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SUCCESS RATE ESTIMATION OF THE GPS AMBIGUITY RESOLUTION TECHNIQUES USING REAL AND SYNTHETIC DATA, CASE STUDY: COMPARISON OF THE LAMBDA AND KTH METHODS*

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Abstract– Fixing integer ambiguities is a non-trivial problem, especially if we aim at computational efficiency and high performance (or success rate). For this reason it has been a rich source of Global Positioning System (GPS) research over the last decade. A brief review of ambiguity resolution using the method of *Least-Squares AMBiguity De-correlation Adjustment* (LAMBDA) and *rapid GPS ambiguity resolution for short and long baseline* (KTH method) is presented in this article. It continues with some numerical comparisons between two methods with real (float) and simulated GPS data. Finally, we end the paper with conclusions and some remarks. This comparison shows that the results of ambiguity resolution for short baselines are exactly the same using the KTH, LAMBDA and *Trimble Total Control (TM)* software. Also, for very long baselines these methods and software were not successful in solving ambiguities. However, the success rate of Trimble Total Control software was lower than for the others. This research also shows the exact effect of ionosphere in ambiguity resolution techniques. Any improvement in this area can improve the quality of ambiguity resolution significantly. More research with extra GPS observations in different conditions must be made for better results in the future.

Keywords– GPS, ambiguity resolution, LAMBDA, KTH

1. INTRODUCTION

Precise GPS positioning requires the use of carrier phase measurements, the data processing of which suffers from having to deal with integer ambiguities. Ambiguity Resolution (AR) is the mathematical process of converting ambiguous ranges (carrier phase measurements) to unambiguous range data with millimetre precision. Many ambiguity resolution techniques using single-frequency or dual frequency measurements have been developed over the last two decades. For some applications, for instance static relative positioning over short distances, the ambiguity resolution techniques are quite mature and are able to deliver centimetre accuracy positioning with high reliability [1]. For kinematics positioning, or long-range static positioning, the integer ambiguities cannot be reliably determined or the process suffers from many limitations.

The probability of correct integer estimation, the success rate, is an important measure when the goal is fast and high precision positioning with a Global Navigation Satellite System. Integer ambiguity estimation is the process of mapping the least-squares ambiguity estimates, referred to as the float ambiguities, to integer values. It is known that the carrier phase ambiguities are integer-valued, and it is only after resolution of these parameters that the carrier phase observations start to behave as very precise range measurements.

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The success rate equals the integral of the probability density function of the float ambiguities over the pull-in region centered at the true integer which is the region that all real values are mapped to this integer. The success rate can thus be computed without actual data and is very valuable as an a priori decision parameter whether successful ambiguity resolution is feasible or not.

This article starts with a brief explanation of the LAMBDA and KTH approaches in section 2. We then compare these two methods, and finally, the success rate of ambiguity resolution using these two approaches is determined.

2. AMBIGUITY RESOLUTION USING LAMBDA METHOD

In this section we present a brief review of the LAMBDA method. This method has been introduced in Teunissen [2-5], at the Delft University of Technology. A detailed description of its implementation is given by DeJonge and Tiberius [6]. LAMBDA is a method to solve the problem of mapping estimated ambiguities from the n-dimensional space of real numbers to the n-dimensional space of integers. It offers a way of finding the integer ambiguities according to the integer least squares criterion. In summary, the LAMBDA-method uses a least square estimator to solve integer ambiguities using the float ambiguities and the corresponding variance-covariance matrix as input.

a) Parameter estimation

The GPS models on which ambiguity resolution is based can all be cast in the following conceptual frame of linearised observation equations:

$$y = Aa + Bb + e \tag{1}$$

where, y is the given GPS data vector, a and b are unknown parameter vectors of order n and p, respectively, and e is the noise vector of order m. The matrices A and B are the corresponding design matrices of order $m \times n$ and $m \times p$, respectively.

The data vector y will usually consist of the "observed minus computed" single- or dual-frequency (*DD)* phase and/or pseudo range (code) observations for all observation epochs. The entries of vector a are then the DD carrier phase ambiguities, expressed in units of cycles rather than range, which are known to be integers. The entries of vector b will consist of the remaining unknown parameters such as, for instance, baseline components (coordinates) and possibly atmospheric delay parameters (troposphere, ionosphere).

The procedure that is usually followed for solving the above model can be divided into three steps (for more details see [6]):

In the first step one simply disregards the integer constraints on the ambiguities and performs a standard adjustment (Eq. (1)). As a result, one obtains the (real-valued) least-squares estimates of a and b, together with their variance-covariance matrix. This solution is often referred to as the "*float*" solution and is denoted by $\hat{a}$ and $\hat{b}$. The corresponding variance-covariance matrices are denoted by $Q_{\hat{a}}$ and $Q_{\hat{b}}$.

$$\begin{bmatrix} B^T Q_y^{-1} B & B^T Q_y^{-1} A \\ A^T Q_y^{-1} B & A^T Q_y^{-1} A \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} B^T Q_y^{-1} y \\ A^T Q_y^{-1} y \end{bmatrix} \tag{2}$$

$$\underbrace{\begin{bmatrix} N_b & N_{ba} \\ N_{ab} & N_a \end{bmatrix}}_{N} \begin{bmatrix} b \\ a \end{bmatrix} = \underbrace{\begin{bmatrix} r_b \\ r_a \end{bmatrix}}_{r} \tag{3}$$

and we can get:

$$\begin{bmatrix} \hat{b} \\ \hat{a} \end{bmatrix}, \begin{bmatrix} Q_{\hat{b}} & Q_{\hat{b}\hat{a}} \\ Q_{\hat{a}\hat{b}} & Q_{\hat{a}} \end{bmatrix} \tag{4}$$

b) Integer ambiguity estimation

The second step consists of:

$$\min_{a} \|\hat{a} - a\|^2_{Q_{\hat{a}}^{-1}} \quad with\, a \in Z^n \tag{5}$$

This minimisation yields the integer least-squares estimates for the vector of ambiguities: $\breve{a}$. For this step we use the LAMBDA method. The two main features of the LAMBDA method are:

The de-correlation of the ambiguities, realised by a re-parameterisation, or as we call it, a *Z* transformation of the original ambiguities *a* to new ambiguities $z = Z^T a$ and, using the actual integer ambiguity estimation through the *Z*-transformation, the variance-covariance matrix is transformed accordingly: $Q_{\hat{z}} = Z^T Q_{\hat{a}} Z$. The efficiency of the method is due to the de-correlation step; the actual integer minimisation is then made upon the transformed ambiguities. In practice, the minimisation of Eq. (3) amounts to a search over grid points inside the *n* dimensional ambiguity hyper-ellipsoid, defined by the variance-covariance matrix of the ambiguities [7]:

$$(\hat{Z} - Z)^T Q_{\hat{Z}}^{-1} (\hat{Z} - Z) \leq x^2 \tag{6}$$

The volume of the ellipsoid (and the number of candidates) can be controlled by setting the value for χ^2 [14]. The input for the integer ambiguity estimation is the vector of real valued ambiguity estimates $\hat{a}$ and the *Cholesky factor* C_a; the factor C_a is the 'split' into *L* and *D*, such that $Q_{\hat{a}} = C_a C_a^T = LDL^T$, where *D* is diagonal and *L* is unit lower triangular prior to the integer estimation. The ambiguities are de-correlated by application of the *Z*-transformation. The *Z*-transformation works directly on the inverse of the factors *D* and *L*:

$$\begin{matrix} D^{-1} \\ L^{-1} \\ \hat{a} \end{matrix} \rightarrow Z - \text{transformation} \rightarrow \begin{matrix} Z\, or\, Z^{-T} \\ \tilde{D}^{-1} \\ \tilde{L}^{-1} \\ \hat{z} \end{matrix}$$

such that $Q^{-1}_{\hat{z}} = \tilde{L}D\tilde{L}^T$. Note that the factors *L* and *D* represent the information of the variance covariance matrix $Q_{\hat{a}}$, which is not explicitly transformed into $Q_{\hat{z}}$, but implicitly via *L* and *D* ($\tilde{L}$ and $\tilde{D}$).

The actual integer minimisation is then carried out on the transformed ambiguities [7]. The output consists of $\breve{z}$ and possibly the second best $\breve{z}'$ together with their respective norms. Using the *Z*-matrix, they can be transformed back to the original ambiguities:

$$\breve{a} = Z^{-T} \breve{z} \tag{7}$$

The integer least-squares estimates for the ambiguities are explicitly computed. The squared distance (or norm) between the real valued $\hat{a}$ and the integer estimate $\breve{a}$ measured in the metric $Q_{\hat{a}}^{-1}$ by

$$t(\breve{a}) = \|\hat{a} - \breve{a}\|^2_{Q_{\hat{a}}^{-1}} \tag{8}$$

may be used *for validation purposes*. The computation of this norm, and possibly $t(\breve{a})$ is integrated in the integer minimisation. It can be easily computed at the moment the grid point is encountered in the search, [8]. Hence, it is based on the transformed ambiguities: $t(\breve{a}) = t(\breve{z})$.

c) Fixed solution

In the final solution $\breve{b}$ the ambiguities are fixed to their integer least square estimates $\breve{a}$. The final estimates for the baseline coordinates are obtained following E*q*s. (2) and (3) in which the vector of integer estimates $\breve{a}$ has been substituted for *a*. Note that the Cholesky factor of the matrix N_b is already available: it is the lower triangular matrix Q_b. The estimate $\breve{b}$ is then easily obtained via forward and backward substitution, or via $\breve{b} = Q_{\breve{b}}(r_b - N_{ba}\breve{a})$ .The variance-covariance matrix $Q_{\breve{b}}$, is computed with the help of the inverse Cholesky factor. The least-squares estimates $\breve{b}$ and $\breve{a}$ are the solution to the constrained minimisation.

In summary, the LAMBDA-method is a method to solve the problem of mapping estimated ambiguities from the n-dimensional space of real values to the n-dimensional space of integers. It offers a way of finding these integer ambiguities according to the integer least squares criterion. It has been found that this problem can be solved in a much more efficient way when the ambiguities are first de-correlated. Therefore, the LAMBDA-method consists of two steps. First, the ambiguities are de-correlated by means of the Z-transformation, then the integer minimisation problem is solved by a discrete search over an ellipsoidal region, the ambiguity search ellipsoid. This also explains the name "LAMBDA", which stands for "Least-squares AMBiguity Decorrelation Adjustment" [9]. In summary, the LAMBDA-method uses the least square estimator to estimate integer ambiguities using float ambiguities and the corresponding variance-covariance matrix as input.

3. AMBIGUITY RESOLUTION USING THE KTH METHOD

The method that is summarized here is based on the works of [10-14]. The basic idea of this method is the use of smoothed pseudo-ranges and a linear combination of phase observables. It is based on the variance analysis of various linear combinations. This method can be used in both short and long baselines and in real-time applications, as it treats each epoch independently of whether the receiver is fixed or moving.

The KTH method uses a similar procedure for solving the ambiguities. Like the LAMBDA method, in the first step it simply disregards the integer constraints on the ambiguities and performs a standard adjustment. As a result, one obtains the (real-valued) least-squares estimates of *a* and b with their variance-covariance matrix. Results of this "float" ambiguity estimate $\hat{a}$ are used to compute the corresponding integer ambiguities. Once the integer ambiguities are computed, they are used in the third step to finally correct the "float" estimate of *b*. The resulting $\breve{b}$ and corresponding $Q_{\breve{b}}$ are referred to as the "fixed" solution. Note that the resulting variance covariance matrix is based on the assumption that after the "fixing step" the ambiguities are known quantities.

a) Parameter estimation

Let us start with four basic codes and phase GPS equations

$$\begin{aligned} \Phi_1 &= \rho + x_1 N_1 + \varepsilon_{11} \\ \Phi_2 &= \rho + x_2 N_2 + \varepsilon_{12} \\ R_1 &= \rho + \varepsilon_{21} \\ R_2 &= \rho + \varepsilon_{22} \end{aligned} \tag{9}$$

where Φ_1, Φ_2, R_1, R_2 are the phase and code observations. L1 and L2 have wavelengths and frequencies $\lambda_1 = 0.1903$, $f_1 = 1575.42$ MHz and $\lambda_2 = 0.2442$, $f_2 = 1227.6$ MHz. ρ is the geometrical distance between the receiver and satellite; N_1 and N_2 are integer ambiguities; and ε are random observation errors. Equation (9) can be written in matrix notation as:

$$L - \varepsilon = Ax \tag{10}$$

where

$$A = \begin{bmatrix} 1 & \lambda_1 & 0 \\ 1 & 0 & \lambda_2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \; x = \begin{bmatrix} \rho \\ N_1 \\ N_2 \end{bmatrix}, \; L = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ R_1 \\ R_2 \end{bmatrix} \tag{11}$$

We assume that the observations are not correlated and that the standard error ratio of the pseudo-range and the carrier phase for each frequency is constant:

$$k = \frac{\sigma_{R_1}}{\sigma_{\Phi_1}} = \frac{\sigma_{R_2}}{\sigma_{\Phi_2}} \tag{12}$$

We also assumed that both carriers have the same phase resolution, i.e. the relation $\sigma_{\Phi_2} = (f_1 / f_2).\sigma_{\Phi_1}$ is valid for standard errors σ_{Φ_2} , σ_{Φ_2} scaled to meters, so that we can write the covariance matrix of these observations as:

$$C_L = 4\sigma_{\Phi_1}^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{f_1^2}{f_2^2} & 0 & 0 \\ 0 & 0 & k^2 & 0 \\ 0 & 0 & 0 & \frac{f_1^2}{f_2^2} k^2 \end{bmatrix} \tag{13}$$

The factor 4 in Eq. (13) reflects double differencing. Now we can compute standard errors of the unknown parameters as:

$$C_x = Q_x.\hat{\sigma}_0^2 = (A^T C_L^{-1} A)^{-1} \hat{\sigma}_0^2 \tag{14}$$

where $\hat{\sigma}_0^2$ is the a post prior standard error of unit weight. For decreasing standard errors and high correlation in this solution, it is useful to use linear combination phase equations instead of original equations as a $N_{ij} = iN_1 + jN_2$. We form the linear combination of the phase observations as $\Phi_{ij} = \rho + \lambda_{ij} N_{ij}$ where:

$$\Phi_{ij} = \left(i\frac{\Phi_1}{\lambda_1} + \frac{\Phi_2}{\lambda_2} \right) \lambda_{ij}, \; \lambda_{ij} = \left(\frac{i}{\lambda_1} + \frac{j}{\lambda_2} \right)^{-1} \tag{15}$$

Eq. (10) can be written in matrix form for any combinations Φ_{ij}, Φ_{mn} [defined in Eq. (17)] of the phase observations as:

$$L_c - \varepsilon_c = A_c x_c, \; L_c = TL, \; x_c = \begin{bmatrix} \rho \\ N_{ij} \\ N_{nm} \end{bmatrix} \tag{16}$$

and

$$L_c = \begin{bmatrix} \Phi_{ij} \\ \Phi_{nm} \\ R_1 \\ R_2 \end{bmatrix}, \quad T = \begin{bmatrix} \frac{i\lambda_{ij}}{\lambda_1} & \frac{i\lambda_{ij}}{\lambda_2} & 0 & 0 \\ \lambda_1 & \lambda_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

The covariance matrix of the transformed observation vector L_c becomes:

$$C_{L_c} = TC_L^{T^T} \tag{18}$$

Based on the Horemuz and Sjöberg [8] investigations, the integer value of $N_{7,-9}$ has the smallest standard error for short baselines in different linear combinations of ambiguities. However, for long baselines the wide-lane ambiguity combination is the optimum combination. By fixing the $N_{7,-9}$ we can estimate $N_{4,-5}$, which makes it possible to estimate $N_{1,-1}$ (wide-lane ambiguity N_w) with a fixed value of $N_{\ ,-}$. Finally, using the fixed wide-lane ambiguity, we can estimate N_1. This procedure can be used to find the resolution of ambiguities epoch by epoch for each satellite independently.

b) Optimum linear combinations

Further, Sjöberg [11] derived the *optimum linear combination* of L_1 and L_2 observations (L can be Φ or R), which *minimizes the mean least square error of this combination.* The linear combination of the phase and codes observations is:

$$\overline{\Phi} = \Phi_1 + (k_1 + m)(\Phi_1 - \Phi_2) = (k_1 + m).(N_1\lambda_1 - N_2\lambda_2) + N_1\lambda_1 + \rho + \overline{\varepsilon} \tag{19}$$

$$\overline{R} = R_1 + (k_1 + m)(R_1 - R_2) = \rho + \overline{\varepsilon} \tag{20}$$

where:

$$m = \frac{4k_2\sigma_{L_1}^2 + 4k_1\sigma_{L_2}^2}{4\sigma_{L_1}^2 + 4\sigma_{L_2}^2 + \Psi^2} \ and\ k_1 = \frac{f_2^2}{f_1^2 - f_2^2},\ k_2 = \frac{f_1^2}{f_1^2 - f_2^2} \tag{21}$$

The number Ψ is the difference between L and L_2 cased by ionosphere bias. If $\Psi = 0$, then the combination $\overline{L}$ becomes the weighted average of L and L_2, and for $\Psi = \infty$ we have the ionospheric free combination of $\overline{L}$.

Horemuz and Sjöberg [8] suggested the following algorithm for rapid ambiguity resolution for both short and long baselines:

- Compute and save the *single difference ambiguities* N_1 and N_2 as a float solution using Eq. (11).
- Compute the averages from the previous epochs.
- Compute $N_{4,-5}$ for very short baselines. Otherwise, compute N_W .
- Round them to the nearest integer, compute N_W and N by Eq. (15) and rounded $N_{4,-5}$ and N_W respectively.
- Choose a reference satellite and compute *double difference ambiguities*.
- Compute the ambiguity candidates for each satellite and test them using Eq. (24) below.

The ambiguity resolution is successful if the test is passed, otherwise take the next observation and go to the first step . The computed ambiguity is valid if the following three conditions are fulfilled [15], and if at least one condition is not met, then the respective set of ambiguity is discarded.

1- Testing prior σ_0^2 and posterior variance factor ($\hat{\sigma}_0^2$) using a x_f^2 (Chi-square) test. Here f denotes degrees of freedom and α is the significance of the level of the test:

$$H_0 : \hat{\sigma}_0^2 = \sigma_0^2 \qquad T_S = \frac{\hat{\sigma}_0^2}{\sigma_0^2} \qquad x_f^2, \alpha/2 \le T_S \le x_f^2, 1-\alpha/2 \tag{22}$$

2- Fisher test between difference of best ($\hat{\sigma}_0^2$) and second best ($\hat{\sigma}_0^{2'}$).

$$H_0 : \hat{\sigma}_0^2 \neq \hat{\sigma}_0^{2'} \qquad T_S = \frac{\hat{\sigma}_0^2}{\hat{\sigma}_0^{2'}} \qquad T_S \le F_{f.f:1-\alpha/2} \tag{23}$$

3- Testing compatibility of the solution between real and fix ambiguities:

$$(X-\hat{X})^T Q_{XX}^{-1}(X-\hat{X}) \le u\hat{\sigma}_0^2 F_{u.f:1-\alpha} \tag{24}$$

Here X and Q_{xx} are unknown vectors of the parameters and their variance-covariance matrices. However, for short baseline it is sufficient to test just one N_1 and N_2 candidate, but for longer baselines, due to the effect of the ionospheric bias, more N and N_2 candidates have to be tested.

In summation, the basic idea of the KTH method is the use of smoothed pseudo-ranges and the linear combination of phase observables. This method is based on the *variance analysis of various linear combinations*. It can be applied for both short and long baselines. For short baselines, it is sufficient to test just one N_1 candidate (estimated with fixed N_w) and Eqs. (9) are used. Then a second adjustment is carried out, where we take advantage of the fixed ambiguity.

However, for the longer baselines, generally the ionospheric bias is not known and more N_1 and N_2 candidates must be tested. The number of candidates must be determined empirically, but it is sufficient to test just the two closest integer candidates around N and N_2 estimated with fixed N_w . As each epoch is treated independently, this method is fast enough for real-time application, thus it does not matter if the receiver is static or moving.

4. COMPARISON OF KTH AND LAMBDA METHODS

In the previous sections we explained some principal aspects with respect to comparison of ambiguity resolution methods. In this section we make some numerical comparisons between the two methods using some real and simulated GPS data. In the first step we need to prepare information including articles, software and some GPS data. We obtained the original *Matlab* (TM) based software and basic articles of the LAMBDA method from the Department of Mathematical Geodesy and positioning of Delft University of Technology. We also recieved the executable files of the KTH method with simulation GPS data generator software and papers from our department.

a) Comparison using real GPS data set

Four data sets were downloaded from http://sopac.ucsd.edu/, collected by receivers of the Southern California Integrated GPS Network (SCIGN). For test purposes, 5 minutes, 10 minutes, 0.5hr and 5hr observation sessions were considered for the individual baselines 37m, 750m, 8km and 2600km baseline data sets, respectively.

Also, in order to test the results of the ambiguity resolution, we use commercial Trimble Total Control (*TTC*) GPS post processing software as the external software. Trimble Total Control is a processing package for GPS and conventional survey measurement data. Recent comparisons of commercial software at UNB indicate that TTC is one of the most flexible when it comes to processing options, and its solutions show a high degree of repeatability. It supports both kinematic and continuous data. On-the-fly ambiguity resolution is available, which processes kinematic data without any period of static initialization. The TCC packages tend to be simpler and more user friendly, but cut some corners, e.g., by allowing only the computation of *vectors*, relative co-ordinate triplets between ground stations, which should then be adjusted to gather in a three-dimensional network. Unfortunately, the detail information about TCC or other commercial software is usually not available.

For ambiguity discrimination, the difference between the best and second best ambiguity combination is crucial. Table 1 shows the statistics F-ratio [16] and the W-ratio [17] chosen for comparison.

From Table 1 there is no significant difference in the F-ratio and W-ratio statistics obtained from the three methods. In terms of the estimated baseline components, the results obtained from the three methods are also essentially identical.

Table 1. Comparison of F-ratio and W-ratio statistics

Baseline	Method	F-ratio	W-ratio
37 m	Lambda	12.76	26.64
	KTH	12.76	26.67
	TTC	12.8	26.71
2km	Lambda	9.21	29.90
	KTH	9.16	29.87
	TTC	9.18	29.85
8km	Lambda	4.54	16.12
	KTH	4.60	16.25
	TTC	4.57	16.19

For a clear investigation concerning the potential of the methods, we compute percentage number of the successful ambiguity resolutions for all epochs of the GPS observations using the Lambda, KTH and TTC softwares. Table 2 and Fig. 1 show the summary of the results of this comparison.

Table 2. Success-rate percentage of the ambiguity resolution of different methods using real (float) GPS data. Columns 5-7 show the agreement between the two selected solutions. (The ambiguity results are the same with a small difference 1 to 3 cycle)

Baselines length (km)	KTH Fix	LAM. Fix	TTC Fix	KTH-LAM.	LAM.-TTC	KTH-TTC
Zero base line	100	100	100	100	100	100
0.037	100	100	100	100	17	17
0.75	100	100	100	100	100	100
2	100	100	100	15	100	15
8	100	100	100	0	0	50
2600	100	100	0	0	0	0
Mean	100	100	71	45	45	40

Unfortunately, the interpretation of the results is not simple. Although for the short baselines the results of the solutions are more or less similar, there are still some exceptions. For example, in the baseline with lengths 37m and 2 km we get different results based on the TTC software and the two mentioned methods. We try this with the LAMBDA method in some *unresolved* epochs of the KTH solution, and the ambiguities are successfully solved.

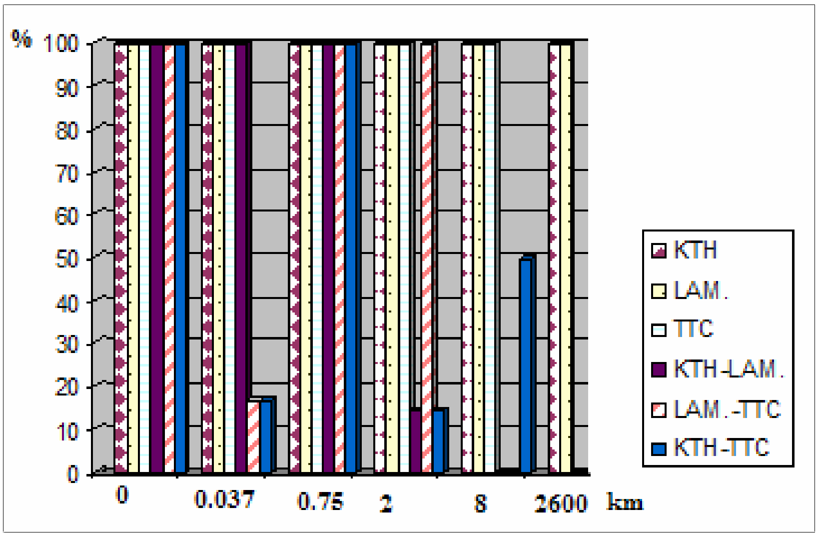

Fig. 1. Success-rate percentage of the ambiguity resolution of different methods using real GPS data

b) Comparison using simulated GPS data set

We cannot get any reasonable conclusion from the real data since there no reference ambiguity solution for our comparison. One way of obtaining the success-rate is using simulation. Using a random generator, a large number of real-valued ambiguity vectors are generated from the origin-centred probability distribution of the ''float'' solution. For each of these generated vectors, the corresponding integer least-squares solution is computed using the LAMBDA method. The percentage of integer solutions that coincide with the origin yields the success-rate. The number of generated samples must be large enough in order to obtain a close enough approximation to the success-rate. In this research, we use this *performance factor* for comparison of the success-rate of results.

There are two main advantages to using simulated data: (a) to evaluate the performance of the proposed algorithm (since it is very difficult to derive highly accurate GPS station coordinates in practice), and (b), to study the impact of incorporating different systematic errors.

A simulation of the raw GPS observations was performed using the *Simulation part* of the *Bernese* software (see Bernes manual reference 4.0 for more details). It can generate simulated GPS and GLONASS observations (code and/or phase, L1 or L1/L2) based on statistical information (RMS for observations, biases, cycle slips). The ionospheric error is produced with respect to the length of the baseline. We use two kinds of simulated data, short baselines (with low ionospheric bias) and baselines with ionospheric error [18]. With respect to ionospheric free solution (short baseline) all of these methods give exactly true solutions for ambiguities, but when ionospheric bias in data is used we get some differences in solutions. The methods give true solutions for the ambiguities, but when we use ionospheric bias in data there are some differences in the solutions.

Different carrier phase observation noises were assigned to different satellites varying from 1mm to 3mm. Four data sets were simulated, a 10 minute session for 12 and 19 meter short baselines and a 0.5hr session for a 32km and a 5hr session for a 57km baseline.

Table 3. Success-rate of the ambiguity resolution of different methods using simulated GPS data. (N/A: Software cannot determine fix solution for ambiguities)

Baseline length (km)	KTH	LAM.	TTC
0.012	100	100	100
0.019	40	50	N/A
7	100	100	100
32	0	100	N/A
57	0	17	N/A
Mean	48	73	40

5. CONCLUSION AND REMARKS

A brief review of the research trends and issues on ambiguity resolution and validation has been presented in this research. At first, we looked at the work carried out by KTH and Delft University research groups. Here we conclude the results of this comparison:

- The LAMBDA software [19] can process epoch by epoch GPS double difference (DD) observations. This requires the float solution of the ambiguities and their variance covariance matrix. However, preparing this input information is not quite as simple since there is little commercial or scientific GPS software able to produce DD ambiguity resolutions with a full matrix of variance covariance. Some routines of the KTH software had to be rewritten and changed for this purpose.

- In contrast to the LAMBDA software, the KTH software is a full ambiguity resolution software because it uses directly original *RINEX* GPS and navigation observations for the full observational period.
- *In the computational view* it is an advantage of the KTH method for GPS users that it does not need any other *external software* for preparing input data and running, it uses whole GPS data and does computation epoch by epoch [20], and determines final ambiguities and position differences with their full variance-covariance matrix automatically; also, it is user friendly.
- The theory of the KTH uses simple mathematical concepts. In practice, it gives *higher computer efficiency* than the LAMBDA method that uses the explicit matrix method.
- With respect to *input data*, however, KTH methods need dual-frequency GPS observations, but the LAMBDA method can be used in triple frequency GPS or other new observation techniques [21]. It should also be emphasized that the KTH method is by now only implemented for baseline observations, while the LAMBDA method works for any type of network. It is obvious that network solutions should be more reliable than baseline solutions.
- Basically, both the KTH and LAMBDA methods use linear combinations of measurement by the Least-squares approximation technique for choosing the best ambiguity solutions in ambiguity space (see Eqs. 1-3), but KTH uses original phase and code observations in the computations. As we mentioned before, LAMBDA used some de-correlation methods and ***Z*** matrix that change the originality of data. However, in the least-squares senses, the estimated quantities of ambiguities are correlated as all the parameters of LS adjustment. While fixing improves the solution, it might worsen the solution if type 1 or type 2 errors are made in this step. De-correlation minimizes these errors, because de-correlation makes the individual parameters as independent as possible from each other. In other words, only the correct values will have the least effect over each other.
- The KTH method uses wide lane combinations in the ambiguities resolutions, and we know that using this combination has a real advantage compared with using the original observation since the ambiguities have to be resolved for a signal with a wavelength four times larger and also ambiguities are solved quickly, independent of the length of the baseline and the ionosphere condition [12]. Although the inventor of the LAMBDA method believes that in the Z matrix this method uses wide lane combination indirectly, we think this is true only for some observations.
- KTH method uses a three step testing procedure in order to control the quality performance of the ambiguity resolution, but LAMBDA uses another performance factor, the ***ambiguity success rate.*** Our computation shows that the KTH and LAMBDA methods have the same success-rate with the simulated data (without ionoshpheric bias) and they solve all ambiguities correctly, however with respect to ionospheric bias simulated data, LAMBDA gave a higher success-rate compared to KTH.

This comparison shows that the results of ambiguity resolution for short baselines are exactly the same in KTH, LAMBDA and Trimble Total Control software. Also, for very long baselines these methods and software were not so successful. However, the success rate of Trimble Total Control software was lower than the others . It is also well known that the LAMBDA is much more efficient than imposing constraint among ambiguities, especially for fixing based on limited observations, for example, for rapid-static, kinematics applications [15].

Also, this research shows again the effect of ionospheric bias in ambiguity resolution techniques. It is clear that any improvement in this area can improve the quality of ambiguity resolution significantly.

More research with extra GPS observations in different conditions must be done for better results in future.

The results of numerical application are not significant enough to determine the superiority of either method over ambiguity resolution. However, this article does offer a basis for further investigations in future.

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