

**“Research Note”**

**A SIMPLE NONLINEAR-ELASTIC MODEL FOR PREDICTION OF LIME  
STABILIZED CLAYEY SAND BEHAVIOR\***

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**Abstract**– This study was undertaken to determine the mechanical behavior of lime- stabilized clayey sands. The variations of compressive and tensile strengths of materials were investigated. Uniaxial and indirect tensile tests were performed and the results of stress-strain diagrams were used to present a simple nonlinear-elastic model. The results show that a simple parabolic elastic model can predict the stress-strain relation before failure. A matrix of elastic coefficients is then presented based on the suggested model and the stress level.

**Keywords**– Clayey sand, lime stabilization, parabolic elastic model

## 1. INTRODUCTION

Stabilization of soils can provide a tremendous economic advantage. Lime stabilization, among other soil improvement methods, is a common and simple technique to make sub layers stronger and less deformable [1, 2]. The objective of this study is to evaluate the potential use of lime in earthwork as a construction material. Hence, knowledge of a complete stress-strain behavior is essential. Application of lime-stabilized clayey sands can be an efficient and cost-effective technology for improving road construction [3, 4]. Meanwhile, there is no standard that specifies the minimum required strength and durability of soil stabilized with lime. It is up to the individual engineer or design agency to determine the strength and durability criteria for highway materials.

## 2. EXPERIMENTAL

A testing program was undertaken using five different clayey sand materials. Percentages of clay content ranged from 5% to 36% by weight. The coarser materials consist of coarse to fine sands. They were washed and then mixed with clay according to their natural condition. Kaolinite clay was used as the fines contents for each mixture. Hydrated lime was used for material stabilization.

The tests were performed on specimens after they reached the laboratory temperature [5]. The results of uniaxial tests on 5 specimens containing 5, 15, 22, 30 and 36 percent clay content, stabilized with 3, 6 and 9 percent lime respectively, are presented in Fig. 1. The initial part of all curves is modified for the erratic stress-strain behavior of samples in the first stages of loading due to no uniform stress distribution on surfaces at the beginning of the test. A digital compressive apparatus with a 0.1kN accuracy, performed Brazilian tests to obtain the tensile strength of specimens. For all cases the Poisson's ratio,  $\nu$ , was obtained by monitoring the vertical and lateral strain of the specimens. It ranged from 0.23 to 0.28, thus a mean value of 0.25 was assumed for further calculations.

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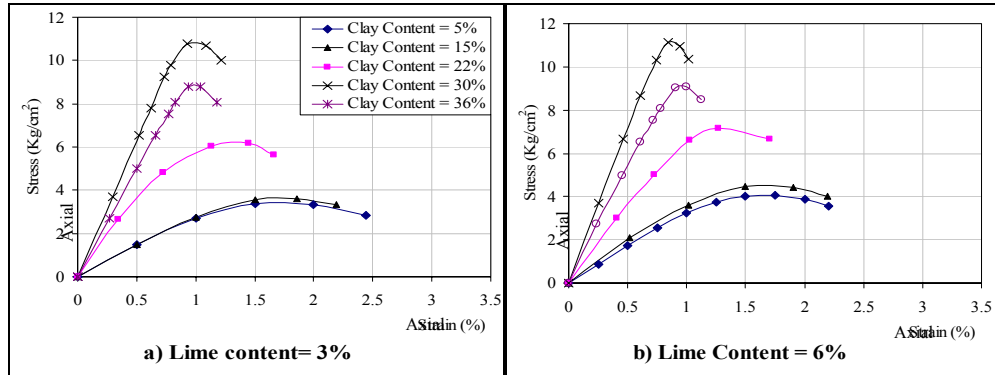


Fig. 1. Stress-strain diagrams for samples stabilized with 3% and 6% lime

### 3. VARIATION OF COMPRESSIVE AND TENSILE STRENGTH

While clay content ranges from 5 to 15% there is either no or very little variation in compressive strength. But for higher clay contents, especially for 20 to 30% clay contents, there is a large variation of compressive strength (e.g. the 5% increase of clay content results in a 30% increase in compressive strength). Fig. 2a shows the variations of the compressive strength of different specimens versus different fines contents. Any more increase in clay content will decrease the compressive strength of the specimens.

Further, for the tensile strength of specimens similar results were observed (i.e. the maximum variations in tensile strength of specimens was observed for clay contents ranging between 15 to 30%). This is called Optimum Clay Content (OCC). Optimum clay content is defined as the optimum percentage of fines contents, which reacts with lime molecules and becomes cemented, providing the maximum strength of lime-stabilized materials. The increase in clay content results in tensile strength becoming almost constant or increasing very slightly in some cases. The variation of tensile strength of the specimens for different lime contents, i.e. 3 and 6%, is presented in Fig. 2b.

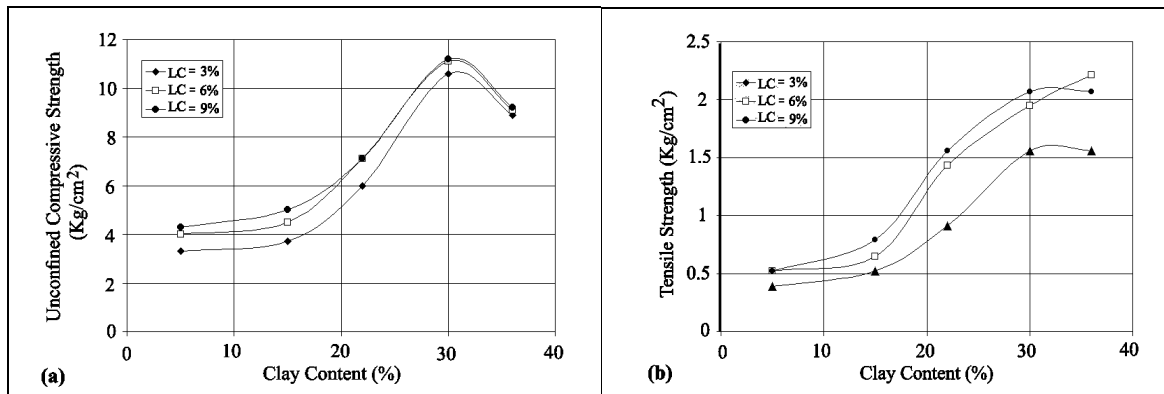


Fig. 2. a) Compressive strength and b) tensile strength vs. clay content for lime-stabilized specimens

### 4. NONLINEAR ELASTIC MODEL AND ELASTIC BEHAVIOR

The results show that as the strength of the material becomes greater a brittle behavior is formed. This brittle behavior is about 3 times greater for the materials with lower clay contents (but with similar lime content). The brittle behavior can be evaluated by the ratio of maximum stress to the maximum strain of the specimens. The behavior of lime-stabilized soils in a simple compressive test resembles concrete behavior (considering the stress-strain diagrams). So, a simple nonlinear elastic model can be used to represent the mechanical properties of such materials [5, 6].

In view of the above, the stress-strain relations obtained from tests can be defined by a simple parabolic function that has a peak positioned on the compressive strength of the specimens. The parabolic function, substituting stress and strain, is defined by Eq. (1):

$$\sigma = -\frac{\sigma_u}{\varepsilon_u^2}(\varepsilon - \varepsilon_u)^2 + \sigma_u \quad (1)$$

Where  $\sigma$  is axial stress,  $\varepsilon$  is axial strain,  $\sigma_u$  is compressive strength and  $\varepsilon_u$  is compressive strain corresponding to  $\sigma_u$ . Therefore, the behavior of materials can be predicted with a simple compressive test if the compressive strength and its corresponding strain are determined. The parabolic function used to define the behavior of materials has been obtained from the test results for the range of stresses from the beginning of loading to the maximum stress level,  $\sigma_u$ .

Assuming a parabolic function for the stress-strain relation, elastic coefficients can be calculated according to predefined definitions:

- a) Initial Elastic Modulus: This modulus can be determined for zero strain at the beginning of loading:

$$E_i = \frac{\partial}{\partial \varepsilon} \left[ -\frac{\sigma_u}{\varepsilon_u^2}(\varepsilon - \varepsilon_u)^2 + \sigma_u \right]_{\varepsilon=0} = \frac{2\sigma_u}{\varepsilon_u} \quad (2)$$

In this equation  $E_i$  is the Initial Elastic Modulus of materials. The value of  $E_i$  can be used for the beginning of loading. Considering the stress-strain diagrams of the samples, the initial value of the elastic modulus is very close to their tangent and secant modules in the early stages of loading, especially for more brittle specimens. However, for ductile specimens with low clay contents (more ductility), the initial and tangent elastic modules are distinctly different.

- b) Tangent Modulus: Similar to Eq. (2) the tangent modulus can be determined in any stage of loading and, at any strain (or stress) condition:

$$E = \frac{\partial}{\partial \varepsilon} \left[ -\frac{\sigma_u}{\varepsilon_u^2}(\varepsilon - \varepsilon_u)^2 + \sigma_u \right] = -\frac{2\sigma_u}{\varepsilon_u^2}(\varepsilon - \varepsilon_u) \quad (3)$$

In which the term  $E$  stands for elastic modulus for any loading level. It is observed that the tangent modulus can be calculated for any stress condition. Rearranging Eq. 1 and replacing  $(\varepsilon - \varepsilon_u)$  in Eq. 3,  $E_t$  can be obtained by Eq. (4):

$$E_t = -\frac{2\sigma_u}{\varepsilon_u^2}(\varepsilon - \varepsilon_u) = \frac{2\sigma_u}{\varepsilon_u} \sqrt{1 - \frac{\sigma}{\sigma_u}} \quad (4)$$

- c) Secant Modulus: For the peak stress of the specimens, from the beginning of loading,  $E_{sec}$  can be determined from Eq. (5) as follows:

$$E_{sec} = \frac{\sigma_u - \sigma_o}{\varepsilon_u - \varepsilon_o} = \frac{\sigma_u}{\varepsilon_u} = \frac{1}{2} E_i \quad (5)$$

It is noticed that the secant modulus is half of the initial tangent modulus if the behavior of the materials is assumed to be parabolic.

As mentioned earlier, for every elastic modulus the mean Poisson's ratio,  $\nu$ , for all specimens, is equal to 0.25. This means a simple nonlinear elastic behavior for the tested materials can be considered. The elasticity matrix that represents the elastic behavior of a certain material, *e.g.*, in a plane strain

problem, can be constructed by substituting the previous equations and Poisson's ratio in Eq. 1, based on the elasticity matrix [5, 7] as follows:

$$D = \frac{1}{\frac{2\sigma_u}{\varepsilon_u} \sqrt{1 - \frac{\sigma}{\sigma_u}}} \begin{bmatrix} 1 & -0.25 & -0.25 & 0 & 0 & 0 \\ -0.25 & 1 & -0.25 & 0 & 0 & 0 \\ -0.25 & -0.25 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.5 \end{bmatrix} \quad (6)$$

The elasticity matrix may be used as a simple model for the mechanical behavior of lime-stabilized clayey sands, using a simple unconfined compressive test. This behavior can be extended to similar clayey sands with similar aggregates and clay contents with a little change in equation.

## 5. CONCLUSIONS

In summary, the compressive strength of clay-lime mixes increases due to chemical reactions that produce a cemented mixture. The variations of compressive and tensile strength of all specimens have their highest value when clay contents ranges in a certain interval, so-called, *optimum clay content (OCC)*. The *OCC* of the investigated materials ranges between 20 to 30% and the maximum strength of lime-stabilized clayey sands is obtained when *OCC* is about 25 to 30%. The reason for such a behavior is the interaction between granular materials and the cemented clay-lime content of the specimens.

Some mechanical properties of clayey sands were investigated and the behavior of these materials was expressed in a simple mathematical equation based on the results of the performed tests on the specimens. A nonlinear elastic behavior can be observed for all specimens in a simple compressive test. Measuring the lateral strain of specimens, and averaging a constant value for Poisson's ratio, mechanical properties of the tested materials were modeled. Thus, the model may be employed for practical purposes.

## REFERENCES

1. Holtz, R. D. and Kovacs, W.D. (1981). *An introduction to geotechnical engineering*. Prentice Hall, 733p.
2. Little, D. N. (1996). Assessment of In Situ Structural Properties of Lime-Stabilized Clay Subgrades. *Transportation Research Record*, TRB, 1546, pp. 13-24.
3. Boardman, D. I., Glendinning, S. and Rogers, C.D.F. (2001) Development of stabilization and solidification in lime-clay mixes. *Geotechnique*, Vol. LI, No. 6, pp. 533-544.
4. Tabtabaei, A. (1982). Application of lime stabilized geomaterials in road construction. *Technical Journal of Tehran University*, No. 44, pp. 47-58.
5. Veis Karami, M. (2004). Aggregation distribution effect of geomaterials on geotechnical properties of lime-stabilized fills. Thesis Submitted for M. Sc. Degree. The Univ. of Guilan.
6. Arabani, M. and Veis Karami, M. (2005). Geomechanical properties of lime stabilized clayey sands. *Proceeding of TREMTI 2005*. Paris, France.
7. Timoshenko, S. P. (1982). *Theory of elasticity*. 3<sup>rd</sup> Ed. McGraw-Hill Company, 567p.