Iranian Journal of Science & Technology, Transaction B, Engineering, Vol. 30, No. B1
Printed in The Islamic Republic of Iran, 2006

RESERVOIR OPERATION BY ANT COLONY OPTIMIZATION ALGORITHMS*

M. R. JALALI,** A. AFSHAR,[1] AND M. A. MARIÑO[2]

[1]Dept. of Civil Eng., Iran University of Science and Technology, Tehran, I. R. of Iran
E-mail: mrjalali@iust.ac.ir
[2]Honory member of ASCE, Hydrology Program and Dept. of Civil and Environmental Eng., University of Calif., Davis, CA 95616

Abstract– In this paper, ant colony optimization (ACO) algorithms are proposed for reservoir operation. Through a collection of cooperative agents called ants, the near optimum solution to the reservoir operation can be effectively achieved. To apply ACO algorithms, the problem is approached by considering a finite horizon with a time series of inflow, classifying the reservoir volume to several intervals, and deciding for releases at each period with respect to a predefined optimality criterion. Three alternative formulations of ACO algorithms for a reservoir operation are presented using a single reservoir, deterministic, finite-horizon problem and applied to the Dez reservoir in Iran. It is concluded that the ant colony system global-best algorithm provides better and more comparable results with known global optimum results. As with any direct search method, the model is quite sensitive to setup parameters, hence fine tuning of the parameters is recommended.

Keywords– Ant colony, optimization, reservoir operation

1. INTRODUCTION

The first ant colony optimization (ACO) algorithm, called ant system [1-3], was inspired by studies of the behavior of ants. Ant algorithms were first proposed by Dorigo *et al*. [3, 5] as a multi-agent approach to different combinatorial optimization problems like the traveling salesman problem and the quadratic assignment problem. The ant-colony metaheuristic framework was introduced by Dorigo and Di Caro [6], which enabled ACO to be applied to a range of combinatorial optimization problems. Dorigo *et al*. [7] also reported the successful application of ACO algorithms to a number of bench-mark combinatorial optimization problems. So far, very few applications of ACO algorithms to water resources problems have been reported [8, 9]. Abbaspour *et al*. [8] employed ACO algorithms to estimate hydraulic parameters of unsaturated soil. Maier *et al*. [9] used ACO algorithms to find a near global optimal solution to a water distribution system, indicating that ACO algorithms may form an attractive alternative to genetic algorithms for the optimum design of water distribution systems. In this paper, a novel way of addressing the optimum reservoir operation problem making use of ACO algorithms is proposed. To do so, the reservoir operation will be structured to fit an ACO model and the features related to ACO algorithms will be introduced. Performance of three different ACO algorithms in the operation of the Dez reservoir in Iran, as well as the influence of the parameter settings on a final selected ACO algorithm, will be compared.

---

*Received by the editors May 11, 2004; final revised form June 12, 2005.
**Corresponding author

2. ANT COLONY BEHAVIOR

Ant colony algorithms have been founded on the observation of real ant colonies. By living in colonies, ants' social behavior is directed more to the survival of the colony entity than to that of a single individual member of the colony. An interesting and significantly important behavior of ant colonies is their foraging behavior, and in particular, their ability to find the shortest route between their nest and a food source, realizing that they are almost blind. The path taken by individual ants from the nest, in search for a food source, is essentially random [5]. However, when ants are traveling, they deposit on the ground a substance called pheromone, forming a pheromone trail as an indirect communication means. By smelling the pheromone, there is a higher probability that the trail with a higher pheromone concentration will be chosen. The pheromone trail allows ants to find their way back to the food source and vice versa. The trail is used by other ants to find the location of the food source located by their nest mates. It follows that when a number of paths is available from the nest to a food source, a colony of ants may be able to exploit the pheromone trail left by the individual members of the colony to discover the shortest path from the nest to the food source and back [6]. As more ants choose a path to follow, the pheromone on the path builds up, making it more attractive to other ants seeking food, and hence more likely to be followed by other ants.

Generally speaking, population based metaheuristic algorithms search for a global optimum by generating a population of trial solutions. Ant colony optimization, as a metaheuristic method, has many features which are similar to genetic algorithms (GAs). Table 1 compares some common and/or similar features of ACO algorithms with those of GAs, as described in detail by Maier *et al.* [9]. The most important difference between GAs and ACO algorithms is the way the trial solutions are generated. In ACO algorithms, trial solutions are constructed incrementally based on the information contained in the environment. The solutions are improved by modifying the environment via a form of indirect communication called stigmergy [7]. On the other hand, in GAs the trial solutions are in the form of strings of genetic materials and new solutions are obtained through the modification of previous solutions [9]. Thus, in GAs the memory of the system is embedded in the trial solutions, whereas in ACO algorithms the system memory is contained in the environment itself.

Table 1. Similarities of ACO and genetic algorithms (Maier *et al*., 2003)

Genetic Algorithm	ACO Algorithm
Population size	Number of ants
One generation	One iteration
Trial solutions utilize the principle of survival of the fittest	It is based on foraging behavior of ant colonies
Probabilistic process is governed by crossover and mutation	Probabilistic process is defined by pheromone intensities and local heuristic information
Encouraging wider search space is achieved by mutation operator	Wider search space is guaranteed by pheromone evaporation

3. ANT COLONY OPTIMIZATION (ACO) ALGORITHMS: GENERAL ASPECTS

In general, ACO algorithms employ a finite size of artificial agents with defined characteristics which collectively search for good quality solutions to the problem under consideration. Starting from an initial state selected according to some case-dependent criteria, each ant builds a solution which is similar to a chromosome in a genetic algorithm. While building its own solution, each ant collects information on its own performance and uses this information to modify the representation of the problem, as seen by the other ants [10]. The ant's internal states store information about the ant's past behavior, which can be employed to compute the goodness/value of the generated solution. In many optimization problems, some paths available to an ant in a given state may lead the ant to an infeasible state, which can be avoided using the ant's memory. Artificial ants are permitted to release pheromone while developing a solution or after a solution has been fully developed, or both. The amount of pheromone deposited is made proportional to the goodness of the solution an artificial ant has developed (or is developing).

Rapid drift of all ants towards the same part of the search space is avoided by employing the stochastic component of the choice decision policy and the pheromone evaporation mechanism. To simulate pheromone evaporation, the pheromone persistence coefficient (ρ) is defined which enables greater exploration of the search space and minimizes the chance of premature convergence to suboptimal solutions (see Eq. 3). A probabilistic decision policy is also used by the ants to direct their search towards the most interesting regions of the search space. The level of stochasticity in the policy and the strength of the updates in the pheromone trail determine the balance between the exploration of new points in the state space and the exploitation of accumulated knowledge [3].

Let $\tau_{ij}(t)$ be the pheromone deposited on path ij at time t, and $\eta_{ij}(t)$ be the heuristic value of path ij at time t according to the measure of the objective function. Defining the transition probability from node i to node j at time period t as [3]

$$P_{ij}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}(t)]^{\beta}}{\sum\limits_{l \in \text{allowed}} [\tau_{il}(t)]^{\alpha}[\eta_{il}(t)]^{\beta}} & \text{if } j \in \text{allowed} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where α and β = parameters that control the relative importance of the pheromone trail versus a heuristic value. Let q be a random variable uniformly distributed over [0, 1], and $q_0 \in [0, 1]$ be a tunable parameter. The next node j that ant k chooses to go is [10]

$$j = \begin{cases} \arg \max\limits_{l \in allowed_k} \left\{ [\tau_{il}(t)]^{\alpha} [\eta_{il}(t)]^{\beta} \right\} & if \;\; q \le q_0 \\ J & otherwise \end{cases} \tag{2}$$

where J = a random variable selected according to the probability distribution of $P_{ij}(t)$ (See Eq. (1)). The pheromone trail is changed globally. Upon completion of a tour by all ants in the colony, the global trail updating is done as follows:

$$\tau_{ij}(t) \xleftarrow{iteration} \rho.\tau_{ij}(t) + (1-\rho).\Delta\tau_{ij} \tag{3}$$

where $0 \le \rho \le 1$; $(1 - \rho)$ = evaporation (i.e., loss) rate; and the symbol $\xleftarrow{iteration}$ is used to show the next iteration.

There are several definitions for $\Delta\tau_{ij}(t)$ [5, 10]. In this paper, we use three algorithms

1. Ant System (AS) algorithm

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{M} \tau m_{ij}^{k}(t) \tag{4}$$

$$\tau m_{ij}^{k}(t) = \begin{cases} 1/G^{k}(m) & if \quad (i,j) \in T^{k}(m) \\ 0 & if \quad (i,j) \notin T^{k}(m) \end{cases} \tag{5}$$

where $G^k(m)$ = value of the objective function for the tour $T^k(m)$ taken by the *k-th* ant at iteration *m*.
2. Ant Colony System–Iteration Best (ACS_{ib})

$$\Delta\tau_{ij}(t) = \begin{cases} 1/G^{k_{ib}}(m) & if \quad (i,j) \in \text{tour done by ant } k_{ib} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where $G^{k_{ib}}(m)$ = value of the objective function for the ant taken the best tour at iteration *m*.
3. Ant Colony System–Global Best (ACS_{gb})

$$\Delta\tau_{ij}(t) = \begin{cases} 1/G^{k_{gb}} & if \quad (i,j) \in \text{tour done by ant } k_{gb} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $G^{k_{gb}}$ = value of the objective function for the ant with the best performance within the past total iterations.

4. ACO ALGORITHMS FOR OPTIMUM RESERVOIR OPERATION

To apply ACO algorithms to a specific problem, the following steps have to be taken: (1) Problem representation as a graph or a similar structure easily covered by ants; (2) Assigning heuristic information to the generated solutions at each time step (i.e., selected path by the ants); (3) Defining a fitness function to be optimized; and (4) Selection of an ACO algorithm to be applied to the problem.

a) Problem representation

To apply ACO algorithms to the optimum reservoir operation problem, it is convenient to see it as a combinatorial optimization problem with the capability of being represented as a graph. The problem may be approached considering a time series of inflow, classifying the reservoir volume to several intervals, and deciding for releases at each period with respect to an optimality criterion. Links between initial and final storage volumes at different periods form a graph which represents the system, determining the release at that period.

b) Heuristic information

The heuristic information on this problem is determined by considering the criterion as minimum deficit

$$\eta_{ij}(t) = 1/([R_{ij}(t) - D(t)]^2 + c) \tag{8}$$

where $R_{ij}(t)$ = release at period *t*, provided the initial and final storage volume at classes *i* and *j*, respectively; $D(t)$ = demand of period *t*; and *c* = a constant to avoid irregularity (dividing by zero in Eq. 8.). To determine $R_{ij}(t)$, the continuity equation along with the following constraints, may be employed as:

$$R_{ij}(t) = S_i - S_j + I(t) - LOSS_{ij}(t) \qquad \forall\, i, j \tag{9a}$$

$$S_{\min} \le S_i \le S_{\max} \qquad \forall\, i \tag{9b}$$

$$S_{\min} \le S_j \le S_{\max} \qquad \forall\, j \tag{9c}$$

where S_i and S_j = initial and final storage volumes (class *i* and *j* of discretized reservoir storage volume), respectively; $I(t)$ = inflow to the reservoir at time period *t*; $LOSS_{ij}(t)$ = loss (e.g., evaporation) at period *t* provided that initial and final storage at classes *i* and *j* respectively; S_{min} and S_{max} = minimum and maximum storage allowed respectively; and *NT*= total number of periods. Using the transition rule (Eq.

(2)), each ant is free to choose the class of final storage (end-of-period storage), if it is feasible through the continuity equation and storage constraints (Eqs. (9)).

d) Fitness function

The fitness function is a measure of the goodness of the generated solutions according to the defined objective function. For this study, taking the objective function as the minimum value of the total square deviation (TSD) from pre-assumed, the fitness function may be presented as:

$$TSD^k = \sum_{t=1}^{NT}\left[R^k(t) - D(t)\right]^2 \tag{10}$$

where $R^k(t)$ = release at period t recommended by ant k.

e) ACO algorithms

Three different ACO algorithms, namely: the Ant System (AS), the Ant Colony System– Iteration Best (ACS_{ib}), and the Ant Colony System–Global Best (ACS_{gb}), have been tested. The so-called solution construction and pheromone trail update rule [10] considered by these ACO algorithms are employed.

The main difference between them is due to global pheromone updating procedures. In the AS algorithm, pheromone updating may be accomplished using all ants upon a tour completion. However, in ACS_{ib}, the ant with the best result in each iteration will be employed for pheromone updating. On the other hand, in ACS_{gb}, the pheromone updating may be left to the ant with the best performance within the past total iterations.

5. MODEL APPLICATION

To illustrate the performance of the model, the Dez reservoir in the southwest of Iran, with an effective storage volume of 2,510 MCM and average annual demand of 5,900 MCM is selected [11]. For illustration purposes, a period of 60 months with an average annual inflow of 5,303 MCM is employed. The reservoir volume is arbitrarily divided into 14 classes with 200 MCM intervals. To limit the range of values of the fitness function, a normalized form of Eq. (10) has been used as:

$$TSD^k = \sum_{t=1}^{NT}\left[(R^k(t) - D(t))/D_{\max}\right]^2 \tag{11}$$

where D_{max} = maximum monthly demand. To start the model, a finite number of ants is randomly distributed in different classes of initial storage volume. It is also assumed that the starting point for ants could be any time along the 60-month operation horizon. Thus, ants are also uniformly randomly distributed along the operation horizon. Feasible paths for ants to follow are constrained by the continuity equation, and the minimum and maximum permitted storage volume (Eqs. (9)). By completion of the first tour by all ants, there will be a finite number of feasible solutions with values for the objective function. Now, realizing the values of the fitness function, the pheromones must be updated to continue the next iteration. To update the pheromones, three previously defined ACO algorithms are employed (Eqs. (4-7)). When the pheromone update is completed, the next iteration begins. A simple flow diagram of ACO algorithms for the optimum reservoir operation is depicted in Fig. 1. The end condition may be defined as the maximum number of iterations.

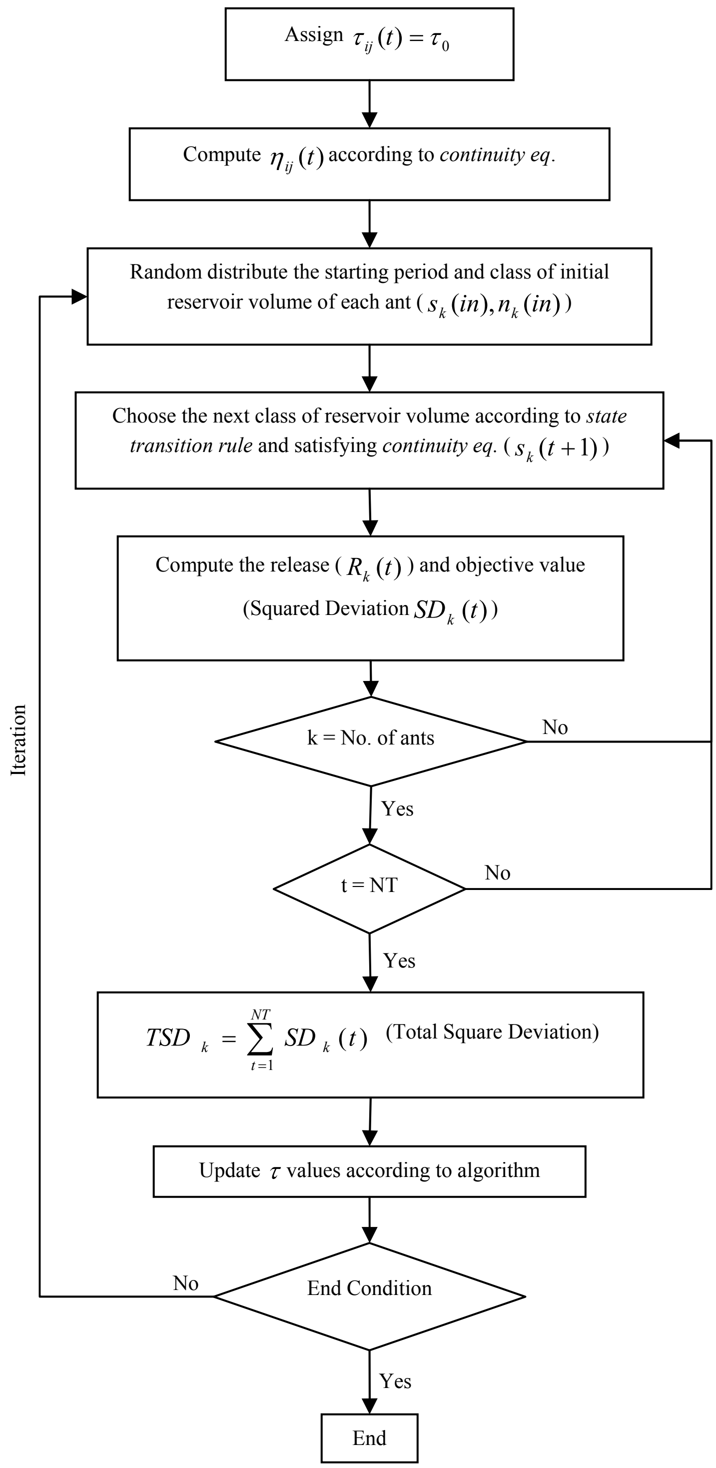

Fig. 1. ACO algorithm for optimum reservoir operation

To compare the performance of different ACO algorithms for updating pheromones in the reservoir operation, three well-known systems will be used.

The model so developed was tested for the Dez reservoir with 10 runs. Results of the model are presented in Table 2. The total number of ants (M) assigned to the problem was 30, with $\rho = 0.9$, $\alpha = 1$, $\beta = 2$, and $q_0 = 0.9$. To start with a generalized initial condition, the pheromone was uniformly distributed all over the defined paths (i.e., $\tau_0 = 1$). To normalize the value of the heuristic function, the parameter c was chosen to be unity (Eq. (8)). The total number of iterations at each run was limited to 200, as a trial value.

Table 2. Comparison of ACO algorithms in optimum reservoir operation (Statistical parameters of TSD resulted from 10 different runs)

	Algorithms		
Parameters	AS	ACS_{ib}	ACS_{gb}
Mean	3.110	2.825	1.889
The Best	2.975	2.334	1.562
The Worst	3.199	3.095	2.097
S.D.*	0.077	0.248	0.163
C.V.**	0.025	0.088	0.086

* Standard deviation
** Coefficient of Variation

Referring to Table 2, ACS algorithms provide better results compared to the AS algorithm. In general, the ACS_{gb} reveals a much better performance, with almost 39 and 33 percent improvement compared to AS and ACS_{ib}, respectively.

As with any search method, the performance of the ACS_{gb} algorithm in a reservoir operation depends on the model parameters. For a given problem, since it was not intended to develop an operation rule for the reservoir under consideration, a short modeling horizon of 60 months, was selected. The model performance was tested against variations of ρ, α, β, and q_0 for M=30. To have a notion on the best possible values of the effective setup parameters (i.e., ρ, α, β, and q_0), a feasible range for each parameter was first defined. Keeping all except one of the parameters unchanged, a variation of mean total square deviation was determined for different values of the changed parameter.

As mentioned earlier, $(1 - \rho)$ is an indication of the pheromone evaporation rate (i.e., losses) and ρ is defined in the literature as the speed of learning. Statistical parameters of the total square deviation from target demand (i.e., objective function) for 10 different runs, considering values of ρ ranging from 0.5 to 0.99, is presented in Table 3. Results of the runs propose a value of 0.75 for speed learning, leading to 0.25 for the pheromone evaporation rate.

Table 3. Influence of parameter ρ on the results of ACS_{gb} algorithm in optimum reservoir operation problem (Statistical parameters of TSD resulted from 10 different runs)

	ρ					
Parameters	0.99	0.95	0.90	0.85	0.75	0.50
Mean	2.678	1.985	1.889	1.790	1.730	1.791
The Best	2.205	1.737	1.562	1.529	1.573	1.639
The Worst	2.864	2.240	2.097	2.014	1.893	1.986
S.D.	0.210	0.156	0.163	0.151	0.089	0.098
C.V.	0.078	0.079	0.086	0.084	0.052	0.055

The significance of pheromone concentration and value of the heuristic function of each path are described by α and β, respectively. Assigning a higher value to β/α will put a higher significance weight on the objective function. However, interaction of the pheromone concentration and the value of the heuristic function may impose a limit on the β/α ratio. In the problem under consideration, keeping $\alpha = 1$, the best result is obtained for $\beta = 4$ (Table 4).

To study the effect of the random proportional rule (Eqs. (1) and (2)), different values of q_0 were examined. For $q_0 = 0$, the next step to be taken by the ants will follow a pure random process according to a predefined distribution function (Eq. (1)). On the other hand, a value of $q_0 = 1$ will entirely eliminate the random component of the decision, which may not necessarily end up to a desirable result (See Eq. (1), and (2)). Therefore, values of 0.8, 0.9, and 1.0 were considered for q_0 and results are displayed on Table 5.

As is clear, $q_0 = 1$ has minimized the standard deviation of the results which is due to random component elimination. A value of 0.9 for q_0 seems to be the best choice for the problem under consideration resulting in a mean total square deviation of 1.62 units.

Table 4. Influence of parameter β on the results of ACS_{gb} algorithm in optimum reservoir operation problem (Statistical parameters of TSD resulted from 10 different runs)

	β				
Parameters	1	2	3	4	5
Mean	1.930	1.730	1.711	1.618	1.653
The Best	1.734	1.573	1.619	1.489	1.514
The Worst	2.199	1.893	1.848	1.791	1.941
S.D.	0.141	0.089	0.082	0.098	0.130
C.V.	0.073	0.052	0.048	0.060	0.078

Table 5. Influence of q_0 on the results of ACS_{gb} algorithm in optimum reservoir operation problem (Statistical parameters of TSD resulted from 10 different runs)

	q_0		
Parameters	0.8	0.9	1.0
Mean	1.860	1.618	1.736
The Best	1.555	1.489	1.627
The Worst	2.024	1.791	1.818
S.D.	0.139	0.098	0.061
C.V.	0.075	0.060	0.035

The effect of the number of iterations and number of ants on mean total square deviation was examined using 50 to 500 iterations and 20 to 100 ants. Results are depicted in Table 6 and Fig. 2. As expected, the results improve as the number of iterations and number of ants increase. However, there seems to be a trade-off between the number of iterations and the total number of ants initially distributed. The best result was observed with 500 iterations and 100 ants, leading to the best total square deviation of 1.296 units. It needs to be mentioned that the best result was obtained for ρ= 0.75, α = 1, β = 4, and q_0 = 0.9, which resulted from parameter-tuning.

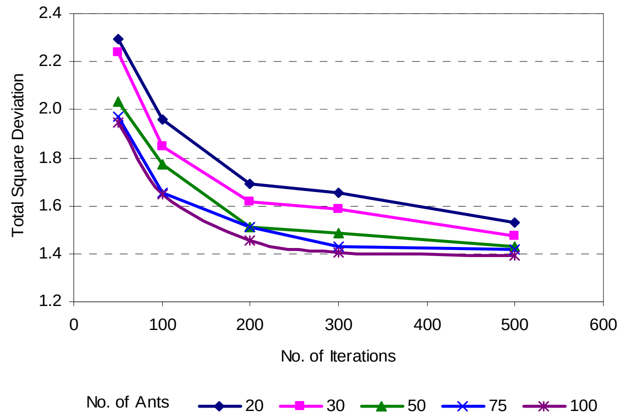

Fig. 2. Mean TSD variation versus number of iterations for different numbers of ants

The best overall result obtained from ACS_{gb} for initial and final storage volumes of 1,430 MCM is 1.296 (TSD). The global optimum with the same initial and final storage volumes resulted in TSD = 1.273. Clearly, the developed model with the ACS_{gb} algorithm for pheromone updating provides comparable results with those of global optimum, and seems promising in the optimum reservoir operation. The fluctuation of reservoir release, taken from two models, is presented in Fig. 3. Except for a few months, reservoir releases resulting from the proposed algorithm follow those of global optimum very well.

Table 6. Influence of number of iterations and number of ants on results of ACS_{gb} algorithm in optimum reservoir operation problem (Statistical parameters of TSD resulted from 10 different runs)

Number of Ants	Parameters	Number of Iterations				
		50	100	200	300	500
20	Mean	2.293	1.960	1.690	1.653	1.527
	The Best	1.959	1.777	1.547	1.519	1.317
	The Worst	2.494	2.209	1.813	1.786	1.714
	S.D.	0.163	0.125	0.085	0.073	0.132
	C.V.	0.071	0.064	0.050	0.044	0.087
30	Mean	2.236	1.847	1.618	1.586	1.473
	The Best	1.913	1.594	1.489	1.424	1.347
	The Worst	2.407	2.167	1.791	1.744	1.577
	S.D.	0.159	0.180	0.098	0.095	0.074
	C.V.	0.071	0.097	0.060	0.060	0.051
50	Mean	2.034	1.771	1.514	1.484	1.428
	The Best	1.827	1.517	1.383	1.413	1.341
	The Worst	2.268	1.999	1.669	1.630	1.515
	S.D.	0.137	0.172	0.090	0.074	0.055
	C.V.	0.067	0.097	0.060	0.050	0.038
75	Mean	1.924	1.653	1.508	1.429	1.394
	The Best	1.669	1.447	1.349	1.348	1.308
	The Worst	2.421	1.884	1.647	1.576	1.540
	S.D.	0.237	0.137	0.094	0.069	0.080
	C.V.	0.123	0.083	0.062	0.048	0.057
100	Mean	1.948	1.650	1.457	1.404	1.417
	The Best	1.795	1.467	1.325	1.322	1.296
	The Worst	2.115	1.743	1.597	1.506	1.592
	S.D.	0.091	0.086	0.083	0.061	0.082
	C.V.	0.047	0.052	0.057	0.043	0.058

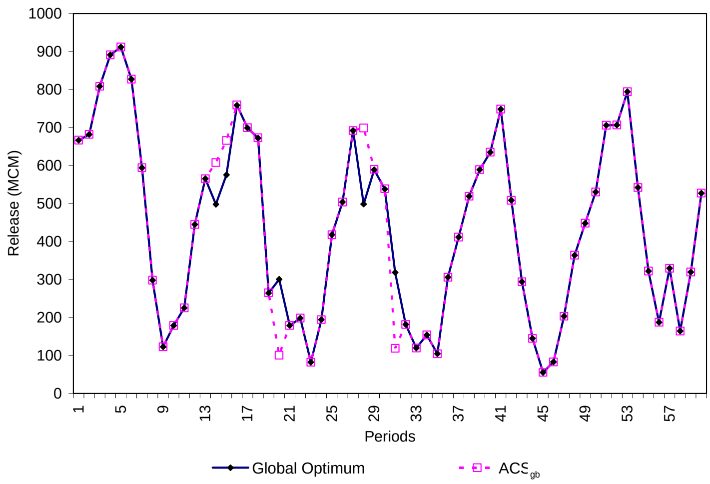

Fig. 3. Comparison of reservoir releases resulting from ACS_{gb} and global optimum

6. CONCLUDING REMARKS

While walking from one point to another, ants deposit a substance called pheromone, forming a pheromone trail. It has been shown experimentally [5] that this pheromone trail, once employed by a colony of ants, can give rise to the emergence of the shortest path. In general, the amount of pheromone deposited is made proportional to the goodness of the solution an ant may build. To apply ACO algorithms to the reservoir operation problem, one may view it as a combinatorial optimization problem. The problem may be approached by considering a time series of inflow, classifying the reservoir volume to several intervals, and deciding on the release at each period with respect to an optimality criterion. Feasible paths for ants to follow may be constrained by the continuity equation, as well as constraints on the storage volume. Upon each tour completion, a finite number of feasible solutions will form, leaving a new value for the pheromone.

Realizing the values of the fitness function, the pheromones will be updated by global and local update rules. Application of the proposed model to the Dez reservoir in Iran provided promising results. From three different pheromone updating algorithms (i.e., Ant System, Ant Colony System-iteration best, Ant Colony System-global best), the ACS_{gb} provides better and comparable results with those of the global optimum in the optimum reservoir operation. As for any search method, the performance of the proposed model is quite sensitive to setup parameters, hence fine tuning of the parameters is recommended.

NOMENCLATURES

ρ	pheromone persistence coefficient
$P_{ij}(t)$	transition probability from node *i* to node *j* at time period *t*
$\tau_{ij}(t)$	pheromone deposited on path *ij* at time period *t*
$\eta_{ij}(t)$	the heuristic value of path *ij* at time period *t*
α, β	parameters that control the relative importance of the pheromone trail versus a heuristic value
q	a random variable uniformly distributed over [0, 1]
q_0	a tunable parameter $\in$ [0, 1]
M	total number of ants
τ_0	initial value of pheromone.
$\Delta\tau_{ij}(t)$	total change in pheromone of path *ij* at time period *t*
$\varpi m_{ij}^{k}(t)$	change in pheromone of path *ij* at time period *t* associated to ant *k*
$G^k(m)$	value of the objective function of ant *k* at iteration *m*
$T^k(m)$	the tour taken by ant *k* at iteration *m*
$G^{k_{ib}}(m)$	value of the objective function for the ant taken the best tour at iteration *m*
$G^{k_{gb}}$	value of the objective function for the ant with the best performance within the past total iteration
$R_{ij}(t)$	release at time period *t*, provided the initial and final storage volume at classes *i* and *j*, respectively
$D(t)$	demand of time period *t*
c	a constant
S	storage
$I(t)$	inflow to the reservoir at time period *t*
$LOSS_{ij}(t)$	loss (e.g., evaporation) at time period *t* provided that initial and final storage at classes *i* and *j* respectively
S_{min}	minimum storage allowed
S_{max}	maximum storage allowed
NT	total number of periods

TSD	total square deviation from target demand
$R^k(t)$	release at time period *t* recommended by ant *k*
D_{max}	maximum monthly demand

REFERENCES

1. Colorni, A., Dorigo, M. & Maniezzo, V. (1991). Ant system: An autocatalytic optimizing process. Tech. Report 91-016, Politecnico di Milano, Italy
2. Colorni, A., Dorigo, M. & Maniezzo, V. (1991). Distributed optimization by ant-colonies. *Proc., 1st European Conf. on Artificial Life (ECAL'91)*, Cambridge, Mass, USA, MIT Press, 134-142.
3. Dorigo, M. (1992). Optimization, learning and natural algorithms. *Ph.D. Thesis*, Politecnico di Milano, Italy.
4. Deneubourg, J. L., Pasteels, J. M. & Verhaeghe, J. C. (1983). Probabilistic behavior in ants: A strategy of errors. *J. Theoretical Biology*, *105*, 259-271.
5. Dorigo, M., Maniezzo, V. & Colorni, A. (1996). The ant system: optimization by a colony of cooperating ants. *IEEE Trans. Syst. Man. Cybern*., *26*(1), 1-13.
6. Dorigo, M., & Di Caro, G. (1999). *The ant colony optimization metaheuristic*. New ideas in optimization, D. Corne, M. Dorigo, and F. Glover, eds., McGraw-Hill, London, 11-32.
7. Dorigo, M., Bonabeau, E. & Theraulaz, G. (2000). Ant algorithms and stigmergy. *Future Generation Comput. Systems*, *16*, 851-871.
8. Abbaspour, K. C., Schulin, R. & Van Genuchten, M. T. (2001). Estimating unsaturated soil hydraulic parameters using ant colony optimization. *Adv. Water Resour., 24*(8), 827-933.
9. Maier, H. R., Simpson, A. R., Zecchin, A. C., Foong, W. K., Phang, K. Y., Seah, H. Y. & Tan, C. L. (2003). Ant colony optimization for design of water distribution systems. *J. Water Resour. Plng. and Mgmt., 129*(3), 200-209.
10. Dorigo, M. & Gambardella, L. M. (1997). Ant colony system: A cooperative learning approach to the traveling salesman problem. *IEEE Transactions on Evolutionary Computation*, *1*(1), 53-66.
11. Mahab Ghodss Consulting Engineers (2002). Bakhtyari dam and power plant. Water Resources Planning Report Tehran, Iran.