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"Research Note"

EVALUATION OF SCALING PARAMETER TO PREDICT SOIL WATER CHARACTERISTIC CURVE USING IMPROVED PARTICLE-SIZE DISTRIBUTION*

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Abstract– Laboratory and field methods that are used to determine the soil water characteristic curve (h-θ function) are expensive and time consuming. A physical model for predicting the h-θ function based on the soil particle-size distribution curve and soil bulk density with different scale parameters (*α, i.e.*, constant, linear, and logistic models) has been proposed in the literature. Unfortunately, many databases do not contain the full particle-size distribution, but instead contain only the sand, silt, and clay mass fractions. A method for estimating the particle-size distribution from clay, silt, and fine plus very fine sand mass fraction (particle radii, between 25 and 125 μm) has been presented in the literature and is improved by using all sand particles (radii between 25 and 999 μm, modified model). The objectives of the present study were to evaluate the predicted soil water characteristic curve for 16 soil samples with different textures based on clay, silt, and sand fractions, and soil bulk density using the improved method for the prediction of particle-size distribution (modified model), different values for the scale parameter as constant, being obtained from linear and logistic models. The results indicated that for clay and silt loam soils using a radius of 999 μm (the modified model), the particle-size distribution, and consequently soil moisture characteristic curves, was predicted more accurately than those obtained by using a radius of 125 μm for the largest particles of the soil. This is specifically shown for *α* determined by the logistic procedure. The values of α based on the logistic model were best suited for the clay, silt loam, and loam soils. The values of *α* based on the linear model were appropriate for the clay and silt loam soils. Further, the values of constant *α* were best suited for the clay soils. However, these results are considered indicative rather than conclusive due to the small size of the data set for clay, silt loam and sandy loam soils. Finally, it is proposed to test the modified model for a wider data set.

Keywords– Scaling parameter, soil water characteristics, particle-size distribution

1. INTRODUCTION

Soil hydraulic properties (*i.e.*, h-θ and *K-θ* functions) are fundamentals for irrigation and drainage modeling, and to study the adverse effect of industrial and municipal activities on subsurface environment [1]. Laboratory and field methods used to determine the soil water characteristic curve (*h-θ* function) are expensive and time consuming. Therefore, researchers have presented methods to estimate *h-θ* function. These are often physical and mathematically based methods [2- 11].

The soil water characteristic curve is closely related to the pore size distribution of soil samples. Soil compaction and porosity (soil bulk density and soil saturation water content) are effective in this relationship. Arya and Paris [4] presented a physical model for predicting the soil water characteristic curve based on the soil particle-size distribution curve and soil bulk density. This model was based on the

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similarity between the shape of the particle-size distribution and soil water characteristic curve. A scale parameter, α, was used in this model and its value was proposed to be 1.35-1.40 (an average of 1.38). Other models, based on particle-size distribution, were proposed by Haverkamp and Parlange [5]. The values of scale parameter α were analyzed by Tyler and Wheatcraft [9]. Later, Arya et al. [10] showed that the values of α varied from 1.1 for fine textured soil samples to 2.5 for coarse textured soil samples. They also revealed that the values of α for various parts of particle-size distribution are not constant.

Three different procedures for the determination of α were tested by Arya *et al*. [10]: (i) the value of α was considered as a constant, (ii) a linear equation for determination of α was obtained, (iii) a logistic equation for the determination of α was proposed. The values for α were proposed as 1.285, 1.459, 1.375, 1.150, and 1.160 for sandy, sandy loam, loam, silty loam, and clay soils, respectively.

Particle-size distribution data may be available from soil databases. Unfortunately, many databases do not contain enough detailed particle-size distribution, but instead contain only the sand, silt, and clay mass fractions. Skaggs *et al.* [11] presented a method for estimating the particle-size distribution from clay, silt, and fine plus very fine sand mass fractions (particle radii, between 25 and 125 μm) [12]. They indicated that their method should not be used when the silt fraction is greater than about 70%. This model is simple to use, but the values of very fine (radius of 25-50 μm) plus fine (radius of 50-125 μm) sand fractions must be available. However, for most conditions, only the value of whole sand subclasses (radius of 25-1000 μm) is available. Therefore, in the present article, it is proposed to use a particle radius of 999 μm (near the radius of 1000 μm corresponding to the extreme value of the radius for sand particle in the USDA system [12]) instead of 125 μm for the extreme value of fine sand fraction. This is called "improved method". The original Skaggs *et al*. [11] model was used by Fooladmand *et al*. (13) to predict the soil water characteristic curve, but the results were poor. Further, Fooladmand and Sepaskhah [14] reported that particle-size distribution was much better when using a radius of 999 μm than using a radius of 125 μm.

The objective of this study was to estimate and evaluate the soil water characteristic curves for 16 soil samples with different textures based on clay, silt, sand fractions, and soil bulk density using the improved Skaggs et al. [11] method (particle radius of 999 μm instead of 125 μm), different values for the scale parameter, α, as constant, and obtained from linear and logistic models.

2. MATERIALS AND METHODS

To evaluate the accuracy of the proposed procedures, soil water characteristic curves for 16 soil samples with different textures reported by Khoshnodi-Yazdi [15] were compiled. Some physical properties of these soils are shown in Table 1.

Table 1. The range of some physical properties of the soils

Soil sample number	Texture	Bulk density g cm^{-3}	Clay %	Silt %	Sand %
10	Loam	1.37-1.54	14.0-26.0	30.0-38.0	36.8-50.0
2	Clay	1.39-1.39	40.8-56.0	29.2-37.2	14.8-22.0
2	Silt loam	1.41-1.45	18.0-18.8	50.0-52.0	29.2-32.0
2	Sandy loam	1.39-1.48	14.0-18.8	28.0-29.2	53.2-56.8

a) Particle-size distribution estimation

Similar to a logistic growth curve with some modification, the soil cumulative particle-size distribution was described by the following empirical model [11]:

$$W=1/\{1+(cl^{-1}-1)\exp[-u(R-1)^{c}]\} \quad \text{for } 1\ \mu m \le R \le 1000\ \mu m \qquad (1.1)$$

$$c=\gamma \ln(v/w) \tag{1.2}$$

$$u=-v^{1-\beta}w^{\beta} \tag{1.3}$$

where:

$$\gamma=1/\ln[(r_1-r_o)/(r_2-r_o)] \tag{1.4}$$

$$\beta=\gamma \ln[(r_1-r_o)/r_o] \tag{1.5}$$

$$v=\ln\{[(cl+si)^{-1}-1]/(cl^{-1}-1)\}, \quad \text{for } v<0 \tag{1.6}$$

$$w=\ln\{[(cl+si+s)^{-1}-1]/(cl^{-1}-1)\}, \quad \text{for } w<0 \tag{1.7}$$

where, the values of γ, β, v, and w are dimensionless (-), r_o is 1 μm (extreme radius of clay), r_1 is 25 μm (extreme radius of silt), r_2 is 125 (extreme radius of very fine sand) or 999 μm (near to 1000 μm, extreme radius of whole mass fraction of sand), *cl* is the mass fraction of the clay particle (-), *cl*+*si* is the cumulative mass fraction of clay and silt (-), *cl*+*si*+*s* is the cumulative mass fraction of clay, silt, and sand (-), *R* is the particle radius in μm, and W is the cumulative mass fraction of particles with a radius smaller than *R* (-). The values of γ calculated by Eq. (1.4) are -0.609 (original model) and -0.268 (improved model) for r_2 of 125 and 999 μm, respectively. In Eqs. (1.1-1.7) the radii of clay, silt, and fine sand as 1, 25, and 125 μm were used by Skaggs *et al*. [11], while in the present study, the radius of coarse sand, i.e., 1000 μm instead of fine sand, was planned to be used. This will result in an unpredictable value for *w* in Eq. (1.7), therefore, a value of 999 μm was used for *R* for coarse sand particle.

b) Soil water characteristic model

For the estimation of the soil water characteristic curve, the model of Arya *et al*. [10] was used. In this procedure, the curve of particle-size distribution was divided into several segments, *i*, for which the mean radius of the particle and the percentage of cumulative mass fraction for particles smaller than a specified radius are determined. On the other hand, the soil saturation volumetric water content is assumed to be equal to the soil porosity, *P* (-), which is obtained from the soil bulk density, ρ_b in g cm^{-3}, ($\theta_s=P=(1-\rho_b/\rho_p)$) where ρ_p is the soil particle density 2.65 g cm^{-3}. Therefore, the soil void ratio, *e* (-), can be calculated as *P*/(1-*P*). By using ρ_p and *e*, the volumetric water content (cm^3 cm^{-3}) in the i^{th} portions of the particle-size distribution curve is obtained as follows:

$$\theta_i=e/(1+e)\Sigma^{i}_{j=1}W_j \tag{2}$$

where, θ_i is the volumetric soil water content for the i^{th} portion of the particle-size distribution curve in cm^{3}cm^{-3} and W_j is the fraction of soil particles mass in g g^{-1}.

For each class range of particle size distribution *i,* the number of spherical particles with a constant radius of R_i in the i^{th} particle range in ideal soil, n_i (g^{-1}) is determined by the following equation [10]:

$$n_i=(3W_i)/(4\pi\rho_p R_i^3) \tag{3}$$

where R_i is the radius of particles in each class range of particle size (cm). The number of spherical particles required to trace the pore length in corresponding natural structured soil, N_i (g^{-1}), and the pressure head, h_i (cm water), by using the capillary equation [$h_i=2\sigma\cos\phi/(\rho_w g r_i)$, where r_i is the radius of pores (cm) in each class range of particle size], can be determined by equations as follows [10]:

$$N_i=n_i^{\alpha} \tag{4}$$

$$h_i=[(N_i\rho_p R_i)/(7.371W_i e)]^{0.5} \tag{5}$$

where, α is the scaling parameter as proposed by Arya *et al.* [10], 7.371 (-) stands for $\rho_w^2 g^2/(8\pi\sigma^2\cos^2\phi)$, ρ_w is the density of water considered to be 1.0 g cm^{-3}, g is acceleration due to gravity, 981 cm s^{-2}, σ is the surface tension of the water, 72.2 dyne cm^{-1}, ϕ is the contact angle considered as zero. Paired values of volumetric water content θ_i, and pressure head, h_i, are used to predict the soil water characteristic curve.

c) Scale parameter

Arya *et al*. [10] proposed three different procedures for the determination of the scaling parameter as constant α value, linear and logistic methods.

d) Constant scale parameter

In this procedure, a constant value for α is considered for each soil texture according to Table 2 [10].

Table 2. Values of coefficients *a* and *b* in linear model for determination scale parameter α, and the constant values of α (after ref. 10)

Soil texture	Linear model coefficients		Constant value
	a	*b*	
Sand	-2.478	1.490	1.285
Sandy loam	-3.398	1.773	1.459
Loam	-1.681	1.395	1.375
Silt loam	-2.480	1.353	1.150
Clay	-2.600	1.305	1.160

e) Linear scale parameter

According to the Eqn (3), there is a linear relationship between Logn_i and Log(W_i/R_i^3). Since N_i shows a larger scale than n_i, there should be a linear relationship between LogN_i and Log(W_i/R_i^3) as follows:

$$\mathrm{Log}N_i = a + b\,\mathrm{Log}(W_i/R_i^3) \tag{6}$$

Combining Eqns (4) and (6) results in:

$$\alpha_i = [a + b\,\mathrm{Log}(W_i/R_i^3)]/\mathrm{Log}n_i \tag{7}$$

The Eqn (7) is used to estimate the values of α. The values for *a,* and *b* for sand, sandy loam, loam, silt loam, and clay soils are given by Arya *et al.* [10] and presented in Table 2.

f) Logistic scale parameter

Arya *et al.* [10], based on plots of Log*Ni* versus Log*ni*, proposed a logistic growth equation as follows:

$$Y = (Y_f Y_{in})/[Y_{in} + (Y_f - Y_{in})\exp(-\mu X) \tag{8}$$

Where, Y is the dependent variable (-), LogN_i, X is the independent variable, Logn_i, μ is the intensity coefficient (-), subscript *in* shows the lower limit of LogN_i, and subscript *f* shows the upper limit of LogN_i. In other words, Y_f and Y_{in} shows the maximum and minimum number of particles in the smallest and the largest portions of the particle-mass distribution curve, respectively. The results of Arya *et al.* [10] indicated that for most soils, the initial values of LogN_i and Logn_i are negative, therefore, the X and Y axis were transferred to the left and down, respectively as:

$$Y + \Delta Y = (Y_f Y_{in})/[Y_{in} + (Y_f - Y_{in})\exp(-\mu(X + \Delta X)] \tag{9}$$

where: $\Delta Y = \Delta$LogN_i and $\Delta X = \Delta$Logn_i. The parameters of Eqn (9) for sand, sandy loam, silt loam, and clay soils are given by Arya *et al.* [10] and presented in Table 3.

Table 3. Coefficients of logistic model for determination of scale parameters α (after ref. 10)

Soil texture	$\Delta \log n_i$	$\Delta \log N_i$	μ	$(\log N_i)_f$	$(\log N_i)_i$
Sand	0.00032	1.734	0.609	16.602	0.996
Sandy loam	1.849	2.492	0.553	16.983	0.559
Loam	1.977	2.242	0.510	16.614	0.628
Silt loam	0.684	1.902	0.457	19.686	0.719
Clay	2.648	4.766	0.289	21.685	1.993

g) Evaluation of the model estimation

The estimated volumetric soil water content was compared with those measured values at a given soil water pressure head. This comparison was evaluated by the standard error between the measured and estimated values as:

$$R_{MSE}=[1/(N_p-1)\Sigma^{n}_{i=1}(\theta_{mi}-\theta_{pi})^2]^{0.5} \qquad (10)$$

where, R_{MSE} is the root mean square of error (cm^3 cm^{-3}), N_p is the number of paired observations, and θ_m and θ_p are the measured and predicted volumetric soil water content, respectively, in cm^3 cm^{-3}.

3. RESULTS AND DISCUSSION

Soil water characteristic curves for 16 soil samples were predicted by using the Skaggs *et al*. [11] method and the fine sand with a radius of 125 μm as the largest particle in the particle-size distribution. The results showed that the logistic procedure for the determination of α resulted in the largest values for R_{MSE} in 88% of soil samples (Table 4). Further, for 31% of the soil samples, the linear method showed R_{MSE} values greater than those obtained with constant value of α. On average, the values of R_{MSE} were higher for the logistic procedure (0.074 cm^3 cm^{-3}) than those obtained for the linear method (0.051 cm^3 cm^{-3}) and constant α (0.049 cm^3 cm^{-3}) (Table 4). These results contradict those of Arya *et al.* [10]. They stated that the estimation of the value of α by logistic procedure is better than the other methods. The differences between predicted and measured soil water characteristic (SWC) curves are more pronounced for the predicted SWC curves obtained by the scaling factor from the logistic method. This contradiction might be due to the fact that Arya et al. (10) used the particle size distribution curves obtained based on the measured particle size fraction of more than three sizes (i.e., clay, silt and sand) for their selected soil samples, while in our study, the curve of particle size distribution was estimated by three particle size fractions (i.e., clay, silt and sand) according to the Skaggs et al. (11) method, which may not be appropriate.

Table 4 . The range of root mean square error for different soil textures and various procedures for scale parameters determination with original particle size distribution

Texture	Logistic	Linear	Constant α
Loam	0.035-0.076	0.012-0.065	0.014-0.061
Clay	0.078-0.110	0.068-0.101	0.080-0.101
Silt loam	0.105-0.120	0.087-0.097	0.074-0.086
Sandy loam	0.060-0.088	0.045-0.057	0.018-0.030
Mean	0.074	0.051	0.049

The samples of predicted and measured soil water characteristic curves using Skaggs *et al*. [11] method and coarse sand with a radius of 999 μm as the largest particle in the particle-size distribution for the same soil textures (modified method) are shown in Figs. 1 and 2. The values of R_{MSE} for different soil textures and various procedures of α determination are given in Table 5. The results indicated that in 69%

of the soils the value of R_{MSE} was the greatest with constant α, in 81% of the soils the value of R_{MSE} was lower with the linear method than those obtained with constant α, and in 81% of the soils, the value of R_{MSE} was the lowest with α obtained by the logistic model. Therefore, using the α as a constant value (with R_{MSE} of 0.069 cm^3 cm^{-3}) is not recommended. Further, the priority order of using α is logistic and linear models with R_{MSE} of 0.053 and 0.064 cm^3 cm^{-3}, respectively.

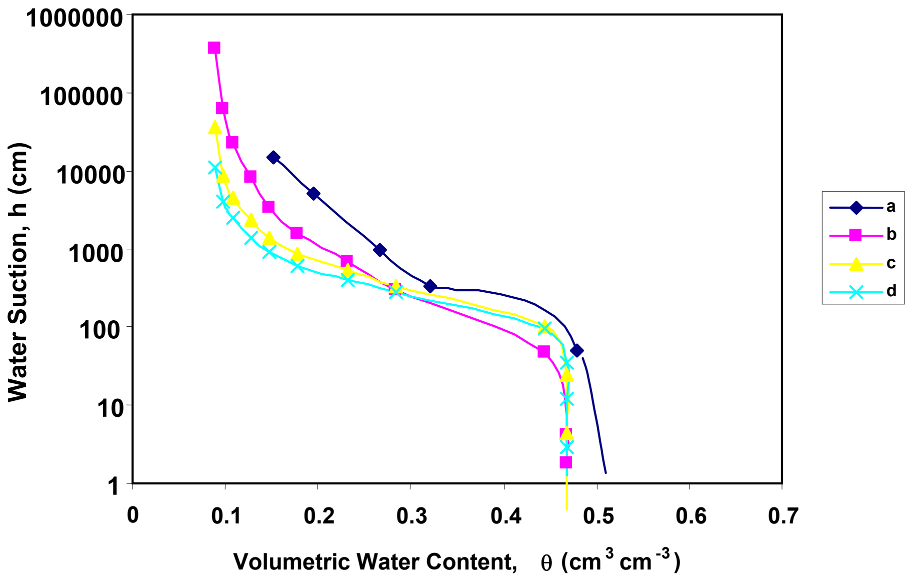

Fig. 1. Measured and predicted soil water characteristic curves for soil no. 15 with modified particle size distribution: a) measured, b) logistic, R_{MSE}=0.044, c) linear, R_{MSE}=0.073, d) constant scale parameter, R_{MSE}=0.092

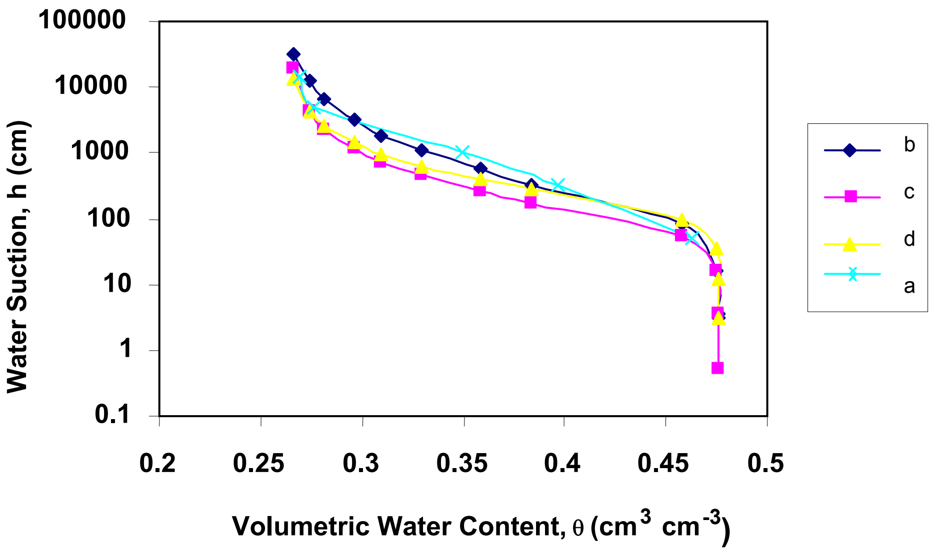

Fig. 2. Measured and predicted soil water characteristic curves for soil no. 13 with modified particle size distribution, a) measured, b) logistic, R_{MSE}=0.012, c) linear, R_{MSE}=0.035, d) constant scale parameter, R_{MSE}=0.024

The values of holistic root mean square error for different soil textures and various procedures for scale parameters determination with the original particle size distribution and modified model are shown in Table 6. The values of α based on the logistic model were best suited for the clay, silt loam, and loam soils. For loam soils, its value was appropriate for 95% of these soils. However, its value was not appropriate for the sandy loam soils. Smaller values of R_{MSE} for the prediction of the soil water characteristic curve for loam soil with a linear model for the determination of α were obtained in comparison with those with constant α procedure. Therefore, the values of α based on the linear model were best suited for the sandy loam soils.

In general, Figs. 1 and 2 indicated that using the particle radius of 999 μm resulted in SWC curves closer to those obtained by coarse particle radius of 125 μm. Further, the mean value of R_{MSE} for logistic procedure is smaller by using the particle radius of 999 μm (Tables 5 and 6).

Table 5. The range of root mean square error for different soil textures and various procedures for scale parameters determination with modified particle size distribution

Texture	Logistic	Linear	Constant α
Loam	0.028-0.076	0.046-0.088	0.049-0.093
Clay	0.012-0.030	0.035-0.051	0.024-0.042
Silt loam	0.044-0.047	0.053-0.073	0.072-0.092
Sandy loam	0.090-0.134	0.071-0.116	0.074-0.119
Mean	0.053	0.064	0.069

Table 6. The values of holistic root mean square error for different soil textures and various procedures for scale parameters determination with original particle size distribution and modified model

Model	Soil texture	Logistic	Linear	Constant α
Original	Loam	0.0625	0.0365	0.0395
	Clay	0.094	0.0845	0.0905
	Silt loam	0.1125	0.092	0.080
	Sandy loam	0.074	0.051	0.024
Modified	Loam	0.0492	0.0623	0.0674
	Clay	0.021	0.043	0.033
	Silt loam	0.0455	0.063	0.082
	Sandy loam	0.112	0.0935	0.0965

4. CONCLUSION

The results indicated that for clay and silt loam soils using a radius of 999 μm (the modified model), the particle-size distribution, and consequently soil moisture characteristic curves, was predicted more accurately than those obtained by using a radius of 125 μm for the largest particles of the soil. This is specifically shown for α determined by logistic procedure. The values of α based on the logistic model were best suited for the clay, silt loam, and loam soils. The values of α based on the linear model were appropriate for the clay and silt loam soils. Further, the values of constant α were best suited for the clay soils. However, these results are considered indicative rather than conclusive due to the small size of the data set for clay, silt loam and sandy loam soils. Finally, it is proposed to test the modified model for a wider data set.

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