

## "Research Note"

# DISCRETE ELEMENT METHOD ANALYSIS OF RETAINING WALL EARTH PRESSURE IN STATIC AND PSEUDO-STATIC CONDITIONS\*

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**Abstract**– Static and pseudo-static analysis based on the Discrete Element Method (DEM) is presented for active and passive earth pressure distribution behind a retaining wall. Soil mass in the present model is treated as comprising blocks which are connected by elasto-plastic Winkler-springs. The solution of this method satisfies all equilibrium and compatibility conditions. Examples are shown to demonstrate the applicability of the method for analyses of earth pressure behind a gravity wall, including pseudo-static pressure of earth and lateral pressure of non-homogeneous soil. The applicability of DEM for analyses of reinforced soil structures and the advantages of this method over the conventional limit equilibrium method are also discussed.

**Keywords** – Retaining wall, discrete element method, lateral earth pressure, limit equilibrium, reinforced soil

## 1. INTRODUCTION

The theories formulated by Coulomb [1] and Rankine [2] were among the first methods used for the analysis of earth-retaining structures. This problem has been solved by many researchers using other methods to model more complicated geometrical and loading conditions.

One of the methods which is extensively used is limit equilibrium. The method has been used to investigate the lateral earth pressure behind a retaining wall in active and passive conditions. In general, the number of static equilibrium equations is insufficient and the problem is solved by means of a different assumption devised for the relations between interslice forces. For example, Shields and Tolunay [3] divided the soil mass into different slices and made assumptions for the direction of interslice forces in passive condition.

The discrete Element Method can be used for the analysis of different geotechnical problems such as retaining wall earth pressure distribution. In this method stresses at all interfaces of blocks are compatible with its deformation and completely satisfy the stress-displacement relationship without any further assumptions.

## 2. DISCRETE ELEMENT METHOD

The method (DEM) was presented first as a new concept by C.S. Chang, for investigating the bearing capacity of foundations [4], slope stability [5] and retaining walls [6].

In this method, a failure surface is assumed for the soil behind the wall. Then the soil mass above the failure surface is divided into several solid slices. These slices are connected together with a Winkler spring to establish a bounded unique system consisting of the retaining wall and the soil wedge behind it. Winkler springs consist of normal (compression-tension) and shearing springs. The normal spring induces

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the rotational stiffness as well as the normal stiffness. During analysis of earth pressure in active and passive conditions, the gravity and inertia (earthquake) forces of the blocks are first applied. In the next stage of the simulation, the wall is moved towards or an outward of the soil until the passive or active condition is reached, respectively.

By increasing the load, the stresses at springs are also increased. When the shear and normal stresses are beyond the admissible stresses at a certain interface, the local factor of safety is calculated to be 1 for the interface and the iteration process redistributes the excess amount of stresses to neighboring slices. The iterative procedure is carried out until the stresses at all interfaces of blocks are compatible with its deformation and completely satisfy the stress-displacement relationship.

The movement of the wall is continued until all shear springs located on soil failure surfaces and the interface of the soil and wall are gone under a plastic condition. The failure of springs on a soil failure surface is based on the properties of the soil medium, and the failure of shear springs at the interface of soil and wall depends on the shear resistance properties between soil and wall. At this moment the earth pressure at limit condition can be obtained from mobilised forces induced in the springs, which are located between the soils and the wall. Several failure surfaces are assumed and the calculations are conducted to determine the active or passive forces at limit conditions. In an active condition, the maximum value of lateral earth pressure obtained in analysis is selected as the active lateral earth pressure, and the corresponding failure surface is assumed to be the critical failure surface. In a passive condition the failure surface providing the minimum value of passive force is assumed to be the critical surface.

Chang [6] applied this model for the analysis of earth pressure behind a retaining wall in a static condition for various modes of wall movement. The method is extended by the authors of this paper to determine lateral earth pressure in a pseudo-static condition, non-homogenous soil and reinforced backfill soil. By extending the method, a computer program is developed. Using the computer program, several examples are presented to demonstrate the applicability of the method.

### 3. RESULTS OF ANALYSIS

Results of several examples are presented to show the applicability of DEM for the analysis of lateral earth pressure. Examples include pseudo-static active earth pressure in different wall movements, pseudo-static passive earth pressure using linear and curved failure surfaces, active and passive earth pressure of non-homogenous soil media and the analysis of a wall with reinforced backfill.

#### *a) Pseudo-static analysis of active earth pressure*

This example considers a retaining wall with the following properties:

$$H=5 \text{ m} \quad c=0 \quad \varphi=30^\circ \quad \delta=20^\circ \quad \gamma=1.8 \text{ t/m}^3 \\ K_h = 0.2 \quad K_v = 0 \quad (\text{Earthquake coefficients})$$

As shown in Fig. 1a, a linear failure surface is adopted. A fine mesh is selected to determine the lateral earth pressure along the wall height. To investigate the effect of the wall displacement mode on resultant earth pressure distribution, analysis with two different displacement modes including translation and rotation was carried out.

Lateral earth pressure distributions for translation and rotation (about the base) modes of wall movement, are shown in Fig. 1b. For rotational wall movement complete consistency between pressure distributions resulting from DEM and that of the Mononobe-Okabe theory [7] exists. For translation wall movement a deviation can be observed, especially at the base of the wall, but overall consistency is obtained.

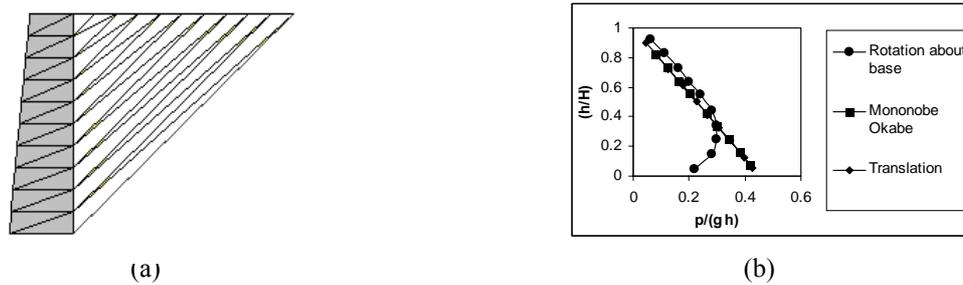


Fig. 1. Pseudo-Static active earth pressure analysis

The resultant earth pressure force in the translation mode is equal to 9.6 t and its point of application is 0.4H (from base). The calculated force in the rotational mode is approximately equal to that of the translation mode, but its point of application is located at 0.33H from the base. There is a little difference between the total lateral force obtained from DEM and that of the Mononobe-Okabe theory, which is computed to be 9.59 t. Other investigations have also concluded that the distribution of earth pressure against a retaining wall depends on the mode of wall movement [8].

Experimental studies also confirmed that the resultant force of pseudo-static active earth pressure is relatively equal to that obtained from Mononobe-Okabe, but the point of application is different. Sherif *et al.* [9] carried out experimental tests on retaining walls in the condition of the translation mode and concluded that the point of application of earth pressure is at 0.45H; while as mentioned before, the resulting point of application from DEM is at 0.4H. Also Ishibashi and Fang [10], based on an experimental study suggested that for the rotational mode of wall movement about the base, the point of lateral force application is at 0.3H, which is comparable with that of the DEM method located at 0.33H.

#### b) Pseudo-static analysis of passive earth pressure

Backfill soil and wall properties in this analysis are the same as those described in Section 3.1. The difference is that the wall was moved towards the soil to provide a passive condition. Also, a curved failure surface was adopted in the passive earth pressure analysis (Fig. 2a). Although in an active condition a linear failure surface was assumed, in a certain conditions like walls with high friction, the real failure surface is not linear. In other words, assumption of a linear failure surface doesn't always coincide with the real failure surface. In the case of a passive condition, the result of analysis is more sensitive to the shape of failure surface. It is believed that a curved failure surface is closer to the reality. For a better comparison of the analysis results with the other published data, the shape of the failure surface is chosen to be exactly the same as that adopted by Shields and Tolunay [3].

Figure 2a shows the failure surface and generated mesh of backfill media. Figure 2b illustrates the distribution of earth pressure along the wall. As can be seen, the distribution of earth pressure is not linear, and the point of force application is located at 0.25H from the base. The resultant force of earth pressure is equal to 98.9 t.

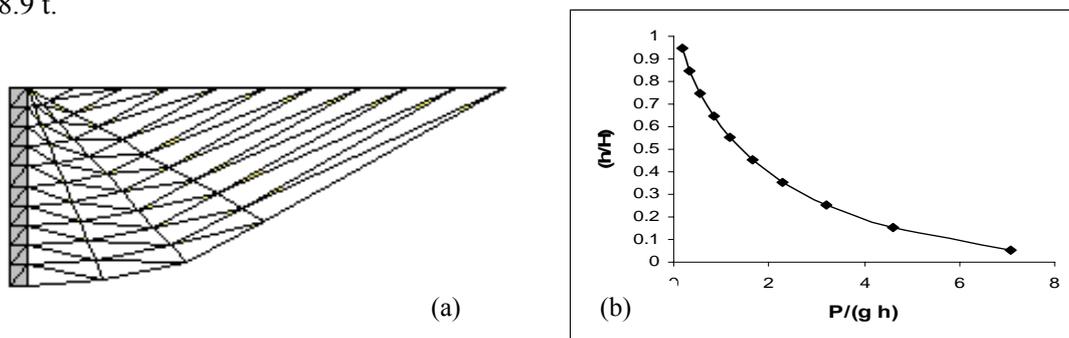


Fig. 2. Pseudo-Static passive earth pressure analysis

For comparison with the Mononobe-Okabe theory [7], this example was solved using a linear failure surface. Figure 3 compares the results of DEM analysis with the Mononobe-Okabe theory. The resultant earth pressure force computed from both methods is relatively the same and equals 105.2 t. It can be concluded that the curved failure surface gives the earth lateral passive force about six percent higher than what is computed for a linear failure surface.

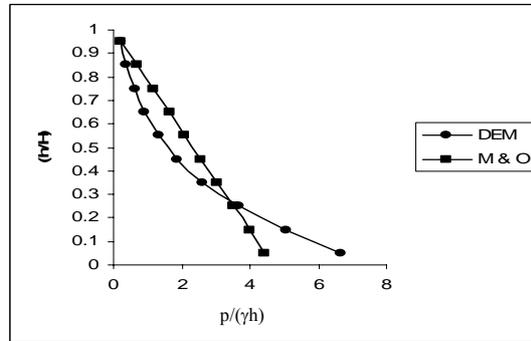


Fig. 3. Linear failure surface for passive condition

**c) Variation of earthquake coefficient along wall height in active and passive conditions**

As mentioned before, the horizontal seismic coefficient for all soil blocks at different depths was chosen to be 0.2 in the analysis presented in previous sections. To investigate the effect of seismic coefficient variation on resultant lateral forces and pressure distribution, analysis has been performed using variable coefficients at different levels. The value of the  $K_h$  was chosen to be linearly varied along the wall height between 0.25 (at the top of the wall) and 0.15 (at the base).

It can be seen from Fig. 4 that there is not a significant difference between earth pressure distributions concluded from both series of analysis, especially for the passive condition. The total lateral force on the passive condition with the varied earthquake coefficient was calculated to be 100.7 tons, which is slightly higher than what was computed for the constant coefficient (98.9 tons). Also the resultant lateral forces in active condition were 9.6t and 10t for constant and varied coefficient states, respectively.

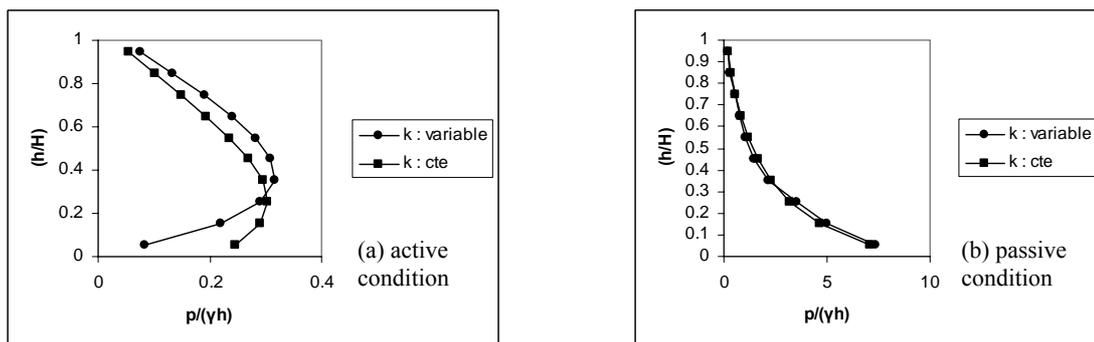


Fig. 4. Effect of variation of seismic coefficient along wall height

**d) Static analysis of active pressure of non-homogenous soil**

By using the DEM it is possible to analyse the lateral earth pressure of non-homogenous soil at a limit condition. This example considers a soil media consisting of two layers with the following properties:

Top layer	: H=2.5 m	c=0	$\phi=20^\circ$	$\delta=0^\circ$	$\gamma=1.8 \text{ t/m}^3$
Bottom layer	: H=2.5 m	c=0	$\phi=30^\circ$	$\delta=0^\circ$	$\gamma=1.8 \text{ t/m}^3$

To compare the results with the classic Rankine method, computation was carried out in the static condition.

In Fig. 5a the linear failure surface and the geometry of blocks employed to model the wall and the backfill soils are shown. Figure 5b presents the distribution of earth pressure along the height of the wall. Resultant lateral earth force calculated by DEM is equal to 8.3 t and that of the obtained from the Rankine theory equals 8.4 t. It can be seen that a good agreement exists.

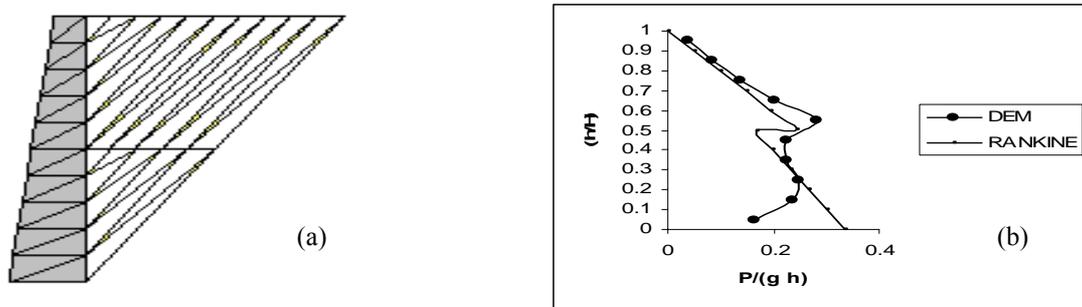


Fig. 5. Static analysis of active lateral pressure of non-homogenous soil

#### e) Pseudo-static analysis of reinforced soil structure

The DEM method is capable of estimating the force mobilized in each reinforcement member. For the analysis of a reinforced soil wall structure, first a failure surface is assumed and then soil media is divided into horizontal slices. At the cross points of failure surface and reinforcement elements normal springs are placed. These springs can tolerate only axial forces in the reinforcement elements at the intersection point.

As can be observed from Fig. 6, the retaining reinforced wall is divided into four trapezoidal-shaped and one triangular-shaped blocks. Five reinforcement members are placed in the soil media, in which four are crossed by the critical failure surface. The critical surface is the surface that induces the maximum resultant force in the reinforcements.

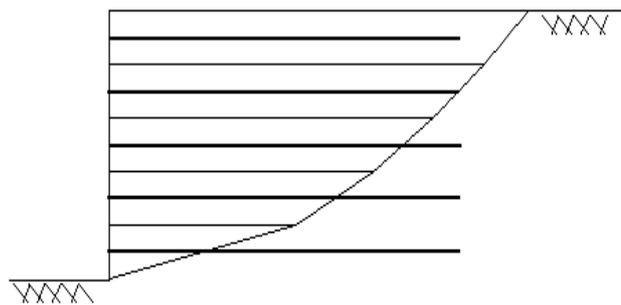


Fig. 6. Discrete element mesh for analysis of reinforced soil structure

The parameters used in this analysis are as follows; height of wall = 5 m,  $\phi=30^\circ$ ,  $c = 0 \text{ t/m}^2$ ,  $\gamma=1.8 \text{ t/m}^3$ ,  $K_h = 0.2$ ,  $K_v = 0.0$ , reinforcement length = 4 m and spaces between reinforcements = 1 m.

The result of the analysis by DEM indicates that the forces of reinforcements from base to top are 1.5, 2.4, 5.2, 0.0 and 0.0 t. The computed total force in the reinforcement elements was equal to 9.05 t. This result is in good agreement with the calculated force by Ling *et al.* [11], which is equal to 10.6 t.

#### 4. CONCLUSIONS

In this study, the DEM was extended to determine lateral earth pressure in a pseudo-static condition, non-homogenous soil and reinforced backfill. To demonstrate the applicability of the method for analyses of earth pressure behind a retaining wall including static and pseudo-static lateral pressure of homogeneous and non-homogeneous soil, several examples are described. The Comparison showed that the results are in good agreement with the other methods and also with the experimental tests. Applicability of DEM for analyses of a reinforced soil structure was also presented.

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