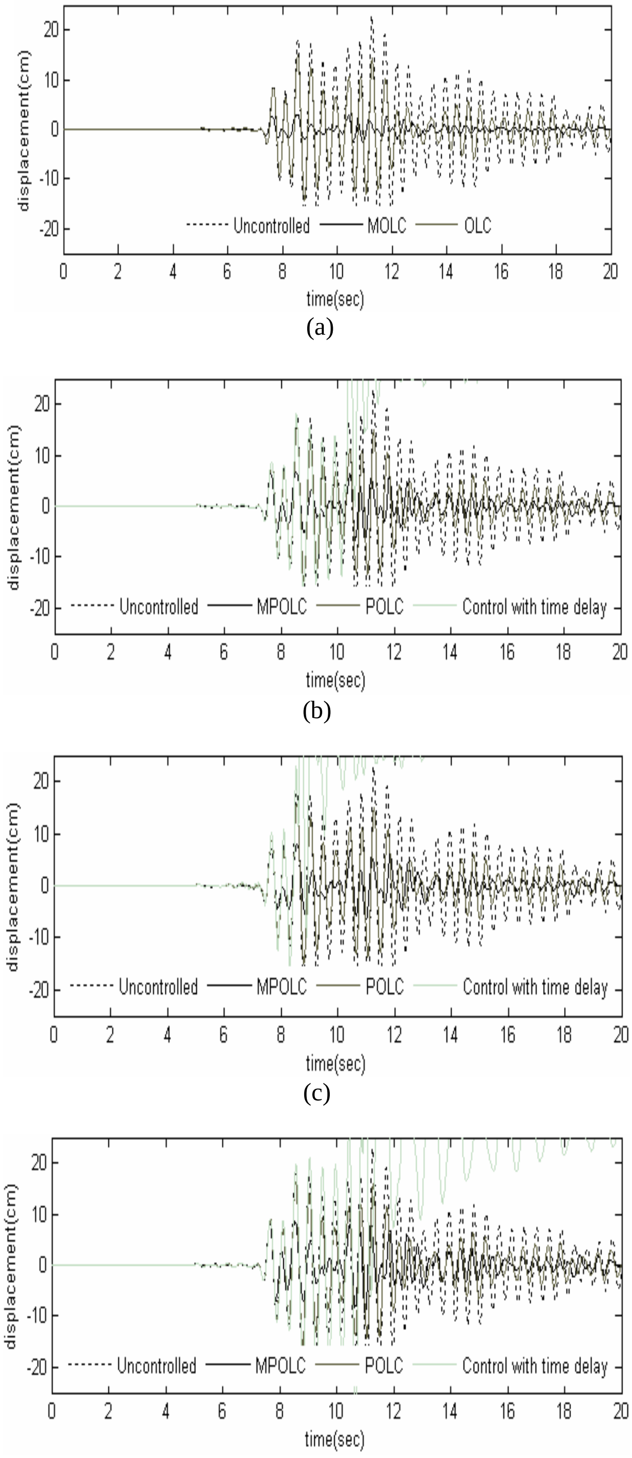


Fig. 5. Top-floor displacement of three-story structure under Kobe earthquake for a) without time delay, b) L=10, c) L=20, d) L=30

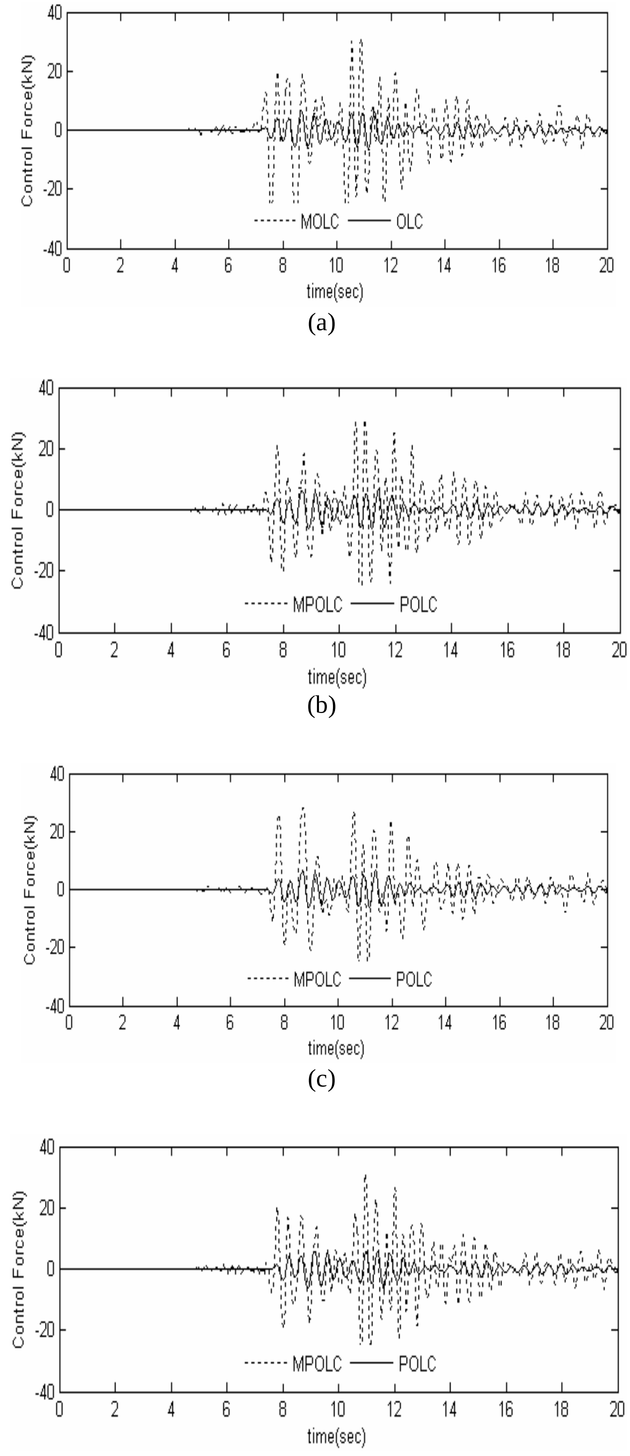


Fig. 6. Control force of three-story structure under Kobe earthquake for a) without time delay, b) L=10, c) L=20, d) L=30

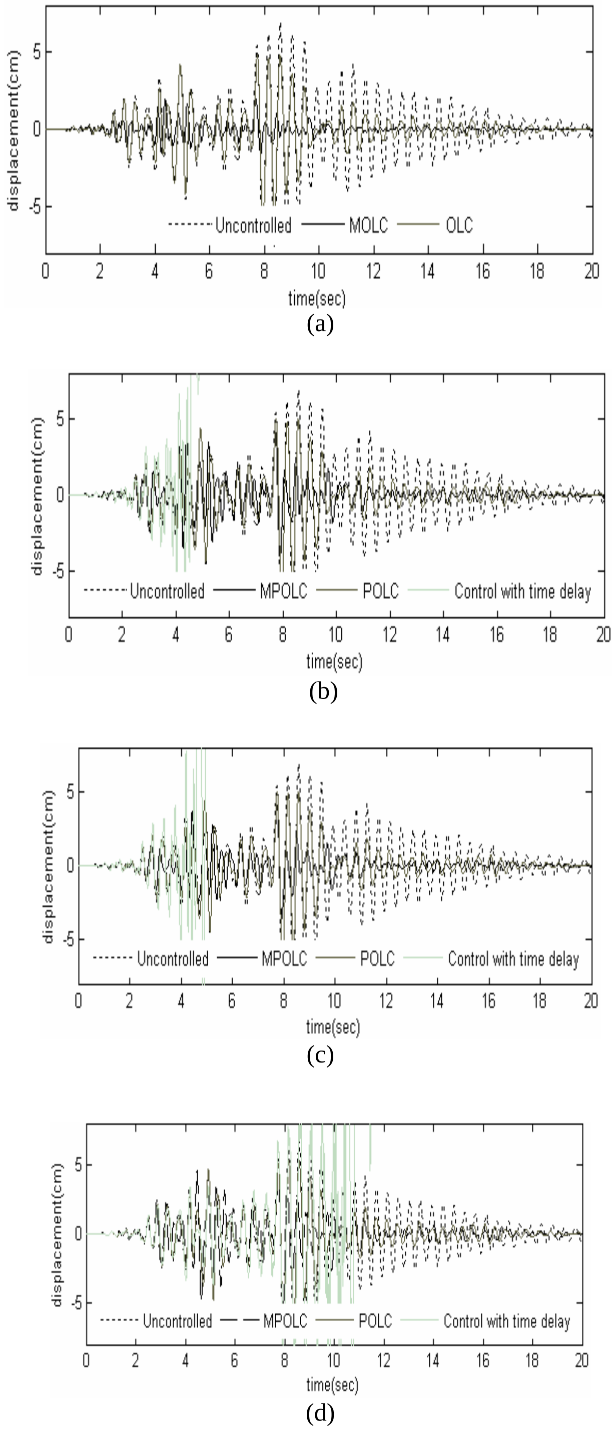


Fig. 7. Top-floor displacement of three-story structure under Northridge earthquake for a)without time delay, b) L=10, c) L=20, d) L=30

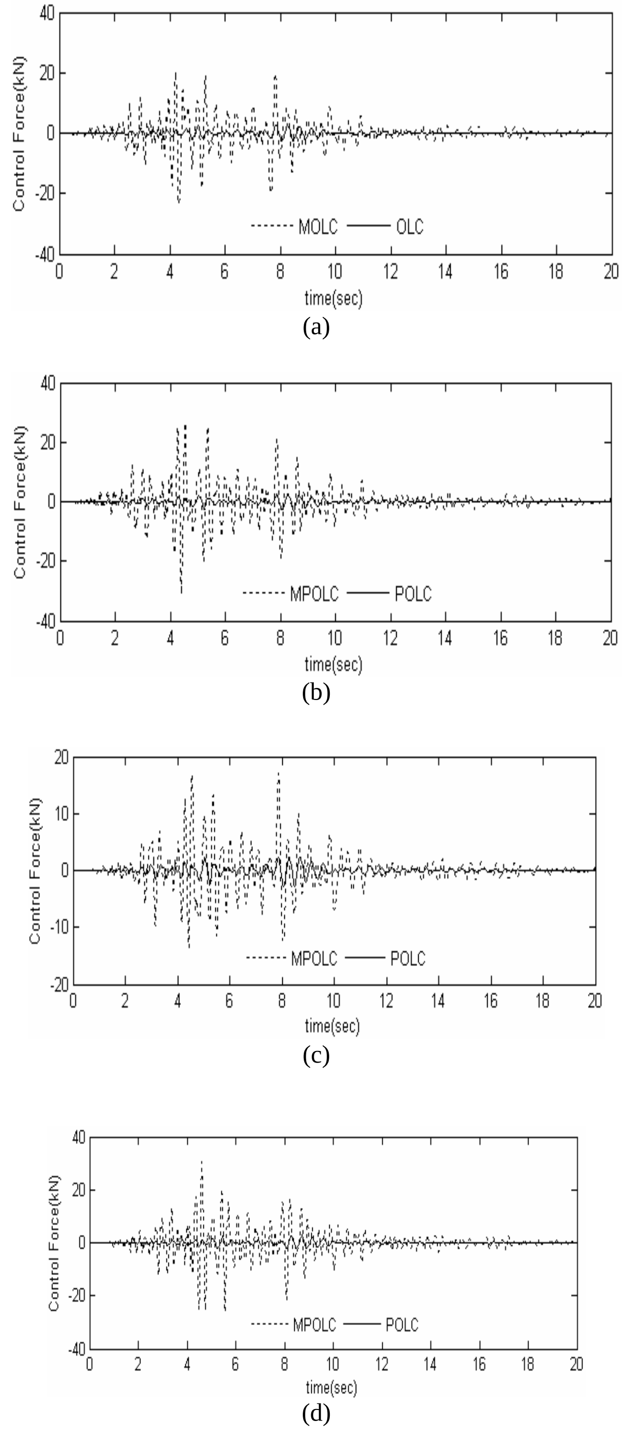


Fig. 8. Control force of three-story structure under Northridge earthquake for a) without time delay, b) L=10, c) L=20, d) L=30

As shown in Figs. 3, 5 and 7, the structural responses using POLC and MPOLC are very close to that of the ideal system with no time delay. While the control system with time delay, which causes the unsynchronized application of the control forces, gives the worst performance and leads to the instability of the structure. Therefore, the structural response after control with time delay is even worse than that before control, so it is better not to put any control action in this case.

Also, it is observed that, both POLC and MPOLC lengthen the acceptable time delay range within which the structure can remain stable.

Figures 4, 6 and 8 show that the modified optimal linear control (MOLC) uses a larger control force than that for optimal linear control (OLC).

As shown in Fig. 9, the peak of the frequency response function for the modified optimal linear control is completely suppressed and the control effectiveness is excellent. Therefore, the modified optimal linear control algorithm is much better in control effectiveness than the optimal linear control. Identical results are obtained for the MPOLC and POLC algorithm and it is observed that the displacement frequency response of MPOLC is much lower than that of POLC for different time delay steps.

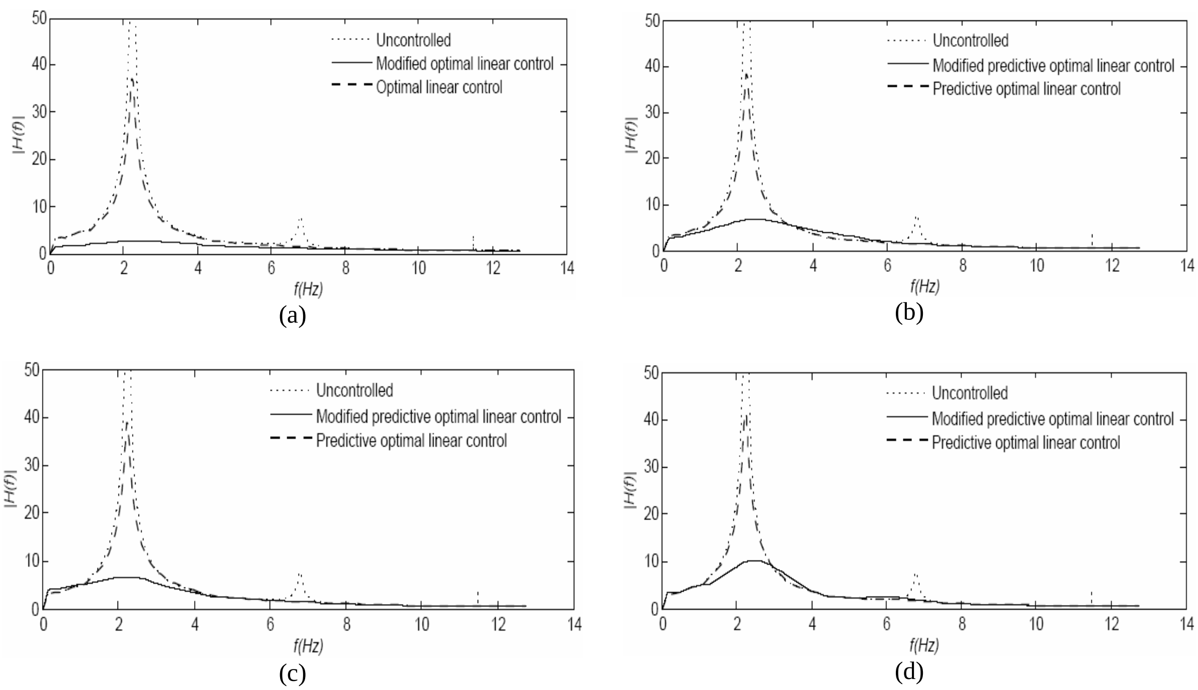


Fig. 9. Top-floor displacement frequency response function of three-story structure for a) without time delay, b) L=10, c) L=20, d) L=30

The control efficiency of the proposed optimal control strategy (MOLC) can be evaluated in several terms of performance criteria. The performance criteria used here are the percentage reduction of root-mean-square (RMS) displacement response $K_d$ and percentage RMS optimal control force $K_u$ relative to the structural total weight:

$$K_d = \frac{RMS_{(\text{uncontrolled displacement})} - RMS_{(\text{controlled displacement})}}{RMS_{(\text{uncontrolled displacement})}} \times 100\% \tag{46}$$

$$K_u = \frac{RMS_{(\text{optimal control force})}}{tr(M)g} \times 100\% \tag{47}$$

tr(M) is the trace operator of mass matrix M and equal to 2943 kg. Higher values of the percentage displacement response reduction $K_d$ and the percentage relative optimal control force $K_u$ indicate more efficient control capability.

Table 2. RMS displacement percentage reduction

| Percentage Reduction $K_d$(%) in RMS responses | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Control Type | Control without Time Delay | | | Predictive Control | | | | | | | | |
| Time Delay | L=0 | | | L=10 | | | L=20 | | | L=30 | | |
| Earthquake | El Centro | Kobe | Northridge | El Centro | Kobe | Northridge | El Centro | Kobe | Northridge | El Centro | Kobe | Northridge |
| OLC | 41.38 | 41.59 | 36 | 38.86 | 39.06 | 33.05 | 38.43 | 38.68 | 32.79 | 34.73 | 34.9 | 28.7 |
| MOLC | 89.09 | 89.12 | 81.4 | 74.58 | 74.06 | 56.42 | 73.56 | 73.03 | 61.35 | 67.13 | 65.45 | 44.6 |

Table 3. Percentage relative RMS control force

| Percentage relative $K_u$(%) in RMS control force | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Control Type | Control without Time Delay | | | Predictive Control | | | | | | | | |
| Time Delay | L=0 | | | L=10 | | | L=20 | | | L=30 | | |
| Earthquake | El Centro | Kobe | Northridge | El Centro | Kobe | Northridge | El Centro | Kobe | Northridge | El Centro | Kobe | Northridge |
| OLC | 2.24 | 4.7 | 1.63 | 2.22 | 4.65 | 1.65 | 2.2 | 4.6 | 1.6 | 2.11 | 4.42 | 1.55 |
| MOLC | 9.49 | 19.88 | 11.59 | 8.93 | 18.26 | 13.47 | 8.52 | 18.25 | 8.87 | 8.75 | 17.84 | 13.42 |

Tables 2 and 3 show, respectively, the RMS displacement percentage reduction ($K_d$) and the percentage relative RMS control force for the two presented control algorithms, i.e. OLC and MOLC under El Centro, Kobe and Northridge earthquakes and with time delay step L=0,10,20,30.

It is seen in Table 2, that the structural response reduction by means of modified optimal linear control (MOLC) is much more than that using optimal linear control (OLC), in both ideal and predictive forms. But by increasing in time delay steps, this factor is reduced for both control algorithms. Also, it is observed from Table 3 that the percentage relative RMS control force applied by MOLC is much more than that applied by OLC for the structure subjected to three earthquake excitations.

Both optimal linear control and modified optimal linear control are effective in the response reduction of structures under earthquakes, but the latter shows a better result for vibration suppression in the time domain and the frequency domain.

## 7. CONCLUSION

If the existence of time delay is neglected, the control system is susceptible to dynamic instability. Therefore, it is better not to put any control into the structural system before time-delay is analyzed and tackled properly. The problem of time delay is solved by developing the prediction model of the dynamic of the structure and the external excitation.

On the other hand, based on the conventional optimal linear control theory, the disturbances and environmental loads are not incorporated into minimizing the process of the cost function, and so, to the control force calculation. The reason is that the environmental loads, e.g. earthquake acceleration, are unknown and not predictable.

In this paper, for the solution of this problem, a modified predictive optimal linear control (MPOLC) algorithm for the active control of structures is proposed, in which the disturbances are effective in minimizing the process of the cost function and the control force calculation. The feasibility of the proposed control algorithm is successfully verified through frequency-domain and time-domain analyses. According to the proposed control algorithm, the control forces are generated from the prediction of the structural states in the last two steps, multiplied by the proportional feedback gains and the control force in the last step. Such a simple on-line calculation of the control forces makes the proposed control algorithm favorable to real-time control implementation. It is observed that the modified optimal control uses larger forces compared to the optimal linear control.

Consequently, it is obvious that the control performance of the modified predictive optimal linear control (MPOLC) is superior to that of the original one (POLC).

This problem, in which all parameters of a structure with a large number of DOF cannot be measured perfectly at instant $k$, is not a disadvantage of the MOLC algorithm. By using methods of estimation for unmeasured parameters of state, in the base of other measured parameters, an estimated full state per instant can be achieved and then applied for the MOLC algorithm. This manner is presented in the LQG method, therefore, the idea of composition LQG with the MOLC algorithm is the subject of the next paper, in which the authors show how these methods can be used for calculating the control force.

In addition to the environmental loads such as wind excitations and earthquake acceleration, it seems that MOLC is capable of compensating most of the other disturbances caused by modeling errors or parametric uncertainties, e.g. the difference between the estimated time delay and an exact time which has to be consumed in processing measured information, performing on-line computation, and executing the required control forces. In such cases, the performance of MOLC can be studied and verified in other research.

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