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A GENETIC APPROACH FOR DETERMINING THE GENERALIZED INTERSLICE FORCES AND THE CRITICAL NON-CIRCULAR SLIP SURFACE [*]

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Abstract– In this paper, a genetic algorithm (GA) method is employed to determine the location of the critical non-circular slip surface giving the minimum factor of safety in conjunction with the magnitude of interslice forces and their points of application. A generalized method of slices satisfying both force and moment equations of equilibrium is adopted for stability calculations. The GA adopted here minimizes an objective function that has three terms, namely, error in equilibrium equations, safety factor and a penalty term. By minimizing this objective function, a critical slip surface (with minimum safety factor) is obtained that satisfies both force and moment equations of equilibrium and is kinematically admissible as well. No assumption is made regarding the location of the thrust line where its position is determined through the GA process. Furthermore, all slip surfaces are kinematically admissible and physically acceptable by considering a suitable penalty term. The proposed algorithm is applied to a number of problems and the results are compared with previous work and discussed in detail.

Keywords– Genetic algorithm, slope stability, interslice force, slip surface, limit equilibrium

1. INTRODUCTION

The conventional limit equilibrium method of slope stability analysis consists of two steps; calculation of the safety factor of a particular trial slip surface, and the other, searching for a critical slip surface with the lowest safety factor value. During the past 30 years considerable attention has been given to the first step. Methods by Bishop [1], Fredlund and Krahn [2], Janbu [3], Lowe and Karafiath [4], Morgenstern and Price [5], and Spencer [6] are among the available methods. Only a small amount of research has been developed for the second step. The use of optimization techniques in locating the critical slip surface is becoming increasingly popular among researchers. The traditional mathematical optimization methods that have been used include dynamic programming [7], conjugate-gradient [8], random search [9-12] and simplex optimization [13-15]. More recently, Pham and Fredlund [16] applied dynamic programming to stability analysis of slopes. In their approach, stresses acting along the critical slip surfaces were computed using a finite element analysis, thus eliminating the need to describe the relationship between interslice forces. Also, Krahn [17] presented a finite element based limit analysis. In that paper, various limit equilibrium methods were compared and discussed in detail, and it was shown that all limit equilibrium methods may fail to satisfy equilibrium conditions for very steep slip surfaces. For instance, with the GLE (Generalized Limit Equilibrium) method it is possible that the interslice forces would have to act outside the slice for the slice itself to be in moment equilibrium. Similarly, fixing the location of the thrust line in the Generalized (Rigorous) Janbu Method may result in a convergence problem. In order to overcome this restriction, in the approach presented in subsequent sections the Generalized Janbu Method was adopted, however, location of the thrust line was not fixed apriori. In other words, the thrust line could assume any

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location along the side of slices, with its optimum location being determined during the optimization process.

The genetic algorithm (GA) differs from other search methods in that it searches among a population of points and works with a coding of the parameter set rather than the parameters themselves. It also uses probabilistic rather than deterministic transition rules. Goh [18] presented GA to locate the critical "circular" slip surface.

This paper presents a method for determining the critical "non-circular" slip surfaces using GA, satisfying both force and moment equations of equilibrium. Fundamental formulations of the generalized (rigorous) Janbu approach were adopted as the limit equilibrium method. The proposed method is capable of determining the interslice forces and their location without any apriori assumption, as well as the safety factor. The robustness of the proposed approach is illustrated via some example problems and the results are compared with conventional methods.

2. GENETIC ALGORITHM

The genetic algorithm developed by Holland [19], relies on the principle of Darwin's theory of survival of the fittest. Solutions to a problem can be obtained through evolution. The algorithm is started with a set of possible solutions. The set of possible solutions is called “population”. Each possible solution within the population is called a “chromosome”. Each chromosome is assigned with a fitness value based on the fitness function. Solutions from one population are taken and used to construct a new population so that the new population (offspring) will be fitter than the old one. This process is repeated until certain criteria are met. For example, the reproduction will stop when the total number of generations reaches a specified maximum value. The basics of GA are described in the following sections:

a) Coding

The first step in GA is to translate the real problem into biological terms. For this purpose, all the variables of the problem are represented by so-called "chromosomes" and the process is called coding. Binary coding is the simplest and most common one. In binary coding, each chromosome is a string of bits, 0 or 1.

b) Selection

In order to reproduce offspring, parents need to be selected. The most commonly used methods are the roulette wheel selection and rank selection. In order to minimize an objective function, the chromosomes are ranked in a descending order based on their fitness values. Next, a probability value, P_j, is assigned to each chromosome j=1,2,…,n, giving higher probability to chromosomes of lower fitness function value. The worst chromosome after ranking is j=1, and its probability value p_1 will be the smallest. The best chromosome is j=n, and its probability value, p_n, will be the largest. The probability values for other chromosomes are linearly interpolated as

$$p_j = p_1 + \frac{(p_n - p_1)}{n-1}(j-1)$$

The summation of probability values for all chromosomes, $\sum_{j=1}^{n} p_j$, is set to unity and the average of probability values for all chromosomes is then 1/n. For this purpose, a value of c/n can be assigned to p_n, so that the probability value for the best chromosome is c times the average, where c>1. The corresponding probability value for the worst chromosome p_1 is then (2-c)/n. To ensure that all probability values are nonnegative, c should be less than or equal to 2. Using the smaller value of c will result in a more robust but slower search [20].

c) Crossover

The crossover strategy determines how the parent chromosomes are combined in order to generate offspring. Crossover is applied to randomly selected pairs of parents with a probability equal to the specified crossover rate. Single-point and two-point crossover, the most popular operators, are explained below.

Single-point crossover: One crossover point is randomly selected along the parent chromosomes. The binary string from the beginning of parent 1 to its crossover point is copied to the new offspring in the same positions. The rest (from the same crossover point of parent 2 to its tail) is copied to the new offspring in the same positions. See example bellow

11001 010 ; 1101 111 ⟹ 11001111 ; 1101010

Two-point crossover: Two crossover points are selected. The binary string from the beginning of parent one to its first crossover point and the binary string from its second crossover point to its end are copied to the new offspring. The rest (the first crossover point of the other parent to its second crossover point) is copied to the new offspring in the same fashion. See example below

11 0010 10 ; 11 011111 ⟹ 11011110 ; 11001011

Similarly, crossover types with more than two points can be used.

d) Mutation

The final genetic operator is that of mutation. The process involves randomly flipping a bit from 0 to 1 or vice versa. A new strategy developed by Pham and Karaboga [21] was used in this study. In this strategy, the mutation probability for each bit in a chromosome depends on the fitness of the chromosome, the generation number and the position of the bit along the particular gene of chromosome.

The mutation rate for bit k (k=1 to l_c) of chromosome i is computed as follows:

$$mutrate\,(i,k) = (\frac{k}{l_c})d \qquad \text{if } g \leq \text{maxgen}/2$$

$$mutrate\,(i,k) = (\frac{1}{l_c})d \qquad \text{if } g > \text{maxgen}/2$$

Where

$$d = (1-\text{fit}(i))(mutrate_{\max} - mutrate_{\min}) + mutrate_{\min}$$

l_c = chromosome length, maxgen = the maximum number of optimization generation to be tried, g = current generation number, $mutrate_{max}$ = maximum mutation rate, $mutrate_{min}$ = minimum mutation rate, fit(i) = fitness of chromosome i.

In short, genetic algorithm is a stochastic process that exhibits variable performances. To enhance the performance, analysis based on different combinations of various parameters such as population size, crossover rate/type, mutation rate and selection methods are necessary. Further details on GA were presented by Goldberg [22] and Michalewicz [23]. A flow chart depicting the major operations of GA is shown in Fig. 1. In the next section, basic formulations for a generalized non-circular surface satisfying both moment and force equations of the equilibrium are derived and the methodology employed to solve the problem using GA is described in detail.

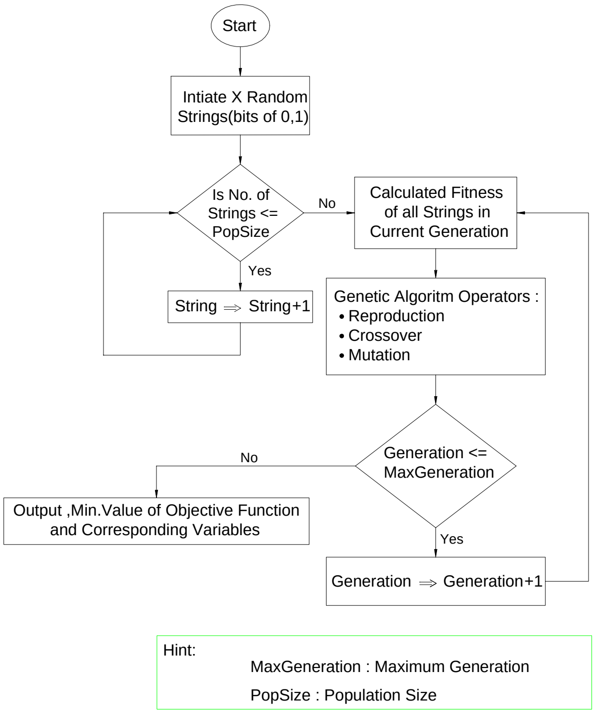


Fig. 1. Flow chart for a genetic algorithm

3. DERIVATION OF EQUILIBRIUM EQUATIONS FOR SLOPE STABILITY ANALYSIS

Figure 2 shows a potential sliding mass along a trial slip surface. The sliding mass is subdivided into a number of vertical slices. Each slice is subjected to a general system of forces as shown in Fig. 3.

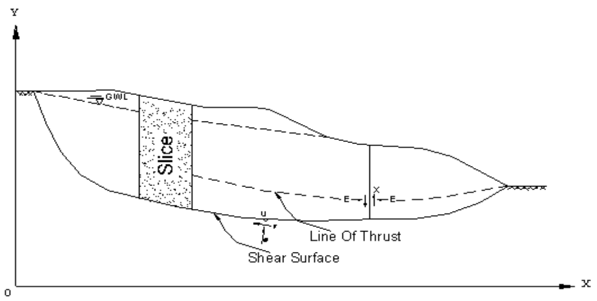


Fig. 2. Typical potential sliding mass

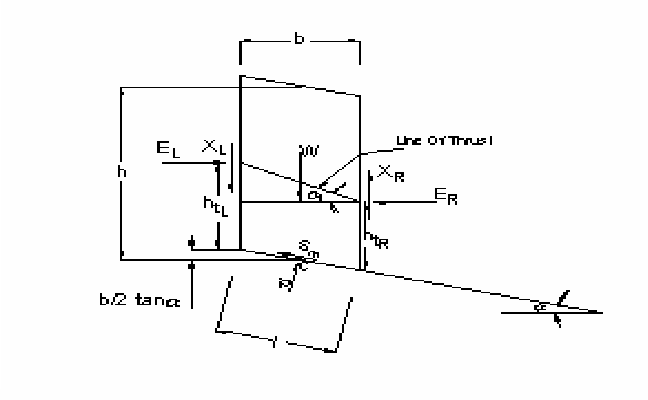


Fig. 3. Forces acting on the boundaries of a single slice

The thrust line connects the points of application of the interslice forces. The location of this thrust line is determined using a generalized method of analysis that satisfies both force and moment equations of equilibrium.

The equilibrium shear stress along the shear surface is given by the equation

$$S_m = \frac{\tau_f}{F_S} \tag{1}$$

Where τ_f is the shear strength and F_S is the safety factor.
Combining Eq. (1) with the Mohr-Coulomb failure criteria yields

$$\tau = \frac{1}{F_S}[c' + (\frac{p}{l} - u)\tan\varphi'] \tag{2}$$

where c' = Effective cohesion value.
φ' = Effective friction angle.
l = The slice base length.
u = Pore water pressure.

Vertical equilibrium for a typical slice gives

$$W + X_L - X_R = P\cos\alpha + S_m \sin\alpha \tag{3}$$

From Eq. (3)

$$P = (W + X_L - X_R)\sec\alpha - S_m \tan\alpha \tag{4}$$

By substituting Eq. (4) in Eq. (2), and after rearranging

$$S_m = \frac{1}{F_S.m_\alpha}\{c'l\cos\alpha + [(W + X_L - X_R) - ul\cos\alpha]\tan\varphi'\} \tag{5}$$

Where m_α is defined as:

$$m_\alpha = \cos\alpha + \frac{1}{F_S}\sin\alpha\tan\varphi \tag{6}$$

Resolving the forces acting on the slice in a tangential direction to the base of the slice

$$S_m = (E_L - E_R)\cos\alpha + (W + X_L - X_R)\sin\alpha \tag{7}$$

The summation of the normal interslice forces should be zero, that is

$$\sum(E_L - E_R) = 0 \tag{8}$$

Substituting ($E_L - E_R$) from Eq. (7) into Eq. (8) yields

$$\sum S_m \sec\alpha - (W + X_L - X_R)\tan\alpha = 0 \tag{9}$$

By inserting the value of S_m from Eq. (5) into Eq. (9), we get

$$\sum\frac{c'l + \{(W + X_L - X_R)\sec\alpha - ul\}\tan\varphi'}{F_S.m\alpha} - (W + X_L - X_R)\tan\alpha = 0 \tag{10}$$

The summation of the moments about the center of the base of each slice must be zero. Therefore, for infinitesimal slices

$$\sum X_R - E_R\tan\alpha_t + (E_R - E_L)\frac{ht_R}{b} = 0 \tag{11}$$

where α_t = the angle between the line of thrust on the right side of the slice and the horizontal direction (Fig. 3).

Equations (10) and (11) are considered to be the fundamental equations which should be solved simultaneously to determine both the location of the thrust line for every slice and the safety factor F_S .

4. APPLICATION OF GA FOR SOLVING THE EQUILIBRIUM EQUATIONS

In order to satisfy both the force and moment equations of equilibrium simultaneously, the following objective function was selected:

$$\text{Objective function} = (Eq.10)^2 + (Eq.11)^2 \quad (12)$$

Using a GA technique, the unknowns are subjected to the following constraints:

$$(\text{lower limit}) \le (X_R)_{lastslice} \le (\text{upper limit}) \quad (13)$$

$$(\text{lower limit}) \le (E_R)_{lastslice} \le (\text{upper limit}) \quad (14)$$

$$(\text{lower limit}) \le r \le (\text{upper limit}) \quad (15)$$

$$(\text{lower limit}) \le F_S \le (\text{upper limit}) \quad (16)$$

where $(X_R)_{lastslice}$ and $(E_R)_{lastslice}$, respectively are the shear and normal forces on the side of the last slice , r is a fraction of height on the right-side of each slice describing the point of application of the interslice forces (position of thrust line), and F_s is the safety factor for a particular slip surface. Upon minimization of the objective function of the form presented, both terms (Eqs. (10) and (11)) are forced to approach zero, thus satisfying both moment and force equations of equilibrium altogether. The square power employed in Eq. (12) avoids the error of each term to be cancelled out by the other if they are of opposite signs. The selected range for the constraints is as follows:

$$(X_R)_{\text{lastslice}} \text{ and } (E_R)_{\text{lastslice}} \ (0, 0.01), r \ (1/6, 5/6) \text{ and } F_s \ (0.1, 3)$$

The GA was used to minimize the objective function given by Eq. (12) subjected to the constraints given by Eq. (13) through Eq. (16). A computer program in Delphi was developed to study the non-circular slip surface. The GA solves for the unknowns in expression 12.

To examine the concept and for the sake of comparison, the following example presents the results obtained from the proposed method, along with those suggested using other equilibrium methods.

Example 1: Figure 4 shows an example problem involving both circular and composite failure surfaces. The results for two cases of homogeneous soil with and without a piezometric line are presented in Table 1. Various methods of limit equilibrium were compared for this case by Fredlund and Krahn [2], and the reported values of the safety factor are given in Table 1. The GA parameters used in this example are: number of generations=2000, population size=50, crossover rate=1 with a single point crossover. Ten runs were made with different initial seeds resulting in different starting populations.

Table 1. Comparison of safety factor for the example problem

Method	Case 1 Simple 2:1 slope 12m high $\varphi = 20^\circ$,c=18.86 kPa	Case 2 Same as 1 except with a piezometric line
Ordinary	1.928	1.693
Simplified Bishop	2.080	1.834
Spencer	2.073	1.830
Janbu's simplified	2.041	1.827
Janbu's rigorous	2.008	1.776
Morgenstern-Price	2.076	1.833
This study	2.079	1.840

Figure 5 shows the best fit in each population versus the number of generations. Although the basic algorithm employed for the stability analysis in the proposed method was based on Janbu's rigorous method, it is interesting to note that the results yield a somewhat higher safety factor than that reported by Fredlund and Krahn [2], adopting the same method of stability analysis. The reason lies in the fact that Fredlund and Krahn [2] assumed a thrust located at an arbitrary distance of one third from the bottom of each slice. The choice of an arbitrary thrust line does not necessarily satisfy the moment equation of equilibrium for the "last slice" as reported by Sarma [24], unless necessary adjustments are made by a trial and error procedure. In the proposed approach, the problem is resolved through the necessary constraints applied to the last slice presented by Eqs. (13) and (14). Furthermore, the position of the thrust line is obtained automatically while varying within a distance of 1/6 to 5/6 from the bottom of each slice.

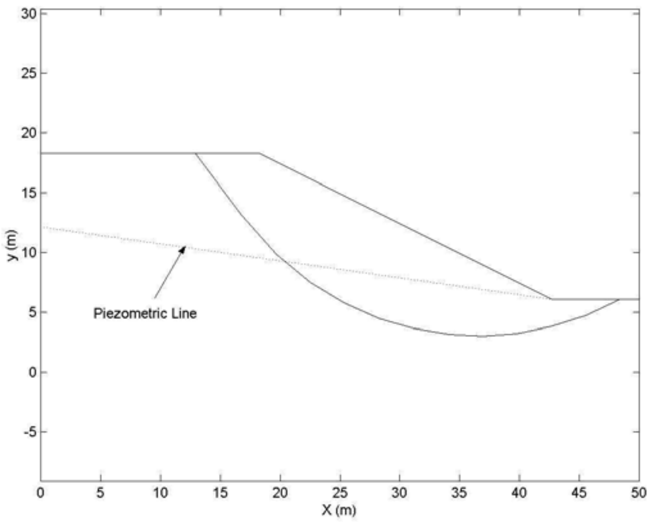


Fig. 4. Cross section of slope in Example1

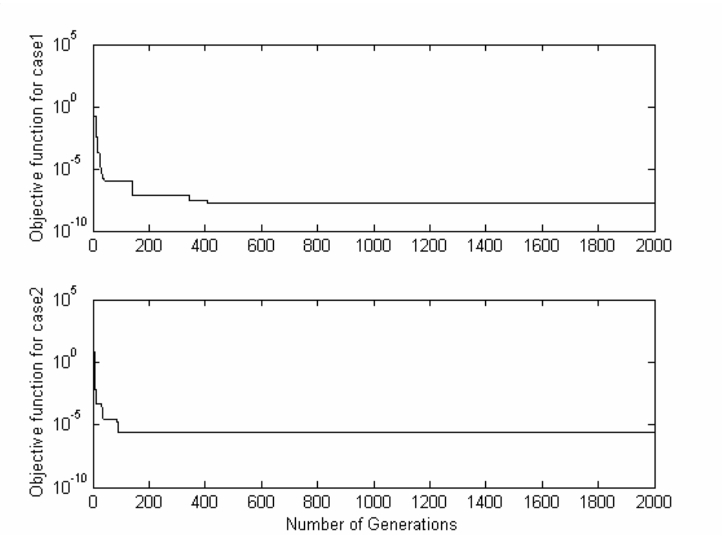


Fig. 5. The best fit versus the number of generations in Example 1

5. APPLICATION OF GA TO FIND THE CRITICAL NON-CIRCULAR SLIP SURFACE

To search for the critical slip surface, the objective function must meet three requirements. First, it must satisfy the equilibrium condition by minimizing the error in the equilibrium equations (Eq. (12)). Second, it must find the slip surface with the minimum factor of safety, and finally, the slip surface must satisfy the conditions of kinematical admissibility, i.e. a failure mechanism is obtained. Therefore, the search for the critical slip surface is mathematically expressed as the minimization of the objective function defined by

$$\text{Objective function} = F_s^2 + \ln(1 + (Eq.12)) + (1 + \psi)^2 \quad (17)$$

Since all the terms in Eq. (17) are positive, during the optimization procedure all the terms are minimized simultaneously. In order to achieve this goal, the terms in the objective function must have the same order of magnitude. Otherwise, one term may outweigh the other, resulting in poor minimization of the term with a smaller order of magnitude. To achieve this goal, various forms of the objective functions were tested and it was concluded that the form presented by Eq. (17) could effectively meet the requirements. The third term in Eq. (17) is a penalty term to force the search toward kinematically admissible slip surfaces. This point is further discussed in subsequent paragraphs.

To make the slip surface kinematically admissible, the segments must meet the following criteria:

$$\alpha_1 > \alpha_2 > \alpha_3 > \ldots > \alpha_{N-1} \tag{18}$$

If the above criterion is not satisfied a failure mechanism can not be attained. Furthermore, Ching and Fredlund [25] demonstrated what is known as "m_α" problem in stability analysis. The problem is that the normal force at the base of a slice sometimes becomes unreasonably large due to unrealistic values computed for m_α using Eq. (6). The difficulties associated with the magnitude of m_α are mainly the result of an inappropriate assumed shape for the slip surface. To alleviate the problem, Ching and Fredlund [25] suggested limiting the inclination of the slip surface in the passive and active zones by

$$\alpha_{all} = 45 - \frac{\phi}{2} \quad \text{For the passive zone} \tag{19}$$

$$\alpha_{all} = 45 + \frac{\phi}{2} \quad \text{For the active zone} \tag{20}$$

Based on the foregoing discussion, a penalty term was introduced in the objective function. The penalty function was defined as

$$\psi = \sum_{i=1}^{P} \left| \left(\frac{\alpha_i}{\alpha_{all}} - 1 \right) \right| \tag{21}$$

Where α_i = Inclination angle of slice base for each segment as shown in Fig. 6, and P=Number of segments on the slip surface to be penalized. The use of the above mentioned penalty function eliminates the "m_α-problem" discussed previously. The effectiveness of the proposed penalty function in generating a kinematically admissible slip surface is shown in Fig. 7.

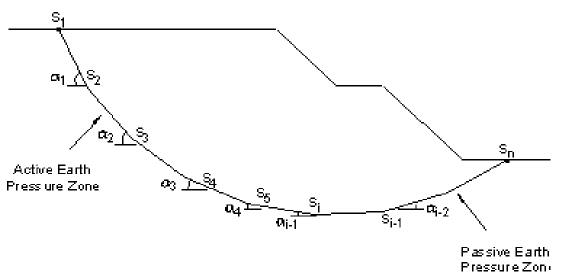


Fig. 6. General cross section of slope

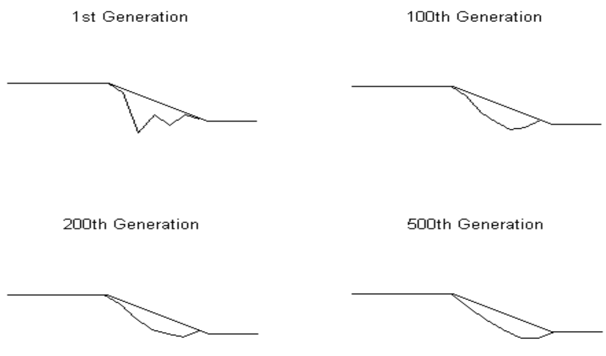


Fig. 7. Evolution of a generated slip surface toward a kinematically admissible slip surface

In order to minimize Eq. (17), a computer program was written in Delphi. In this program, binary coding was used to represent the coordinates of points specifying the slip surface, as well as the unknowns used in Eq. (12). The length of the binary string depends on the required accuracy. For this study, the length of the binary code was selected such that an accuracy of two decimal points could be achieved. In the genetic algorithm used for all the forthcoming examples, a population size=50 was selected. A sensitivity analysis of the GA for various crossover rates and types was carried out. The crossover rate was also varied at 0.5 increments from 0.8 to 1.0, while the crossover type was varied from a single-point to a four-point crossover with a step of 1.0. The following six examples serve to illustrate the effectiveness and accuracy of the GA in finding the critical non-circular slip surface and the interslice forces as well as their location.

Example 2: Yamagami and Ueta [14] used different minimization procedures, BFGS [26-29], DFP [30, 31], to locate the critical slip surface and its associated minimum safety factor using the Morgenstern-Price method for the homogeneous slope shown in Fig. 8. The geotechnical parameters are: $\varphi = 10^\circ$, c=9.8 kPa and $\gamma = 17.64 kN/m^3$.

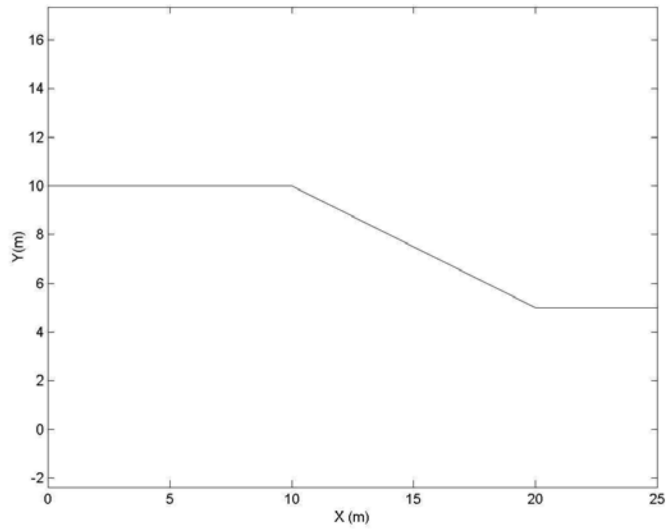


Fig. 8. Cross section of slope in Example 2

Similarly, Greco [11] used the Spencer method in combination with a Pattern-Search and Monte Carlo for the same problem. Malkawi *et al*. [12] employed the limit equilibrium-based methods (i.e., Ordinary method of slice, Bishop's method, Janbu's method, Morgenstern-Price's method and Spencer's method) combined with the Monte Carlo technique. Table 2 presents the results obtained with the proposed approach compared with the results obtained by different investigators .The GA parameters used in this example were as follows: number of generation=2000, population size=50, crossover rate=0.95, two-point crossover. The plot of the cost function versus the number of generations is shown in Fig. 9. Summary plots from the computer program are shown in Fig. 10.

Table 2. Minimum safety factor given for Example 2

Optimization Method	Limit Equilibrium Method	Slip Surface	Safety Factor	Reference
BFGS DFP Powell(1964) Simplex	Morgenstern-Price	Non-Circular	1.338 1.338 1.338 1.339-1.348	Yamagami and Ueta [14]
Pattern-Search Monte Carlo	Spencer	Non-Circular	1.327-1.33 1.327-1.333	Greco [11]
Monte Carlo	Ordinary	Non-Circular	1.238	Malkawi *et al.* [12]
Genetic Algorithm	Generalized	Non-Circular	1.38	This Study

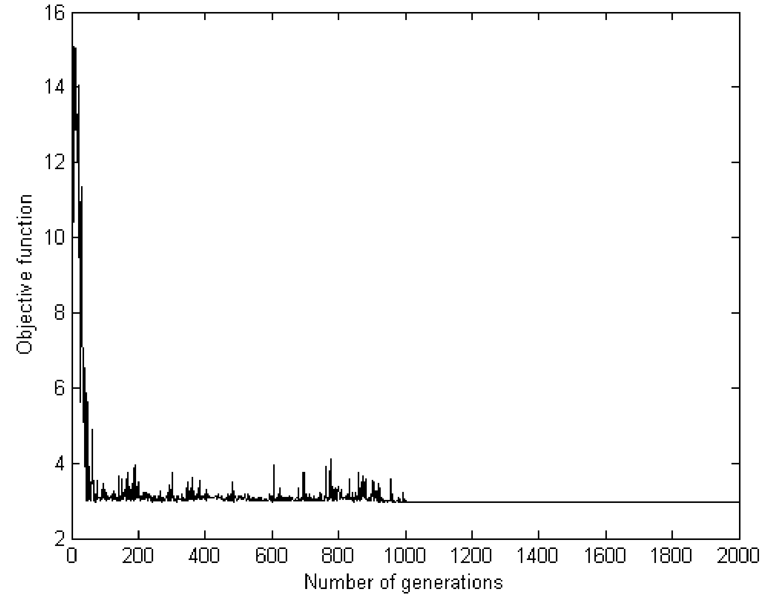


Fig. 9. Objective function versus the number of generations for Example 2

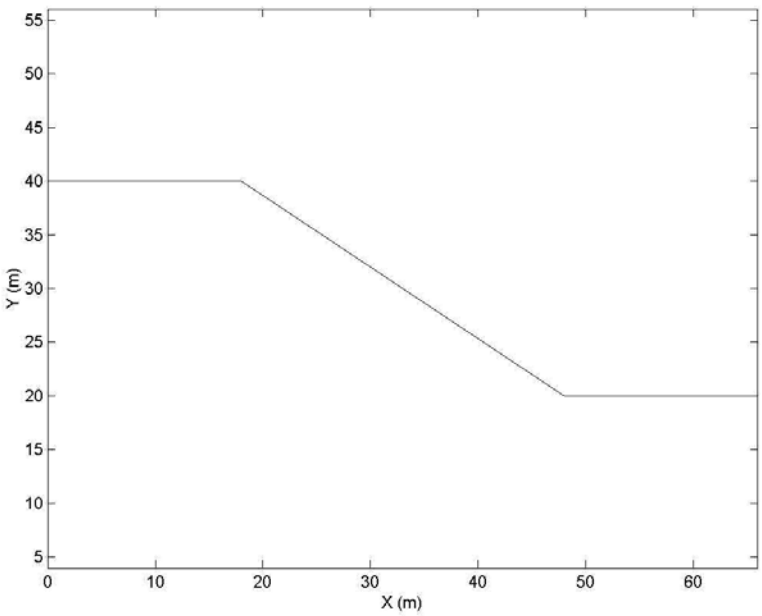


Fig. 10. Summary plots for Example 2

Example 3: A simple slope of a homogeneous soil is shown in Fig. 11. The geotechnical properties are $\varphi = 15^\circ$, $c = 41.7kPa$ and $\gamma = 18.8kN/m^3$. Arai and Tagyo [8] used Janbu's simplified method in combination with the conjugate-gradient method to find the critical slip surface.

This problem was examined by using the proposed method. Table 3 presents the safety factor obtained by Arai and Tagyo [8] compared with the results obtained from the proposed approach. The GA parameters chosen in this example were as follows: number of generation=2500, population size=50, crossover rate=0.95 and two-point crossover. Summary plots of the results are shown in Fig. 12. The plot of the objective function versus the number of generations is shown in Fig. 13.

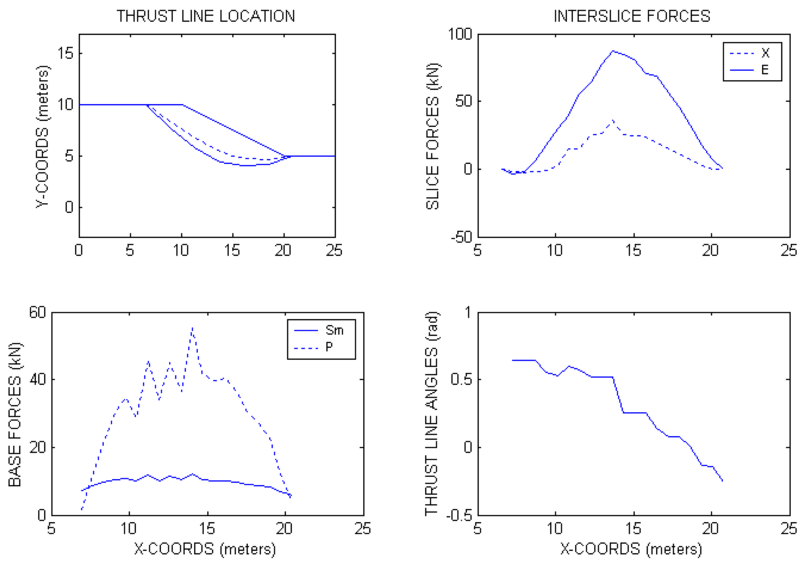


Fig. 11 Cross section of slope Example 3

Table 3. Minimum safety factor given for Example3

Optimization Method	Method	Slip Surface	Safety Factor	Reference
Conjugate Gradient	Simplified Janbu	Non-Circular	1.357	Arai and Tagyo(1985)
Genetic Algorithm	Generalized Limit Equil.	Non-Circular	1.41	This Study

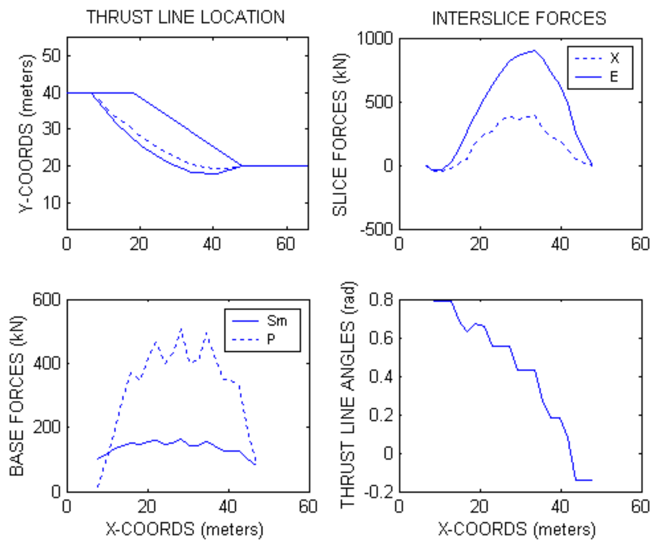


Fig. 12. Summary plot for Example 3

Example 4: A multilayer slope was analyzed by Yamagami and Ueta [14]. A cross section and soil properties are presented in Fig. 14. Yamagami and Ueta [14] solved this example using the Morgenstern-Price method, and also by employing different minimization procedures. Similarly, Greco [11] used the Spencer method in combination with a Pattern-Search and Monte Carlo technique. Malkawi *et al*. [12] employed the limit equilibrium-based methods (i.e, Ordinary method of slice, Bishop's method, Janbu's method, Morgenstern-Price's method and Spencer's method), combined with the Monte Carlo technique.

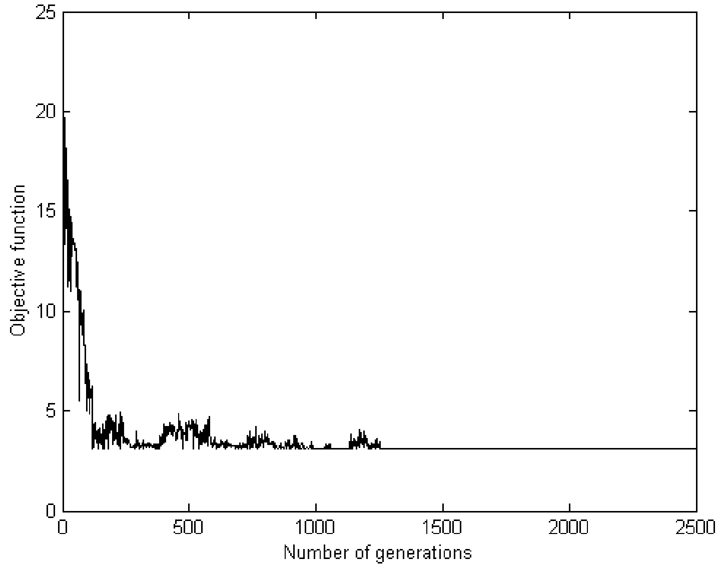
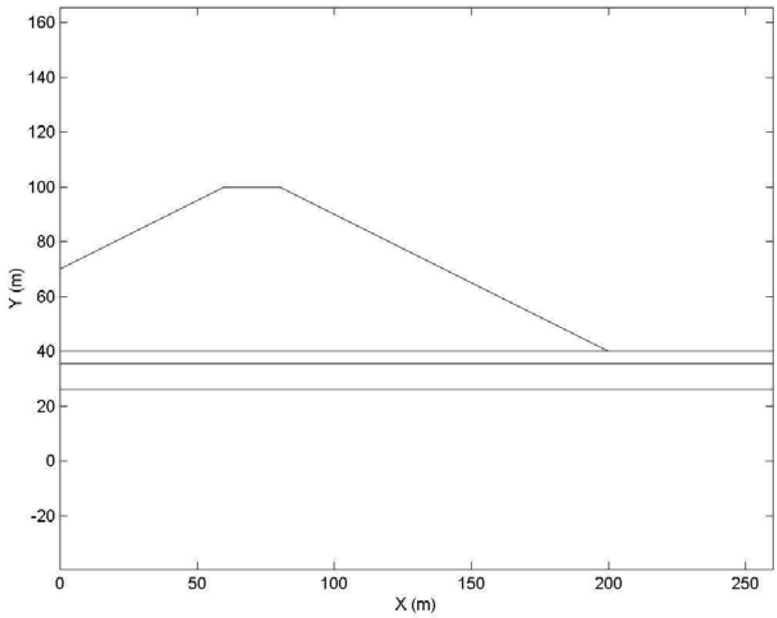


Fig. 13. Objective function versus the number of generations for Example 3



Layer	$c = kPa$	$\varphi(\circ)$	γ kN/ m^3
1	49	29	20.38
2	0	30	17.64
3	7.84	20	20.38
4	0	30	17.64

Fig. 14. Cross section and geotechnical properties of Example 4

The results obtained using the proposed procedure, as well as those obtained by other investigators are presented in Table 4. The GA parameters used in this example were as follows: number of generation=2000, population size=50, crossover rate=1, four-point crossover.

Table 4. Minimum safety factor for Example 4

Optimization Method	Limit Equilibrium Method	Slip Surface	Safety Factor	Reference
BFGS DFP Powell(1964) Simplex	Morgenstern-Price	Non-Circular	1.423 1.453 1.402 1.405	Yamagami and Ueta [14]
Pattern-Search Monte Carlo	Spencer	Non-Circular	1.400 1.401	Greco [11]
Monte Carlo	Ordinary	Non-Circular	1.33	Malkawi *et al.* [12]
Genetic Algorithm	Generalized	Non-Circular	1.46	This Study

The plot of the objective function versus the number of generations is shown in Fig. 15. Summary plots from the computer program are shown in Fig. 16.

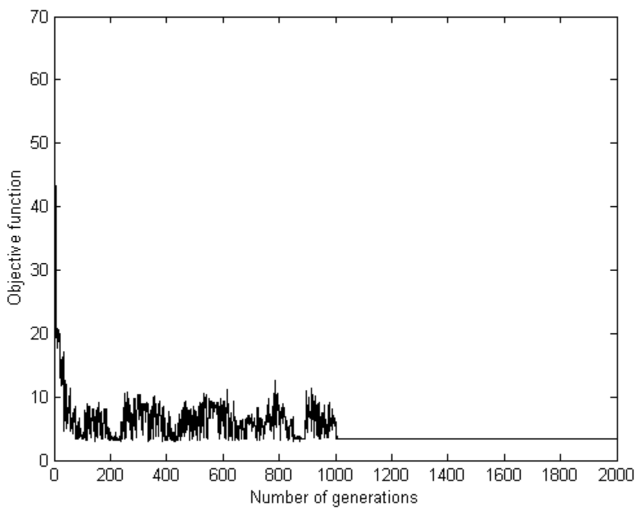


Fig. 15. Objective function versus the number of generations for Example 4

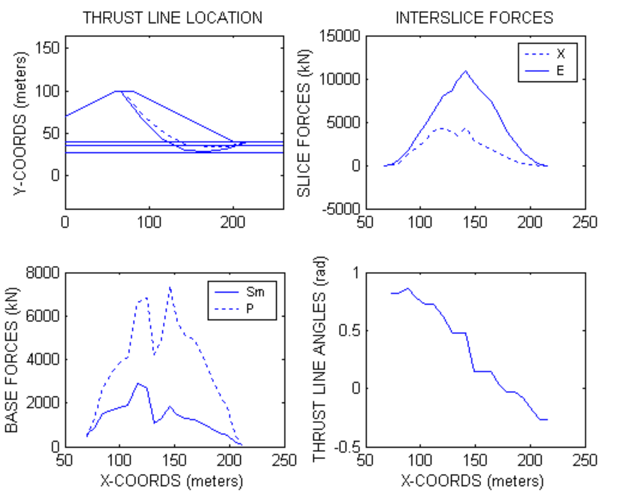


Fig. 16. Summary plots for Example 4

Example 5: This example has been analyzed by Goh [18]. The soil properties and a cross section are presented in Fig. 17. Table 5 presents the result of Goh [18] compared with this study. Goh [18] solved this example using the simplified Bishop's method and employing the GA method for searching critical circular slip surface. The GA parameters chosen in this example were as follows: number of generation=2000, population size=50, crossover rate=1 and four-point crossover. The plot of the objective function versus number of generations is shown in Fig. 18. Summary plots are shown in Fig. 19.

Table 5. Minimum safety factor for Example 5

Optimization Method	Limit Equilibrium Method	Slip Surface	Safety Factor	Reference
Genetic Algorithm	Simplified Bishop	Circular	1.435	Goh [18]
Genetic Algorithm	Generalized	Non-Circular	1.45	This Study

Layer	$c = kPa$	$\varphi(^\circ)$	γ kN/ m^3
1	30	0	18
2	20	0	18
3	150	0	18

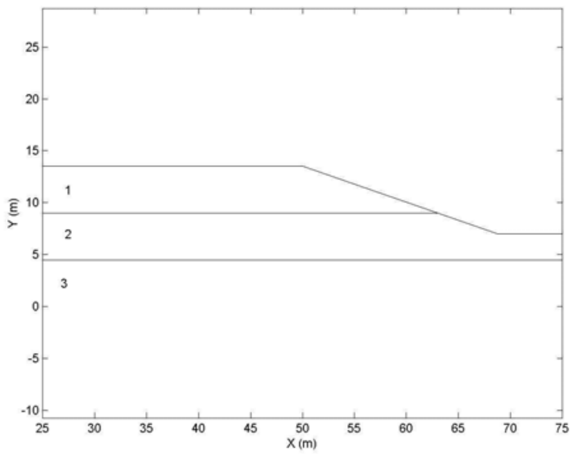


Fig. 17. Cross section and geotechnical properties of Example 5

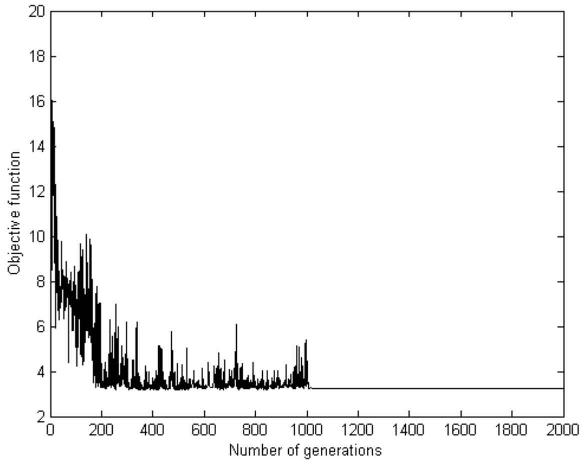


Fig. 18. Objective function versus the number of generations for Example 5

Example 6: Baker [7] used a dynamic programming technique to locate the critical slip surface and its associated safety factor using Spencer's method for the homogeneous slope with a piezometric line as shown in Fig. 20.

The geotechnical parameters are

$$\varphi = 20^\circ,\ c = 28.4kPa \text{ and } \gamma = 18.8kN/m^3.$$

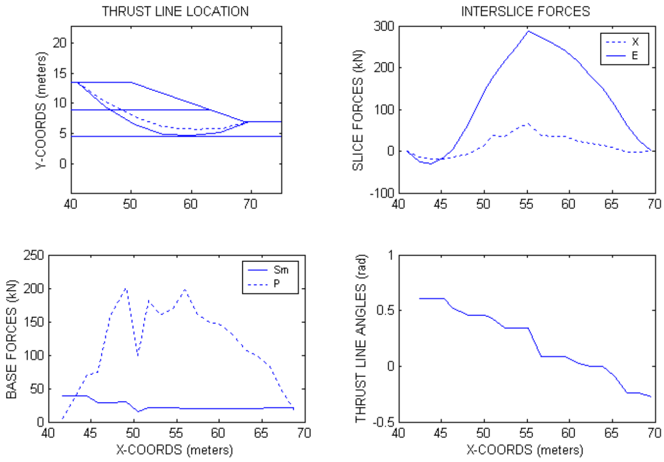


Fig. 19. Summary plots for Example 5

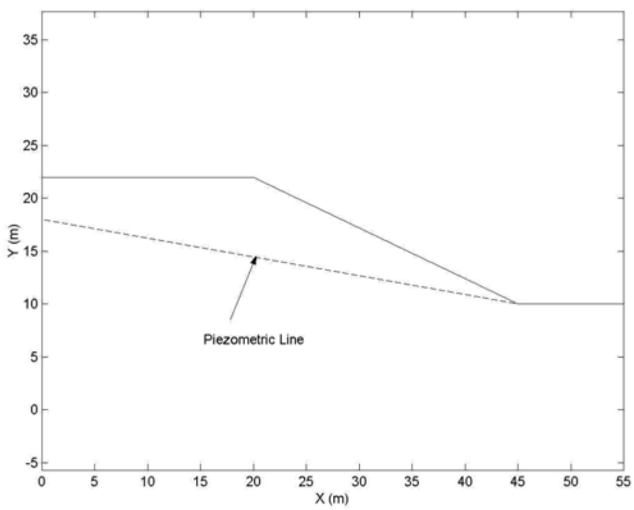


Fig. 20. Cross section of slope in Example 6

Similarly, Greco [11] employed pattern-search and Monte Carlo methods to solve the same example and so did Malkawi *et al*. [12], who employed the limit equilibrium-based methods (i.e, Ordinary method of slice, Bishop's method, Janbu's method, Morgenstern-Price's method and Spencer's method) combined with the Monte Carlo technique.

The problem was analyzed using the proposed method and the results are presented in Table 6. The GA parameters used in this example were as follows: number of generation=3000, population size=50, crossover rate=1, two-point crossover. The plot of the objective function versus number of generations is shown in Fig. 21. Summary plots from the computer program are shown in Fig. 22.

Table 6. Minimum safety factor for Example 6

Optimization Method	Limit Equilibrium Method	Slip Surface	Safety Factor	Reference
Pattern-Search Monte Carlo	Spencer	Non-Circular	1.744-1.745 1.744-1.751	Greco [11]
Dynamic Programming	Spencer	Non-Circular	1.77	Baker [17]
Monte Carlo	Ordinary	Non-Circular	1.502	Malkawi *et al.* [12]
Genetic Algorithm	Generalized	Non-Circular	1.80	This Study

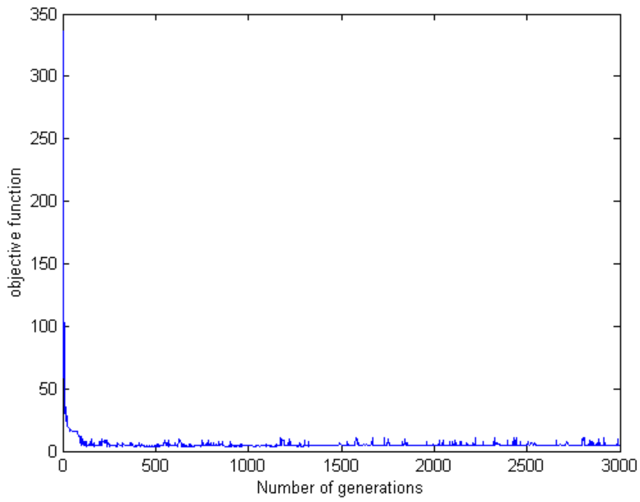


Fig. 21. Objective function versus the number of generations for Example 6

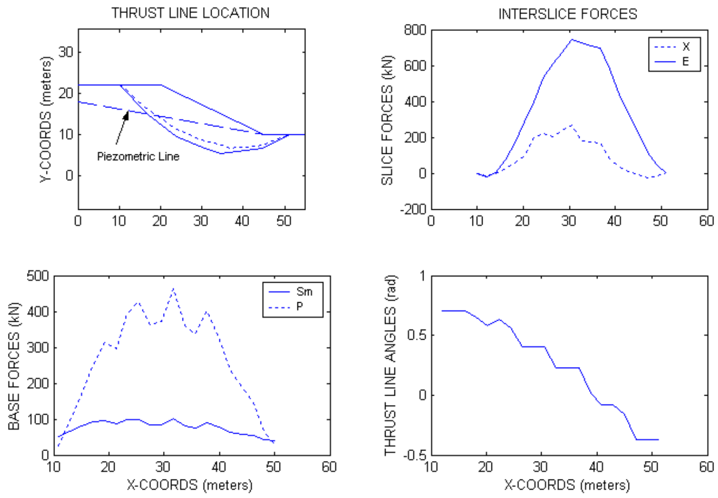


Fig. 22. Summary plots for Example 6

Example 7: The last example (Fig. 23) investigated is a homogeneous slope with a piezometric line. This example has been analyzed by Arai and Tagyo [8]. They used Janbu's simplified method in combination with the conjugate-gradient method. The geotechnical parameters are

$$\gamma = 18.8\ kN/m^3, C = 41.7\ kPa, \phi = 15^0 .$$

This problem was examined using the proposed method. Table 7 presents the results of Arai and Tagyo [8] compared with this study. The GA parameters chosen in this example were as follows: number of generation=2000, population size=50, crossover rate=0.95 and four-point crossover. The plot of the objective function versus the number of generations is shown in Fig. 24. Summary plots of results are shown in Fig. 25.

Table 7. Minimum safety factor for Example7

Optimization Method	Limit Equilibrium Method	Slip Surface	Safety Factor	Reference
Conjugate Gradient	Simplified Janbu	Non-Circular	1.071	Arai and Tagyo [8]
Genetic Algorithm	Generalized	Non-Circular	1.09	This Study

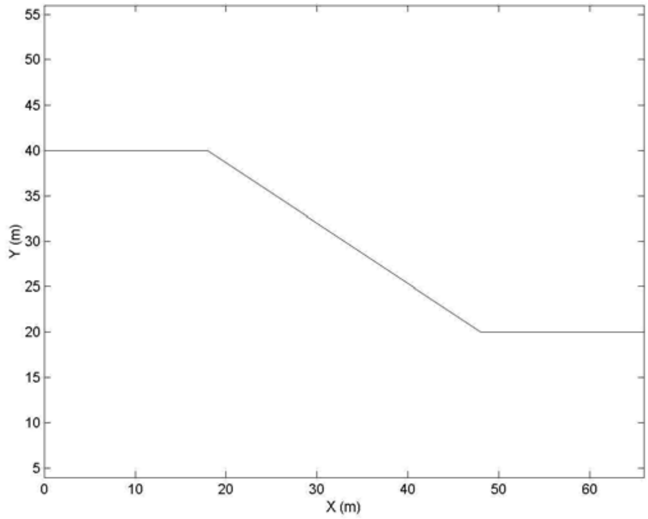


Fig. 23. Cross section of slope in Example 7

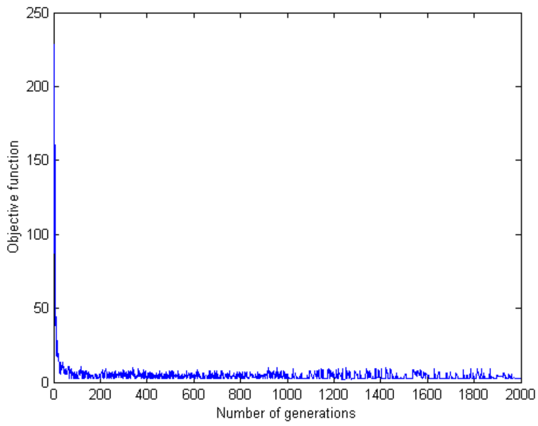


Fig. 24. Objective function versus the number of generations for Example 7

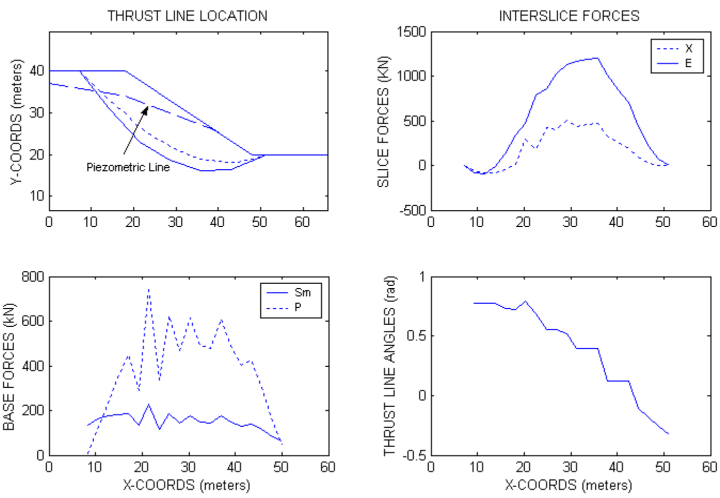


Fig. 25. Summary plot for Example 7

6. DISCUSSION OF RESULTS

The first example illustrated the capability of the proposed method to determine the safety factor for a specified non-circular slip surface. The value of the safety factor obtained was very close to the

Morgenstern-Price method. Examples 2-7 examined the capability of the proposed method in locating the critical non-circular slip surface and the results were compared with those reported by other investigators. From these examples, it may be concluded that the proposed method gives a relatively higher value for the safety factors. This may be attributed to the following:

- It is common to obtain a relatively higher factor of safety when employing limit equilibrium methods that satisfy both force and moment equations of equilibrium as opposed to the methods that satisfy only force or moment equations of equilibrium. Therefore, it is not surprising to arrive at a higher factor of safety for the proposed method, which satisfies both equations of equilibrium compared to methods like Simplified Bishop, Simplified Janbu, and Ordinary Fellenius, which satisfy only force or moment equation of equilibrium.
- The second group of limit equilibrium methods includes Morgenstern and Price, Spencer and the Generalized (Rigorous) Janbu method of slices, all satisfying both moment and force equations of equilibrium simultaneously. Therefore these methods are more accurate than the aforementioned methods. In the Spencer method, a constant ratio of the interslice normal to shear force is assumed for all the slices. In other words, the resultant interslice forces have the same direction for all slices. In the Morgenstern and Price method, an arbitrary function is assumed to describe the direction of the interslice forces. Hence, the Spencer method can be considered as a special case of Morgenstern and Price where a constant function is used for the interslice forces. The generalized Janbu method assumes an arbitrary location for the position of the thrust line. Within this group of limit equilibrium methods, the variation in the safety factor reflects the kind of assumption(s) made in each method. In the proposed approach the generalized method was adopted with no prior assumption regarding the location of the thrust line. The location of the thrust line was determined automatically through the GA process in order to satisfy the conditions of equilibrium for the last slice, which is a handicap of the conventional method as reported by Sarma [24]. Consequently, the proposed method is more robust and is believed to give a more realistic value for the factor of safety.

The optimal GA parameters for cases involving a soil layer are: P_c=0.93 with two point crossover, population size=50, No. of generations≤2000. The same parameters for multilayer soil conditions are: P_c=1.0 with a four-point crossover, population size=50 and No. of generations≥2000.

7. CONCLUSION

The present paper suggests a genetic algorithm method for locating the critical non-circular slip surface in conjunction with the interslice forces and their position for slope stability analysis. When minimized, the proposed objective function yields the critical non-circular slip surface that satisfies both force and moment equations of equilibrium. The method does not make any assumption regarding the location of the interslice thrust line. In other words, the thrust line is determined so that the equilibrium equations are satisfied for all the slices. This procedure eliminates some of the problems associated with limit equilibrium methods where the location of the thrust line (or interslice forces in GLE) is fixed at the start of the analysis. Furthermore, the method yields surfaces that are kinematically admissible and physically acceptable through a suitable penalty function. The method does not require derivatives of the objective function. The proposed technique is simple in structure and easily programmable. In addition, some examples have been investigated to demonstrate the capabilities of the proposed approach. From the examples considered it may be concluded that a population size of 50, a crossover rate of 0.95 and 1, crossover mode single point and two points together with a ranking method and a variable mutation technique can locate the critical non-circular surface and yield the generalized interslice forces. The present approach does not require any assumption regarding the interslice forces. Based on results presented in this paper, the proposed method gives a somewhat higher safety factor compared to the results obtained by other investigators. The reasons may be any of the following:

1. The proposed approach uses a generalized method of stability analysis, and therefore both force and moment equations of equilibrium are satisfied. This fact, in most cases, results in a higher safety factor compared with methods that satisfy only the moment equilibrium equation such as the simplified Bishop method.
2. The conventional methods of slope stability make some assumptions regarding the position of interslice forces, their ratio, their magnitude, etc. However, the proposed approach does not make any assumption regarding the interslice forces.
3. As stated before, the objective function for searching the critical non-circular slip surface consists of three distinct terms. The relative magnitude of each term with respect to other terms can influence the optimization procedure. In this study, various forms of the objective function were considered in order to remove this effect. However, further research in this area is encouraged.

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