Iranian Journal of Science & Technology, Transaction B, Engineering, Vol. 31, No. B5, pp 459-471
Printed in The Islamic Republic of Iran, 2007

PARAMETRIC STUDY ON MOMENT REDISTRIBUTION IN CONTINUOUS RC BEAMS USING DUCTILITY DEMAND AND DUCTILITY CAPACITY CONCEPT *

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Abstract– Experimental studies show that an indeterminate structure or a continuous concrete beam does not fail when critical sections reach their ultimate strengths. Therefore, if a structure has adequate ductility, stress and moment redistribution will take place in the flexural members by developing plastic hinges at critical sections. This causes the other points of beams to achieve their ultimate strengths and capacities. Besides, moment redistribution allows designers to adjust the bending moment diagram computed by elastic analysis. The usual result is a reduction in the values of negative moments at the support face as well as an increase in the values of positive moments along the span.

In the current investigation, a parametric study on moment redistribution in continuous RC beams with equal spans under uniform loading was performed. First, the governing equation for the allowable percent of moment redistribution was extracted using ductility demand and ductility capacity concepts. The effects of different parameters such as the concrete compressive strength, the amount and the strength of reinforcing steel, the magnitude of elastic moment at the support and the ratio of the length to the effective depth of the continuous beam on moment redistribution were then investigated. Furthermore, the allowable moment redistributions were calculated according to the regulations of different codes in each case. The results showed that, whereas the permissible moment redistribution in continuous reinforced concrete beams based on the relevant rules in the current codes is not in a safe margin in some cases, it is rather conservative in most cases.

Keywords– Moment redistribution, reinforced concrete, ductility demand, ductility capacity

1. INTRODUCTION

Elastic analysis of continuous reinforced concrete beams is not able to demonstrate a realistic behavior of the structure at ultimate loads. According to the codes, continuous members must be designed to resist more than one configuration of live loads. An elastic analysis is performed for each loading arrangement and an envelope moment value is obtained for the design of each section. Therefore, for any loading patterns, certain sections in a given span will reach the ultimate moment, while the full capacity in the other sections is not utilized. Experimental tests have shown that a structure can carry some additional load in excess to its elastic capacity if the sections that reach their moment capacities continue to rotate as plastic hinges and redistribute the moments to other sections until a collapse mechanism forms. Such additional load capacity provides the possibility of moment redistribution in concrete structures [1].

Moment redistribution permits designers to use the bending moment diagram computed by elastic analysis and modify it to account for plastic behavior. Usually this redistribution is carried out by decreasing the negative moment at the first plastic hinge region, with corresponding changes in the positive moments required by the equilibrium. The changes in moments may be such as to reduce both the

*Received by the editors April 16, 2006; final revised form December 16, 2006.
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maximum positive and negative moments in the final envelope diagram. Accordingly, moment redistribution may avoid reinforcement congestion in the negative moment regions without any increase in the reinforcement of the positive moment region.

The primary experiments regarding the moment redistribution in continuous reinforced concrete beams were performed by Mattock in 1959, and Cohn in 1964 [2, 3]. Their works indicated that the redistribution of the design bending moments up to 25 percent does not significantly change the curvature and the crack widths in a continuous RC beam designed by the elastic theory.

In 1993 Scholz investigated and verified the influence of the varying beam slenderness and stiffness on the moment redistribution in continuous RC beams using the ductility concept. He compared his proposed method with the allowable moment redistribution given in the Canadian code, and concluded that his method predicts more realistic results for moment redistribution compared to the results of the Canadian code (CAN3-A23.3) [4].

In 2000 Lin and Chien studied the effect of longitudinal and transverse reinforcement on the ductility and moment redistribution of 26 continuous RC beams. They concluded that the transverse reinforcements confine the concrete and increase the ductility and moment redistribution in the continuous flexural members [5].

The aim of the current study is to determine the allowable moment redistribution using ductility demand and ductility capacity concepts. Furthermore, the influences of different variables on the allowable moment redistribution are investigated and discussed. The results of this study show that the allowable values extracted from the ductility-based relationships are less than those permitted by codes in some cases.

2. RESEARCH SCOPE

The current standards in the world state the permissible moment redistribution in various forms. Some of the codes, i.e. CAN-A23.2, AS 3600, BS 8110 and CEB-FIP, define the allowable moment redistribution directly on the basis of the ratio of the depth of the neutral axis, c, to the effective depth of beam, d [6-9]. ACI 318-99 have limited the percentage of the moment redistribution to the maximum value of $20\{1-(\rho-\rho')/\rho_b\}\%$, where ρ, ρ' and ρ_b are the ratio of the tensile reinforcement , $\rho = A_s/bd$ the ratio of the compressive reinforcement, $\rho' = A'_s/bd$, and the reinforcement ratio corresponding to the balance condition, respectively; while the redistribution is limited to the condition that ρ or $\rho-\rho'$ are not greater than $0.5\rho_b$ [10]. However the ACI 318-05 defines the allowable moment redistribution in terms of net tensile strain in extreme tensile steel, ε_t, and expresses it as $1000\,\varepsilon_t$ percent. According to this code, the moment redistribution is permitted if the critical sections have adequate ductility; i.e. ε_t is equal or greater than 0.0075 at the sections under consideration [11].

Using equilibrium equations in a beam section, the permissible moment redistribution in any code could be expressed as a function of c/d . Such relationships are shown in Fig. 1 for six different codes. It could be observed from this figure that ACI 318 and CAN-A23.2 have limited the maximum redistribution to the value of 20%, while the other 4 codes have limited the redistribution of the moments to the maximum value of 30%.

On the other hand, the ratio of c/d could be easily expressed as a function of the net tensile strain in the extreme tensile reinforcement, ε_t, using the compatibility equations in the strain diagram of a beam section. Such relationships are shown in Fig. 2 for the relevant expressions in the aforementioned 6 codes. It could be seen from this figure that ACI 318 and CAN-A23.2 have considered a more conservative safety margin for moment redistribution compared to the other 4 codes.

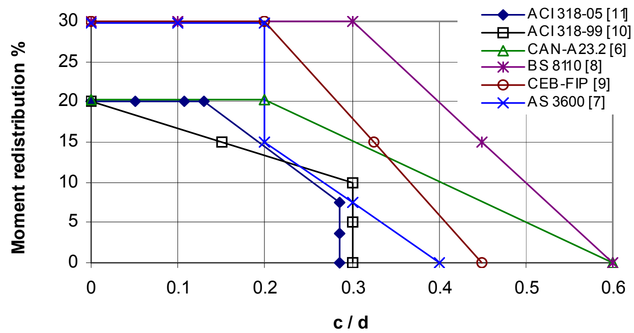

Fig. 1. Permissible moment redistribution versus the ratio of c/d based on different codes

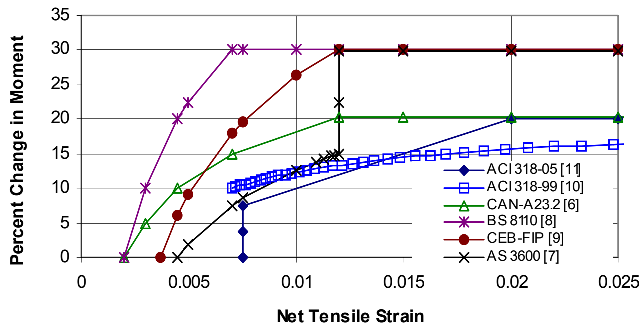

Fig. 2. Permissible moment redistribution versus the net tensile strain based on different codes

In the current research, it is intended to study the influences of different parameters on moment redistribution, i.e. the effects of the compressive strength of concrete, yield strength of steel, the amount of tensile and compressive steel, the magnitude of the elastic moment of the support and the ratio of the span to the effective depth of the beam. To do so, the governing equation of the allowable moment redistribution in continuous RC beams is obtained with regard to the minimum rotational capacity and the required ductility in the plastic hinge region. Afterwards, the effects of the aforementioned parameters on the amount of moment redistribution are investigated and the results are compared with the limitations of the moment redistribution provided by ACI 318-05 and the relevant requirements in some other codes.

3. CONVENTIONAL MOMENT REDISTRIBUTION

Figure 3 illustrates an internal span of a continuous reinforced concrete beam with length L, subjected to uniformly distributed load W. The maximum moments at the support and the mid-span of the beam obtained from the elastic analysis for a particular loading configuration are M_e, and M'_e, respectively. As shown in Fig. 3 these values will be respectively changed to M_u and M'_u after completion of redistribution. The percentage of moment redistribution in the negative moment region, *R*, is defined as follows:

$$R = 100\left(M_e - M_u\right)/M_e \tag{1}$$

From Eq. (1), M_u can be expressed in terms of the percentage of moment redistribution, *R*, and the elastic moment at the support, M_e;

$$M_u = M_e(1 - R/100) \tag{2}$$

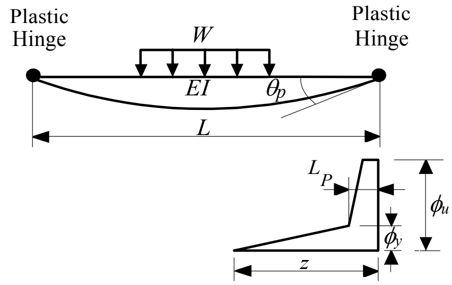

Fig. 3. Elastic and inelastic moments

4. DUCTILITY DEMAND

Assume that the continuous beam in Fig. 4 has a constant stiffness *EI*, and primarily the plastic hinge takes place at the support; also assume an elasto-plastic bilinear moment-curvature for the beam. Therefore, the demand value of plastic hinge rotation using the moment area method as shown in Fig. 5 can be expressed by;

$$\theta_p^d = (\theta'_p - \theta''_p) = \frac{L}{2EI}\left(\frac{WL^2}{12} - M_u\right) \tag{3}$$

where *E* is the modulus of elasticity, *I* is the moment of inertia of the beam, and θ'_P and θ''_P are illustrated in Fig. 5.

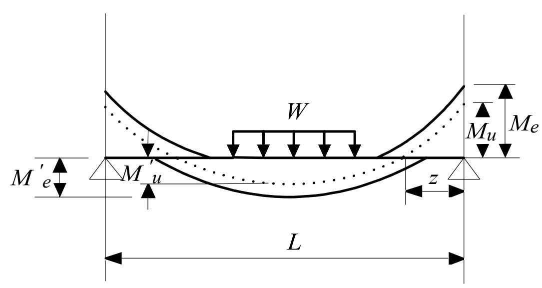

Fig. 4. Plastic hinge rotation and curvatures

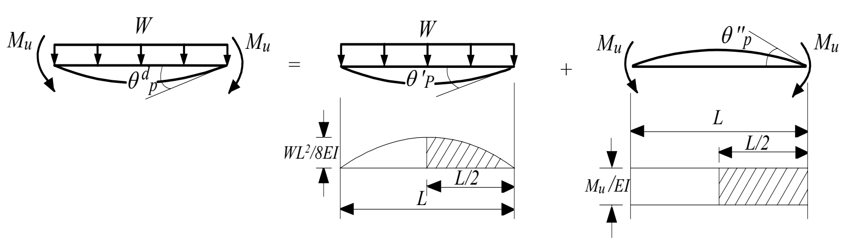

Fig. 5. Demand rotation using moment area method

The relationship between the available plastic hinge rotation and the curvature of a critical section is given by [12];

$$\theta_p^a = (\phi_u - \phi_y)L_p \tag{4}$$

where ϕ_u is the curvature at ultimate, ϕ_y is the curvature at yield, and L_P is the effective length of the plastic hinge (Fig. 4). Besides, based on the bi-linear moment-curvature diagram for the beam section, it is possible to express the section curvature at yield as;

$$\phi_y = M_u / EI \tag{5}$$

To have sufficient ductility for the occurrence of full moment redistribution in a continuous beam, θ_p^a must be equal or greater than θ_p^d. To meet this requirement and substituting the *EI* value from Eq. (5) into Eq. (3), it can be stated;

$$(\phi_u - \phi_y)L_p \geq \frac{L}{2}\left(\frac{WL^2\phi_y}{12M_u} - \phi_y\right) \tag{6}$$

$$\therefore \frac{\phi_u}{\phi_y} \geq 1 + \frac{L}{2L_p}\left(\frac{WL^2}{12M_u} - 1\right) \tag{7}$$

Substituting M_u from Eq. (2) into Eq. (7) will result in:

$$\frac{\phi_u}{\phi_y} \geq 1 + \frac{L}{2L_p}\left(\frac{(WL^2/12)}{M_e(1 - R/100)} - 1\right) \tag{8}$$

where ϕ_u/ϕ_y is the ductility demand curvature.

5. DUCTILITY CAPACITY

The curvature of a beam section at its ultimate behavior with respect to the strain diagram at failure is;

$$\phi_u = \varepsilon_{cu} / c \tag{9}$$

with $\varepsilon_{cu} \cong 0.003$ at the ultimate strain of concrete. The curvature at yield is obtained as;

$$\phi_y = \left(f_y / E_s\right) / \left[d(1-k)\right] \tag{10}$$

where f_y and E_s are yield stress and modulus of elasticity of steel, respectively; d is effective depth. The value of k, the ratio of the depth of the neutral axis to an effective depth of a beam section at the verge of the yield of reinforcement could be obtained as follows [12]:

$$k = \left[(\rho + \rho')^2 n^2 + 2(\rho + \rho' d'/d)n\right]^{1/2} - (\rho + \rho')n \tag{11}$$

with $n = E_s / E_c$ and d' equal to the distance from compression steel to the extreme compression fiber.

Dividing Eq. (9) by Eq. (10), the ductility capacity index will be calculated as follows:

$$\eta_\phi = \frac{\phi_u}{\phi_y} = \frac{\varepsilon_{cu}(1-k)}{(f_y/E_s)(c/d)} \tag{12}$$

The relationship between c/d_t and ε_t using compatibility equation of strains, as shown in Fig. 6, can be indicated as;

$$c/d_t = \varepsilon_{cu} / (\varepsilon_t + \varepsilon_{cu}) \tag{13}$$

where d_t is the distance from the extreme compression fiber to the extreme tension steel. Furthermore, the ratio of c/d could be obtained as follows:

$$c/d = (d_t/d)[\varepsilon_{cu}/(\varepsilon_t + \varepsilon_{cu})] \tag{14}$$

Substituting the value of c/d in Eq. (12), the ratio of ϕ_u/ϕ_y will be found to be as follows:

$$\eta_\phi = \frac{\phi_u}{\phi_y} = \frac{(\varepsilon_t + \varepsilon_{cu})(1-k)}{(d_t/d)(f_y/E_s)} \tag{15}$$

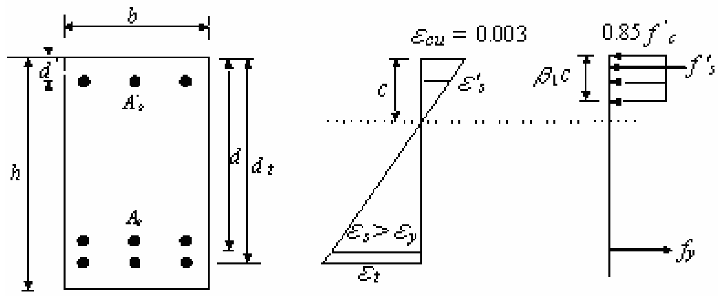

Fig. 6. Stress and strain distribution of beam section at ultimate

6. ALLOWABLE MOMENT REDISTRIBUTION

Sufficient rotational capacity and adequate ductility for moment redistribution of flexural members will be provided, if ductility capacity would be at least equal to ductility demand. Equating the right side of inequality (8) and Eq. (15), the allowable moment redistribution R is extracted as:

$$R = 100\left(1 - \frac{WL^2/12}{M_e\left(\frac{2L_p}{L}\left[\frac{(\varepsilon_t + \varepsilon_{cu})(1-k)}{(d_t/d)(f_y/E_s)} - 1\right] + 1\right)}\right) \tag{16}$$

The variation in the allowable moment redistribution versus the net tensile strain in extreme tensile steel of the beam section is shown in Fig. 7. For a continuous beam with equal spans, $L/L_P = 38$, $f_y = 400$ MPa, $\varepsilon_{cu} = 0.003$, $d_t = d$ and the elastic moment of support $M_e = WL^2/12$. Figure 7 illustrates adequate agreement between the moment redistribution curve obtained from Eq. (16) and reference [13]. It could be seen from this figure that the ACI 318-05 code provides an adequate margin of moment redistribution, while the other standards present excessive and higher values of redistribution; for example AS 3600 and CEB-FIP show high values of moment redistribution for $\varepsilon_t = 0.012 - 0.02$ and $\varepsilon_t = 0.005 - 0.02$, respectively. Note that Eq. (16) could also be directly used for moment redistribution in continuous reinforced concrete beams provided for the control of serviceability requirements.

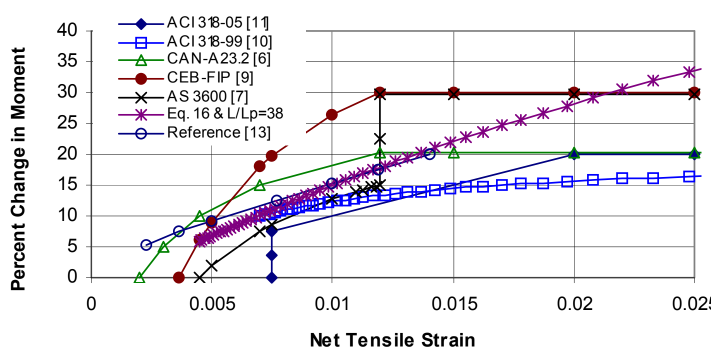

Fig. 7. Variation of permissible moment redistribution versus net tensile strain in extreme tensile steel based on different codes

7. EFFECT OF L/d RATIO ON MOMENT REDISTRIBUTION

As indicated in Eq. (16), the permissible moment redistribution is related to L/L_p. To investigate the effect of the ratio of the beam span to the effective depth of the beam, L/d, on the amount of the moment redistribution in a continuous beam, the length of the plastic hinge , L_p, must be quantified.

a*) *The plastic hinge length

Various expressions have been proposed for determining the equivalent length of the plastic hinge; some of them are summarized as follows [12]:

- Baker, 1964:

$$L_P = k_1 k_3 (z/d)^{1/4} d \quad \text{[mm]} \tag{17}$$

where k_1 is taken equal to 0.7 for mild steel, k_3 is taken equal to 0.6 when $f'_c = 35.2$ MPa, and 0.9 when $f'_c = 11.7$ MPa; d is the effective depth of the beam; and z is the distance of critical section from the point of inflection in the bending moment diagram.

- Sawyer, 1964

$$L_P = 0.075z + 0.25d \quad \text{[mm]} \tag{18}$$

- Mattock, 1967

$$L_p = 0.05z + 0.5d \quad \text{[mm]} \tag{19}$$

- Paulay and Priestley, 1992 [14]:

$$L_p = 0.08z + 0.022 f_y d_b \geq 0.044 f_y d_b \quad \text{[mm]} \tag{20}$$

where f_y is the yield strength of steel and d_b is the diameter of longitudinal steel.

- Leman *et al*, 1998 [15]:

$$L_P = 0.5\alpha\, z + 1.2\alpha\,(f_u / 4\sqrt{f'_c})d_b \quad \text{[mm]} \tag{21}$$

$$\alpha = \frac{M_u - M_y}{M_u} \tag{22}$$

where f_u is the ultimate strength of steel, f'_c is the compressive strength of concrete, M_y is the moment at the first yield of tensile steel and M_u is the ultimate moment of beam section.

- Panagiotakos and Fardis, 2001 [16]:

$$L_P = 0.12z + 0.014 f_y d_b \quad \text{[mm]} \tag{23}$$

It could be seen that the length of the plastic hinge in Eq. (17) to (19) is defined in terms of z and effective beam depth, d ; while L_p in Eqs. (20) to (23) is comprised of the effect of the parameter z and the anchorage slip of longitudinal reinforcing bars.

b*) *The relationship between L/d and L/L_p

To determine relation between L/d and L/L_p , it is required to first calculate z . For practical cases, in a continuous beam with equal spans under uniform loading, z varies from $0.15L$ to $0.20L$ for $M_u = WL^2/16$ to $M_u = WL^2/12$, respectively. Besides, the ACI 318-05 code limits the ratio of length to the effective depth of the continuous beam, L/d , to the maximum value of 21 for deflection. Since h varies from $1.1d$ to $1.2d$ in conventional beams; therefore, L/d will be limited to the maximum value of 25. Hence, for calculating L_p based on Eq. (17), substituting $k_1 = 0.7$ and assuming $z = 0.2L$ and an average value of 0.75 for k_3 , the plastic hinge length, L_p , will be equal to

$$L_P = (0.525)(0.2L)^{1/4} d^{3/4} \tag{24}$$

$$L/L_P = 2.8483(L/d)^{3/4} \qquad \text{(Baker's equation)} \tag{25}$$

Furthermore, assuming $z = 0.2L$ in Eqs. (18) and (19), L/L_p will be as follows for Sawyer's and Mattock's expressions, respectively;

$$L/L_P = 1/(0.015 + 0.25\,d/L) \qquad (26)$$

$$L/L_P = 1/\,(0.010 + 0.5\,d/L) \qquad (27)$$

Similar equations could be obtained for L/L_p, with the assumption of $z = 0.15L$; for instance using Eq. (18), L/L_p will equal to;

$$L/L_P = 1/(0.01125 + 0.25\,d/L) \qquad \text{(Sawyer's equation)} \qquad (28)$$

The amounts of L/L_p have been calculated and presented in Table 1 for $L/d = 15,\ 20$ and 25, according to Baker's, Sawyer's, and Mattock's equations and for $z = 0.15L$ and $0.20L$. Furthermore, assuming $L = 8000$ mm, $d_b = 20$ mm and $f_y = 400$ MPa, the ratio of L/L_p has been presented in this table, based on Eqs. (20) and (23). As shown in Table 1, except for Sawyer's equation that indicates higher values for L/L_p, the other predictions of L/L_p are close together.

Table 1. Ratios of L/L_p for different values of z

z	L/d	L/L_p Eq. (25)	L/L_p Eq. (26)	L/L_p Eq. (27)	L/L_p Eq. (20)	L/L_p Eq. (23)
0.20L	15 20 25	22 27 32	32 36 40	23 29 33	26	26
0.15L	15 20 25	23 29 34	36 42 47	25 31 36	29	31

Figure 8 illustrates the variation of the permissible moment redistribution in terms of ε_t, based on Eq. (16), assuming $f_y = 400$ MPa and $M_e = WL^2/12$, for $z = 0.2L$ and for different values of L/d. To draw Fig. 8, Eq. (26), which is based on Sawyer's equation, was used for the relationship of L/L_p and L/d. As shown in Fig. 8, the permissible moment redistribution decreases when the ratio of L/d increases. Furthermore, it could be observed in this figure that CAN-A23.2 and AS 3600 codes imply excessive redistribution for $\varepsilon_t < 0.015$ and $\varepsilon_t = 0.012 - 0.025$, respectively.

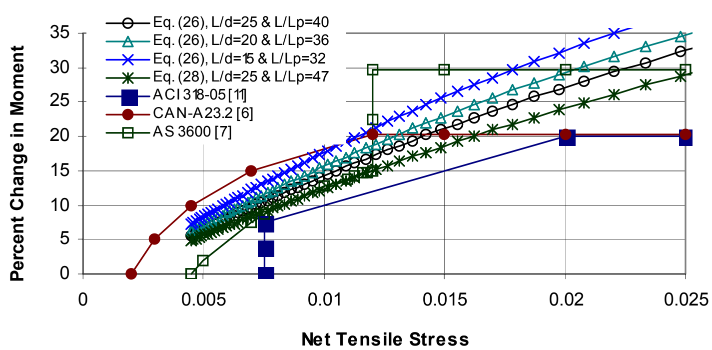

Fig. 8. Allowable moment redistribution for different ratio of L/d

The variation of permissible moment redistribution for $z = 0.15L$, $L/d = 25$ and $L/L_p = 47$, has been also indicated in Fig. 8. It could be seen from this figure that for high levels of L/L_p, e.g.

$L/L_p = 47$, ACI 318-05 provides a safety margin for moment redistribution, while other codes like CAN-A23.2 and AS 3600 give higher amounts of redistribution in the same condition.

8. EFFECT OF CONCRETE COMPRESSIVE STRENGTH

To evaluate the effect of the compressive strength of concrete on moment redistribution in continuous beams, it is required to determine the relation of ε_t and f'_c. As shown in Fig. 9, assuming $\rho' = 0$ and using the strain compatibility principal, Eq. (14) can be rewritten as below:

$$c/d = (d_t/d)[\varepsilon_{cu}/(\varepsilon_t + \varepsilon_{cu})] = (\rho f_y)/(0.85 f'_c \beta_1) \tag{29}$$

$$(\varepsilon_t + \varepsilon_{cu})/(d_t/d) = (0.85 \beta_1 f'_c \varepsilon_{cu})/(\rho f_y) \tag{30}$$

Substituting Eq. (30) into Eq. (16) and assuming $\varepsilon_{cu} = 0.003$ and $E_s = 2\times10^5$ MPa, the permissible moment redistribution is obtained in terms of f'_c;

$$R = 100\left(1 - \frac{(WL^2/12)}{M_e\left(\frac{2L_p}{L}\left[\frac{510\beta_1 f'_c(1-k)}{\rho(f_y)^2} - 1\right] + 1\right)}\right) \tag{31}$$

Figure 9 shows the permissible moment redistribution in terms of ε_t for various amounts of f'_c, assuming $f_y = 400$ MPa, $L/L_p = 38$ and $M_e = WL^2/12$. It could be seen from Fig. 9 that higher values of f'_c provide higher amounts for possible moment redistribution. Such a result is applicable for concrete strengths which comply with the codes' assumptions on compression stress block parameters. Furthermore, Fig. 9 shows that for low values of concrete compressive strength, i.e. $f'_c = 17$ MPa, ACI 318-05 overestimates the permissible moment redistribution; also the other current codes like CAN-A23.2., AS 3600 and CEB-FIP, allow for higher amounts of moment redistribution, compared to the quantities taken from theoretical analysis.

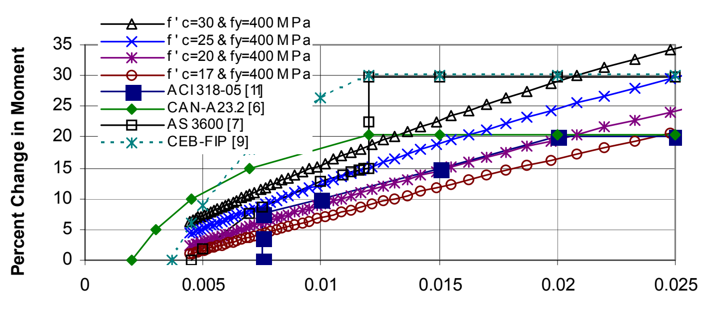

Fig. 9. Permissible moment redistribution versus net tensile strain of steel for various concrete compressive strength

9. EFFECT OF YIELD STRENGTH OF STEEL

The yield strength of steel is counted as an influential parameter on moment redistribution in RC continuous beams. Assuming $M_e = WL^2/12$ and $L/d = 22$, the permissible moment redistribution in terms of strain ε_t, for $f_y = 400$ and 550 MPa is indicated in Fig. 10. It could be seen from Fig. 10 that as long as the yield strength of reinforcing steel is not greater than that of conventional steel, i.e. $f_y \le 420$ MPa, the ACI 318-05 code permits for moment redistribution in a safe margin, while the other standards, i.e. CAN-A23.2 and AS 3600 allow for higher amounts of redistribution. However, for higher values of f_y, all codes may overestimate the amount of permissible moment redistribution, especially when the beam is designed in such a way that its tensile reinforcement is not much tensioned.

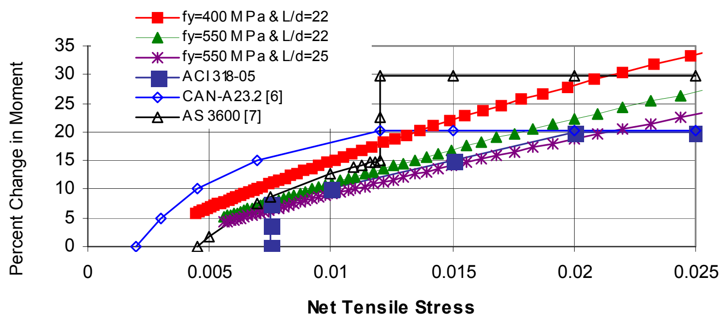

Fig. 10. Permissible moment redistribution for different values of yield strength of reinforcing steel

10. EFFECT OF THE AMOUNT OF REINFORCING STEEL

The amount of tensile and compressive reinforcing steel in a continuous beam also affects on the possible moment redistribution. For an under-reinforced concrete section which is shown in Fig. 6, using equilibrium and compatibility equations and solving the resulting equation with respect to the neutral axis depth, c, the following equation could be obtained;

$$c = \frac{(\rho f_y - 0.003\rho' d E_s) + \sqrt{(\rho f_y - 0.003\rho' E_s)^2 + 3.4 f'_c \beta_1 (0.003\rho' E_s d'/d)}}{1.7\beta_1 f'_c} d \qquad (32)$$

where ρ and ρ' are the tensile and compressive reinforcement ratios, respectively; and d and d' are respectively the distances of tensile and compressive steel from the extreme compression fiber in concrete. Substituting c from Eq. (32) into Eq. (14) and using the balanced steel ratio as $\rho_b = 0.85\beta_1 \frac{f'_c}{f_y}\frac{600}{600+f_y}$, the following equation is extracted;

$$\frac{\varepsilon_t + \varepsilon_{cu}}{d/d_t} = \frac{2\rho_b (f_y + 600)(f_y/600)}{(\rho f_y - 0.003\rho' E_s) + \sqrt{(\rho f_y - 0.003\rho' E_s)^2 + 3.4 f'_c \beta_1 (0.003\rho' E_s\, d'/d)}} \qquad (33)$$

Equation (33) accompanied by Eq. (16) can be used to show the effect of the amount of steel reinforcement on the possible percentage of moment redistribution. Using these equations and assuming $f_y = 400$ MPa, $L/L_p = 38$ and $M_e = WL^2/12$, the permissible moment redistribution in terms of ρ/ρ_b is calculated and illustrated in Fig. 11 for different ratios of compressive and tensile steel.

Figure 11 shows that increase in the ratio of tensile reinforcement to the balanced reinforcement, ρ/ρ_b, decreases the allowable moment redistribution. Furthermore, increasing the amount of compressive reinforcement, ρ', increases the amount of the moment redistribution. It could be observed from the figure that ACI 318-05, despite the other codes, provides acceptable amounts of moment redistribution for different combinations of ρ/ρ_b and ρ'/ρ. Nevertheless, the procedure of

ACI 318-05 leads to very conservative values of moment redistribution for low ratios of ρ/ρ_b and high ratios of ρ'/ρ.

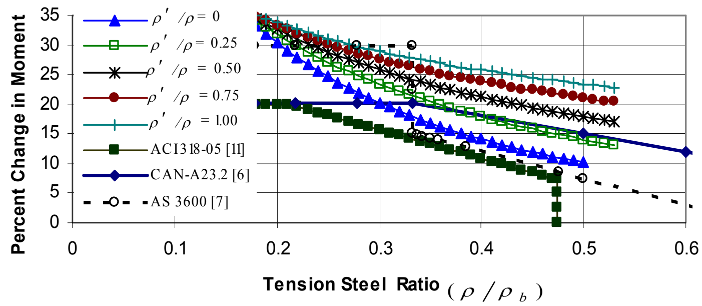

Fig. 11. Permissible moment redistribution versus the tensile steel ratio, for different values of compressive steel

11. EFFECT OF THE ELASTIC MOMENT OF SUPPORT

To investigate the effect of the amount of elastic moment of support on the moment redistribution based on Eq. (16), the variation of the permissible moment redistribution in terms of ε_t for $L/L_p = 38$, $M_e = WL^2/11$ and $M_e = WL^2/12$ is shown in Fig. 12. Note that in conventional continuous beams, the negative elastic moment at the face of internal supports normally varies within $WL^2/11$ and $WL^2/12$. It could be seen from Fig. 12 that if the elastic moment in the support region of a reinforced concrete beam increases, the permissible moment redistribution in the beam would also increase. Figure 12 shows that for a particular net tensile strain of $\varepsilon_t = 0.02$, the percentages of redistribution in moments are 28% and 34% for elastic moment of the support equal to $WL^2/12$ and $WL^2/11$, respectively. However most codes including ACI 318-05 and CAN-23.2 limit the maximum permissible moment redistribution to 20%, probably to prevent the severe propagation of the cracks under serviceability condition. Nevertheless, some other codes including AS 3600, BS 8110 and CEB-FIP allow for redistribution of the moments up to 30% [7-9].

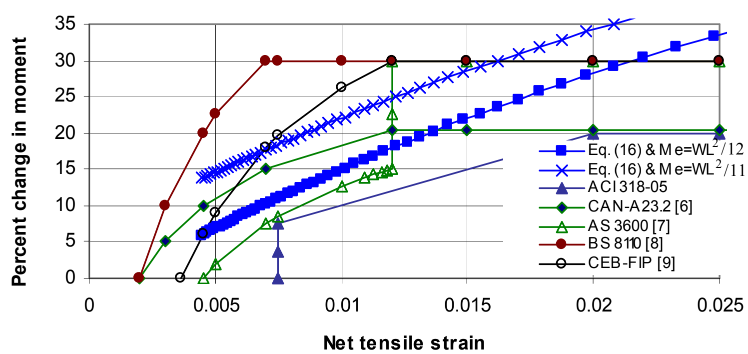

Fig. 12. Permissible moment redistribution for different values of support elastic moment

12. SUMMARY AND CONCLUSIONS

In the current study, using ductility demand and ductility capacity concepts, the effects of different parameters on the moment redistribution of reinforced concrete continuous beams with equal spans, under uniform loading were investigated. Providing minimum rotational capacity in the plastic hinge regions of the beam, the constitutive equation of permissible moment redistribution in terms of net tensile strain, ε_t, as well as in terms of ratios of tensile and compressive reinforcement was obtained. The variations of percent changes in moment redistribution with regard to some parameters were drawn and compared with the provisions of some current standards including ACI 318-05, CAN-A23.2, AS 3600, BS 8110 and CEB-FIP codes in some cases. The results of this study within the assumptions which were made in the analysis are summarized as follows:

1. Increasing the net tensile strain in extreme tensile steel increases the permissible moment redistribution values.
2. Increasing the ratios L/d and L/L_p, and utilizing the conventional equations for the estimation of the plastic hinge length, decreases the permissible moment redistribution.
3. The permissible moment redistribution is decreased by decreasing the concrete compressive strength; also, it is decreased by the increase of yield strength of tensile reinforcement. Furthermore, increase in tensile reinforcement ratio, decreases the value of permissible moment redistribution.
4. Compressive steel advantageously affects on moment redistribution enhancement. The higher ratios of compressive to tensile reinforcement in the beam section augment the possible moment redistribution.
5. The permissible moment redistribution would enhance by increase in the elastic moment of support.
6. Although the ACI 318-05 code exhibits a safe margin for permissible moment redistribution in most cases: the code provisions lead to higher amounts of moment redistribution in some cases; for instance, in continuous beams with high ratios of L/d and steel yield strength, or in beams with low compressive strength of concrete. Furthermore, other current codes, i.e. Canadian, Australian and European standards provide excessive values of moment redistribution for the cases when $\varepsilon_t < 0.02$, $\varepsilon_t > 0.012$ and $0.005 < \varepsilon_t < 0.025$, respectively.
7. Provisions of the current codes including ACI 318-05, CAN-A23.2 and AS 3600 lead to very conservative values for moment redistribution in some cases, i.e. in continuous reinforced concrete beams with tensile and compressive reinforcement with the ratio of $\rho'/\rho > 0.25$.
8. Compared to the provisions of the codes, higher values of moment redistribution could be obtained for continuous reinforced concrete beams with high values of ε_t at ultimate; however, it may not be advisable to utilize these values due to severe crack propagation at the serviceability stage.

REFERENCES

1. Notes on ACI 318-02, (2002). Building code requirements for structural concrete with design applications. *PCA*, USA.
2. Mattock, A. H. (1959) Redistribution of design bending moments in reinforced concrete continuous beams, *Proceedings*, *Institution of Civil Engineers* (London), Vol. *13*, pp. 35-46.
3. Cohn, M. Z. (1964). Rotational compatibility in the limit design of reinforced concrete continuous beams. *Proceedings*, *International Symposium on the Flexural Mechanics of Reinforced Concrete, ASCE-ACI*, Miami, pp. 359-382.
4. Scholz, H. (1993). Contribution to redistribution of moments in continuous reinforced concrete beams, *ACI Struct. J.*, Vol. *90*, No. 2, pp. 150-155.
5. Lin, Chien-Hung, & Chien, Yu-Min. (2002). Effect of section ductility on moment redistribution of continuous concrete beams, *Journal of Chinese Institute of Engineers*, *Transactions of the Chinese Institute of Engineers,* Vol. 23, No. 2, pp. 131-141.
6. CAN-A23.2. (1994). Code for the design of concrete structures for buildings, *Canadian Standards Association*.

7. AS 3600, (1994). Australian concrete structures standard. *Standards Australia*, Sydney, Australia.
8. BS 8110, (1995) *Structural Use of Concrete-part 1*, British Standards Institution.
9. CEB-FIP, (1990). International system of unified standard codes of practice for structures. Model Code for Concrete Structures, Europe.
10. ACI 318-99, (1999). Building code requirements for structural concrete (ACI 318-99) and commentary-ACI 318 RM-99. *ACI*, Farmington Hills, MI, USA.
11. ACI 318-05, (2002). Building code requirements for structural concrete (ACI 318-02) and commentary-ACI 318 R-05. *ACI*, Farmington Hills, MI, USA.
12. Park, R. & Paulay, T. (1975). *Reinforced concrete structures*. John Wiley & Sons.
13. Mast, R. F. (1992). Unified design provisions for reinforced and prestressed concrete flexural and compression members. *ACI Struct. J.*, Vol. 89, No. 2, pp. 185-199.
14. Paulay, T. & Priestley, M. J. N. (1992). *Seismic design of reinforced concrete and masonary buildings.* John Wiley & Sons, New York.
15. Leman, D. E., Calderone, A. J. & Moehle, J. P. (1998). Behavior and design of slender columns subjected to lateral loading. *6th US National Conference on Earthquake Engineering*, EERI, Seattle, Washington, May 31.
16. Panagiotakos, T. & Fardis, M. N. (2001). Deformation of RC members at yeilding and utimate. *ACI Struc. J.*, Vol. 98, No. 2, pp. 135-148.