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DISCRETE INSTANTANEOUS OPTIMAL CONTROL METHOD*

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Abstract – A control algorithm based on the instantaneous optimal control method is presented for on-line control of structures subjected to earthquake excitations. This algorithm employs the digital state-space equation to discretize the continuous dynamical equation of motion, and named *discrete instantaneous optimal control method*. Based on the Lyapunov stability method, a procedure to obtain a discrete stable weighting matrix is developed. To demonstrate the precision and the efficiency of the proposed control algorithm an *8*-story shear-type building frame equipped with one active mass damper/driver (*AMD*) mechanism is used. Behavior of different weighting matrices is also examined.

Keywords – Active control, instantaneous optimal control, digital state-space equation, discrete stable weighting matrix, active mass damper/driver

1. INTRODUCTION

Successful performance of a control system with high efficiency mainly depends on the employed control algorithm. From the viewpoint of the practical application, a control algorithm has to be stable and simple to use. Since 1960 when Kalman, by using the performance function of a quadratic form and optimizing it, systemized the controlled system [1], many control algorithms such as pole assignment, instantaneous optimal control, independent modal space control, bounded state control, sliding mode control, intelligent control and so on, are proposed.

In the 1960's, the modern control theory became popular and was widely used in automobile, aeronautics and aerospace engineering. As the control technique extended from the stationary control process to the dynamic control process, the civil engineering field was influenced as well. Since 1972, when Yao [2] laid down the more rigorous control-theory based concept of structural control, many control algorithms have been presented. Yang [3, 4] proposed the *instantaneous optimal control algorithm* to improve the classical optimal control algorithm based on the condition that the entire earthquake ground acceleration history was not known *a priori*. However, this algorithm is sensitive to change of time increment. In other words, by changing the time increment the designer must use a new weighting matrix, which is difficult for this algorithm. To overcome these difficulties Yang *et al.* [5] by using the Lyapunov direct method, proposed a stable weighting matrix for this algorithm that greatly improved improved its efficiency.

Based on the instantaneous optimal control technique, Chang and Yang [6] derived a new control algorithm in *2nd*-order form using the Newmark integration scheme. They selected the weighting matrices so that the algorithm was unconditionally stable. Bahar *et al.* [7, 8] proposed a new instantaneous control algorithm using the unconditionally stable *Wilson-θ* method. To enhance serviceability of the structural system for occupant's comfort by adding the acceleration term, they modified the time-dependent

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performance index and obtained a complete feedback control. But this algorithm, like the other algorithms in the class of instantaneous optimal methods, is sensitive to change of time increment.

In this paper, to obtain the discretized form of the dynamical equation of motion, the definition of digital control state-equation is used. A first-order stable control algorithm based on a specific definition of time-dependant performance index is then established. This new algorithm is named *the discrete instantaneous optimal control method*. Also a new procedure to obtain a proper *stable weighting matrix* that enhances the efficiency of the control system is presented. This procedure is based on *the Lyapunov direct method*. By using this stable weighting matrix, the new algorithm will no longer be sensitive to any change in the time increment. Therefore, a major difficulty of *the class of instantaneous methods* will be resolved.

In the following sections, formulation of the discrete instantaneous optimal control is presented and by studying a numerical example, the precision of the new algorithm is demonstrated.

2. DIGITAL STATE-SPACE EQUATION

Consider a structure that is controlled by an *AMD* system and subjected to one-dimensional earthquake ground acceleration $\ddot{x}_0(t)$. The matrix equation of motion of the entire structural system (the building and active system), which is idealized by a linear system with *n*-degrees-of-freedom can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{M}\boldsymbol{e}\ddot{x}_0(t) \tag{1}$$

in which *M*, *C*, and *K* are $n \times n$ mass, damping, and stiffness matrices, respectively. $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$, and $\ddot{\boldsymbol{x}}(t)$ are *n*-dimensional displacement, velocity, and acceleration vectors, respectively. *D* is an $n \times r$ matrix that specifies the locations of r active controllers, $\boldsymbol{u}(t)$ is an *r*-dimensional control force vector and $e = [-1 -1 .... -1]^{\mathrm{T}}$ is an *n*-dimensional vector which defines the ground acceleration influence on masses of the entire building.

The first order digital state-space equation of motion of such structural system is defined as follows:

$$\boldsymbol{z}_{k+1} = \boldsymbol{A}_d \boldsymbol{z}_k + \boldsymbol{B}_d \boldsymbol{u}_k + \boldsymbol{w}_{1d} \ddot{x}_{0k}\,, \qquad \boldsymbol{z}(t_0) = \boldsymbol{z}_0 \tag{2}$$

where t_0 is the initial time and subscript *k* refers to time instant *t*, such that $\mathrm{t} = \mathrm{k}\Delta\mathrm{t}$, and $\Delta\mathrm{t}$ is the time increment. Vector z_k is the state vector in time instant *t* and is defined as follows:

$$\boldsymbol{z_k} = \begin{bmatrix} \boldsymbol{x_k} & \dot{\boldsymbol{x}}_{\boldsymbol{k}} \end{bmatrix}^T \tag{3}$$

Matrices $\boldsymbol{A}_d$, $\boldsymbol{B}_d$, and $\boldsymbol{w}_{1d}$ are the transition matrices corresponding to $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{w}_1$, respectively and are defined as follows:

$$\boldsymbol{A}_d = \exp(\boldsymbol{A}\Delta\mathrm{t}), \quad F = \int_0^{\Delta\mathrm{t}} \exp(\boldsymbol{A}\,\eta)\,\mathrm{d}\eta, \quad \boldsymbol{B}_d = \boldsymbol{F}\boldsymbol{B}, \quad \boldsymbol{w}_{1d} = \boldsymbol{F}\boldsymbol{w}_1 \tag{4}$$

$$\boldsymbol{A} = \begin{bmatrix} 0_{n\times n} & \boldsymbol{I}_{n\times n} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{0}_{n\times r} & \boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix}^T, \quad \boldsymbol{w}_1 = \begin{bmatrix} \boldsymbol{0}_{n\times 1} & \boldsymbol{e} \end{bmatrix}^T \tag{5}$$

In the next section, by employing this discretized form of the dynamical equation of motion, a new instantaneous optimal algorithm based on discrete state-space will be derived.

3. DISCRETE INSTANTANEOUS OPTIMAL CONTROL METHOD

For the new procedure, the common quadratic time-dependant performance index *J*(*t*), is modified in discrete form based on the instant control force and the resulting responses of the structure in the next time as follows:

$$J(t) = \frac{1}{2}(\boldsymbol{z}_{k+1}{}^T \boldsymbol{Q} \boldsymbol{z}_{k+1} + \boldsymbol{u}_k{}^T \boldsymbol{R} \boldsymbol{u}_k) \tag{6}$$

in which $2n \times 2n$ positive semi-definite Q matrix and $r \times r$ positive definite R matrix are weighting matrices related to the state variables and the control force, respectively. To minimize the time-dependant performance index subject to the constraint Eq. (2) at each time instant, the Lagrange function $L(t)$, is formed as follows:

$$L(t) = \frac{1}{2}(\boldsymbol{z}_{k+1}{}^T \boldsymbol{Q} \boldsymbol{z}_{k+1} + \boldsymbol{u}_k{}^T \boldsymbol{R} \boldsymbol{u}_k) + \boldsymbol{\lambda}_{k+1}{}^T (\boldsymbol{A}_d \boldsymbol{z}_k + \boldsymbol{B}_d \boldsymbol{u}_k + \boldsymbol{w}_{1d} \ddot{x}_{0k} - \boldsymbol{z}_{k+1}) \tag{7}$$

where $\boldsymbol{\lambda}_{k+1}$ is the Lagrange multiplier vector. By taking the first variations of the Lagrangian with respect to the state vector in the next time $\boldsymbol{z}_{k+1}$ and control variables in the present time $\boldsymbol{u}_k$, and then equating them to zero, the necessary conditions are found as follows:

$$\boldsymbol{\lambda}_{k+1} - \boldsymbol{Q} \boldsymbol{z}_{k+1} = 0 \tag{8}$$

$$\boldsymbol{u}_k = -\boldsymbol{R}^{-1} \boldsymbol{B}_d{}^T \boldsymbol{\lambda}_{k+1} \tag{9}$$

Substituting Eqs. (2) and (8) into Eq. (9), the *close-open loop control force vector* is obtained as follows:

$$\boldsymbol{u}_k = -[\boldsymbol{R} + \boldsymbol{B}_d{}^T \boldsymbol{Q} \boldsymbol{B}_d]^{-1} \boldsymbol{B}_d{}^T \boldsymbol{Q} (\boldsymbol{A}_d \boldsymbol{z}_k + \boldsymbol{w}_{1d} \ddot{x}_{0k}) \tag{10}$$

Focusing on Eq. (10), one can see that this equation is equivalent to the first two terms of the control force vector in the close-open loop classical optimal control method [9]. The only difference is that in Eq. (10) the weighting matrix $\boldsymbol{Q}$ appears (instead of the Riccati matrix) in the closed-open loop classical optimal control method. Hence, this control method can affect the system matrix and simultaneously alleviate the external excitation load transmitted to the structure.

By substituting Eqs. (8) and (9) into Eq. (2), a simple relation to evaluate the state variable z in the next time increment is as follows:

$$\boldsymbol{z}_{k+1} = [\boldsymbol{I} + \boldsymbol{B}_d \boldsymbol{R}^{-1} \boldsymbol{B}_d{}^T \boldsymbol{Q}]^{-1} (\boldsymbol{A}_d \boldsymbol{z}_k + \boldsymbol{w}_{1d} \ddot{x}_{0k}) \tag{11}$$

Equations (10) and (11), which are the control force vector and the state variable, respectively, are the base equations in the *discrete instantaneous optimal control method*. In order to obtain a stable weighting matrix, a new procedure in discrete form based on the Lyapunov direct method is proposed in the next section.

4. STABLE WEIGHTING MATRIX

The performance index $J(t)$ for the instantaneous optimal algorithm is a time-dependant quadratic function of responses and control force. In the previous section, the differential equation of motion was approximated by the discrete state transition equations, which express the response of the structure at time $t+\Delta t$ in terms of the response at time t. Such an approximation does not guarantee the stability of the controlled structure. The approximation improves as Δt becomes smaller; however the control force, which is a function of Δt, should be finite. As a result, the weighting matrix $\boldsymbol{Q}$ in addition to being positive semi-definite, should also guarantee the stability of the controlled structure.

To determine a stable weighting matrix $\boldsymbol{Q}$ for the continuous differential equations of motion, Yang *et al.* [5] have presented a procedure using the Lyapunov direct method. However, extensive analysis shows that efficiency of the control system employing discrete control formulation while using a stable weighting matrix obtained from a continuous procedure is not high. For this reason a new procedure to obtain a stable weighting matrix is presented, which is formed based on the Lyapunov direct method in discrete state-space.

Lyapunov stability analysis- Based on the Lyapunov direct method [9] a system defined by

$$\boldsymbol{z}_{k+1} = \boldsymbol{A}_d \boldsymbol{z}_k \tag{12}$$

is stable if a scalar Lyapunov function $\boldsymbol{V}(z) > 0$ for $z \neq 0$, $\boldsymbol{V}(z) = 0$ for $z = 0$, and $\boldsymbol{V}(z) \to \infty$ as $z \to \infty$ exists, such that its first direct difference (in discrete systems) is negative semi-definite for all z, *i.e.* $\Delta\boldsymbol{V}(z) \leq 0$.

Consider a positive semi-definite matrix $\boldsymbol{Q}$, such as

$$\boldsymbol{V}(\boldsymbol{z}_k) = \boldsymbol{z}_k^T \boldsymbol{Q}\, \boldsymbol{z}_k \geq 0 \tag{13}$$

which is a possible Lyapunov function. Now, consider the structural system defined as follows:

$$\boldsymbol{z}_{k+1} = [\boldsymbol{A}_d - \boldsymbol{B}_d \boldsymbol{G}]\boldsymbol{z}_k \tag{14}$$

where $\boldsymbol{G} = (\boldsymbol{R} + \boldsymbol{B}_d^T \boldsymbol{Q}\boldsymbol{B}_d)^{-1} \boldsymbol{B}_d^T \boldsymbol{Q}\boldsymbol{A}_d$. In Eq. (14) the excitation terms are not present because they are not relevant to the stability of the structure. Based on Eq. (13), the first direct difference of the Lyapunov function in discrete form is given as follows:

$$\Delta\boldsymbol{V}(\boldsymbol{z}) = \boldsymbol{z}_{k+1}^T \boldsymbol{Q}\, \boldsymbol{z}_{k+1} - \boldsymbol{z}_k^T \boldsymbol{Q}\, \boldsymbol{z}_k \tag{15}$$

By substituting Eq. (14) into Eq. (15) and using the definition of G from Eq. (14), we have

$$\Delta\boldsymbol{V}(\boldsymbol{z}) = \boldsymbol{z}_k^T [\boldsymbol{A}_d^T \boldsymbol{Q}\boldsymbol{A}_d - \boldsymbol{A}_d^T \boldsymbol{Q}\boldsymbol{B}_d (\boldsymbol{R} + \boldsymbol{B}_d^T \boldsymbol{Q}\boldsymbol{B}_d)^{-1} \boldsymbol{B}_d^T \boldsymbol{Q}\boldsymbol{A}_d - \boldsymbol{Q} - (\boldsymbol{A}_d - \boldsymbol{B}_d \boldsymbol{G})^T \boldsymbol{Q}\boldsymbol{B}_d \boldsymbol{G}]\boldsymbol{z}_k \tag{16}$$

If the weighting matrix $\boldsymbol{Q}$ is selected so that the bracket in Eq. (16) will be negative semi-definite, it is a stable weighting matrix.

As a sufficient condition, we can assume that the sum of the first three terms of the bracket in Eq. (16) is equal to a negative semi-definite matrix, - $\boldsymbol{I}_0$ that $\boldsymbol{I}_0$ is an arbitrary positive semi-definite matrix. Using this definition we get

$$\boldsymbol{A}_d^T \boldsymbol{Q}\boldsymbol{A}_d - \boldsymbol{A}_d^T \boldsymbol{Q}\boldsymbol{B}_d (\boldsymbol{R} + \boldsymbol{B}_d^T \boldsymbol{Q}\boldsymbol{B}_d)^{-1} \boldsymbol{B}_d^T \boldsymbol{Q}\boldsymbol{A}_d - \boldsymbol{Q} + \boldsymbol{I}_0 = 0 \tag{17}$$

This is *the discrete Riccati matrix equation*. Now, Eq. (16) is reduced to

$$\Delta\boldsymbol{V}(\boldsymbol{z}) = \boldsymbol{z}_k^T [-\boldsymbol{I}_0 - (\boldsymbol{A}_d - \boldsymbol{B}_d \boldsymbol{G})^T \boldsymbol{Q}\boldsymbol{B}_d \boldsymbol{G}]\boldsymbol{z}_k \tag{18}$$

By selecting a positive semi-definite matrix $\boldsymbol{I}_0$, Eq. (17) is solved and the weighting matrix $\boldsymbol{Q}$ is obtained. Now, if the bracket in Eq. (18) is a negative semi-definite matrix, the computed weighting matrix $\boldsymbol{Q}$ is a discrete stable weighting matrix for the differential equation of motion in Eq. (14).

5. NUMERICAL EXAMPLE

To study the behavior of the proposed algorithm, an eight-story shear-type building frame with similar story properties is investigated. The structural properties of each story are as follows; floor mass is *345.6 tons*, elastic stiffness is *3.404e5 kN/m*, and internal damping coefficient is *2937 tons/sec*, which corresponds to a *2%* damping for the first vibration mode of the building without a control system. The N-S component of the *1940 El Centro* earthquake record with a maximum acceleration of about one-third of ground acceleration is used as input excitation. The time length of the acceleration record is about *54 sec*.

An active mass damper/driver (*AMD*) system is installed on the top floor of the building. The properties of *AMD* are as follows: the mass is *29.63 tons*, the frequency is *98%* of the first vibration mode of the building, the damping is *25 tons/sec*, so that the damping ratio of the *AMD* is approximately *7.3%*. When an *AMD* is installed on the *8th* floor, the displacement vector is defined as follows

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_8 & x_m \end{bmatrix}^T \tag{19}$$

in which x_i's and $x_{\boldsymbol{m}}$, are displacement of the *ith* floor and the driver mass, respectively. The members in the *9th* row and the *9th* column of the matrices of the entire building are related to the properties of the mass driver.

In the following sections, efficiency of the control system using a different arrangement of the weighting matrix and comparison between different optimal methods will be investigated.

a) Influence of time increment

To assign an adequate time increment so the results of the analysis converge, four different time increments from 1/2 to 1/20 of the smallest period of the building, *i.e. 0.10 sec* that is related to the last mode of the building, are selected. The responses of the building are shown in Table 1. It is seen that the *0.02 sec* time increment has a good convergence for without control and passive control mode responses of the building. Therefore, it seems 1/5 of the last period of the building is a useful criterion for selecting an adequate time increment.

b) Usual weighting matrix

To consider the performance of the active control system with respect to the time increment change, we need to select a proper weighting matrix. If the $2n \times 2n$ weighting matrix **Q** is partitioned into four equal size matrices, the two sub-matrices on the main diagonal of the **Q** matrix are related to the displacement and velocity state vectors of the entire building, respectively. There are many different options for selecting these sub-matrices based on usual matrices of the dynamical system. These matrices include mass matrix **M**, stiffness matrix of the primary building $\mathbf{K}_1$, stiffness matrix of the entire building **K**, identity matrix **I**, and a special matrix with all of its elements equal to one. These weighting matrices are referred to as "usual weighting matrix" in this paper.

Table 1. Maximum responses of the *8th* floor using discrete instantaneous optimal control method with respect to the different time increments

	Without control		Passive control		Active control		
Time Increment(Sec)	Displ. (cm)	Accel. (m/sec ^{2})	Displ. (cm)	Accel. (m/sec ^{2})	Displ. (cm)	Accel. (m/sec ^{2})	Max. Force (kN)
0.05	17.7	7.43	13.9	6.00	6.9	2.92	1472
0.02	19.2	7.92	15.2	6.39	9.2	4.17	760
0.01	19.2	7.91	15.2	6.39	11.8	4.77	366
0.005	19.2	7.91	15.2	6.39	14.1	5.89	159

By using the time increment and the 1×1 control force related weighting matrix equal to *0.02 sec*, and *0.001*, respectively a proper **Q** matrix with a highly efficient control system can be selected. Many different cases have been analyzed. Among them, a combination of the **K** and **M** matrices produce admissible performances of the control system. To achieve a proper efficiency of the control system, we have assumed some constraints as follows: the displacement reduction of the floors and the maximum length stroke are about *50%*, and *1.5 m*, respectively. Moreover, the average required control force, which is a fixed value equal to *72.68 kN*, is determined as follows:

$$\mathrm{ACF} = \frac{1}{t_f}\int_0^{t_f} \left| \boldsymbol{u}(t) \right| \mathrm{d}\tau \tag{20}$$

where t_f is the terminal time. After extensive analysis, the elements of the proper weighting matrix are selected as follows:

$$\begin{bmatrix} \alpha \boldsymbol{K}_\gamma & \boldsymbol{O}_{9\times9} \\ \boldsymbol{O}_{9\times9} & \beta \boldsymbol{M} \end{bmatrix} \quad \text{where} \quad \boldsymbol{K}_\gamma = \begin{bmatrix} \boldsymbol{K}_{11}(8\times8) & \boldsymbol{K}_{12}(8\times1) \\ \boldsymbol{K}_{21}(1\times8) & \gamma \end{bmatrix} \tag{21}$$

in which $\boldsymbol{K}_{ij}$ in matrix $\boldsymbol{K}_\gamma$ is a partitioned sub-matrix of the entire stiffness matrix of the building and γ is an arbitrary scalar value. The value of γ is selected as small as possible positive value such that the Lyapunov stability conditions are satisfied. Based on the above assumptions, the coefficients α, β, and γ are equal to

4×10³, *32.275*, and *21*, respectively. This case, named *Case I*, produces the displacement reductions of the *8th* and the *1st* floors equal to *52%* and *43.9%*, respectively, and the acceleration reductions of the *8th* and the *1st* floors are *47.3%* and 7*.1%*, respectively.

The resulting responses of the controlled building for different time increments using *Case I* are determined and summarized in the last three columns of Table (1). It can be observed that by decreasing the time increment, the efficiency of the control system decreases, *i.e.* the required control force is decreased and the responses of the building are increased monotonically. Later it will be shown that by employing a new discrete stable weighting matrix, the proposed algorithm, named *discrete instantaneous optimal control method*, will be a stable method and the time increment problem will be overcome.

Changing coefficients in the weighting matrix. By changing the coefficients α and β, and fixing γ to 21 in Eq. (23), other cases can be achieved in such a way that their required average control forces are equal to 72.68 kN. Among these, to compare with Case I, the results of another case named Case J, are presented in Table 2 and Fig. 1. Specifications of this case are as follows:

Case J : $\alpha = 1\times 10^4$, $\beta = 92.025$, $\gamma = 21$

Table 2. Comparison of the control system performances with usual weighting matrix using discrete instantaneous optimal control method

	Driver mass responses				
	Displacement (m)	Velocity (m/sec)	Acceleration (m/sec^2)	Max. control Force (kN)	Max. base shear reduction (%)
Case I	1.22*	7.08*	49.2	760.76	43.4
Case J	1.17*	6.66*	47.7	738.01	42.9
Case K	1.89*	11.23*	67.5	573.47	35.9

* Relative to the 8th floor responses

Comparison between these cases shows that increasing the coefficient α results in: (a) decrease in the acceleration and velocity responses of all floors except the acceleration response of the first floor; (b) small increase in the displacement responses of the floors; (c) decrease in the driver mass responses; (d) decrease in the maximum required control force.

Based on these observations, the authors recommend the use of a large value for coefficient α, related to the displacement state vector in the **Q** matrix. This will result in saving energy and decreasing responses of the driver mass and floors, however the displacement responses of the floors will increase slightly.

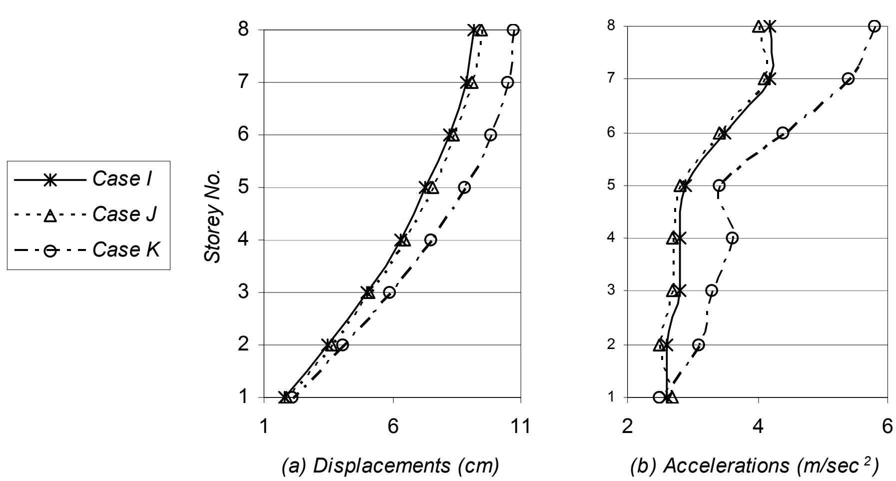

Fig. 1. Comparison of the responses of Cases I, J, and K using discrete instantaneous optimal control method

Special weighting matrix Q. If either α or β in Eq. (23) is assumed to be zero, a special **Q** matrix is obtained which is related to one state variable of the entire building. Analysis indicates that when α is assumed to be zero, i.e. the **Q** matrix only includes matrix related to the velocity state vector, the required control force is limited to a small value. In this case, the performance of the control system is very weak and is very similar to the performance of the passive mechanism.

On the other hand, the efficiency of the control system by selecting β equal to zero, i.e. the **Q** matrix, includes only a matrix related to the displacement state vector acceptable. Analysis shows that the following case, named Case K, has a proper efficiency.

$$\text{Case K}: \ \boldsymbol{Q} = \begin{bmatrix} \alpha \boldsymbol{I}_\gamma & \boldsymbol{0}_{9\times9} \\ \boldsymbol{0}_{9\times9} & \boldsymbol{0}_{9\times9} \end{bmatrix} \qquad \text{where} \qquad \boldsymbol{I}_\gamma = \begin{bmatrix} \boldsymbol{I}_{8\times8} & \boldsymbol{0}_{8\times1} \\ \boldsymbol{0}_{1\times8} & \gamma \end{bmatrix}$$

In which $\alpha = 9.3116 \times 10^6$, $\gamma = 0$, and the average required control force is 72.68 kN. The results of the analysis of Case **K** are shown in Fig. 1 and Table 2. The results show that the efficiency of this case is very low so that the responses of the floors in Case **K** are greater than other cases. Increased responses of the driver mass in this case cannot cause remarkable reduction of the responses of the building.

e) Stable weighting matrix

Using stable weighting matrix to achieve high efficiency of the control system is a major advantage of the control design of a building to resist excitation loads specially earthquake loads. But this matrix is not always achievable because a routine procedure that satisfies all different conditions of a controlled system does not exist, yet. Nevertheless, some procedures have been introduced to produce matrices that are close to the actual stable weighting matrix. In the next part, weighting matrices using a continuous procedure and a new discrete procedure based on Lyapunov direct method are obtained and compared with each other.

i. Continuous procedure: In 1992, Yang *et al*. [5], by using the Lyapunov direct method, proposed a stable weighting matrix. This matrix is obtained by solving a continuous Riccati matrix equation based on the classical optimal algorithm. Since the coefficient of the state vector in the control law of our proposed method, Eq. (10), is identical to the classical one, this continuous stable weighting matrix (CSWM) can be used for the proposed method.

Analysis shows that the matrix in Eq. (23) is an adequate positive semi-definite matrix. By specifying coefficient β to 100 and the value of γ to 21, coefficient α is assigned to be 876.2 such that the average required control force is fixed to 72.68 kN. The resulting responses of this case, named Case M, are shown in Table 3 and Fig. 2. It can be seen in Fig. 2 that the displacement responses of the floors are smaller, the velocity responses are similar, and the acceleration responses are slightly greater than Case J. The responses of the driver mass shown in Table 3 are remarkably greater, while the maximum required control force of Case M is remarkably smaller than Case J.

Table 3. Comparison of the control system performances with stable weighting matrix using discrete instantaneous optimal control method

	Driver mass responses				
	Displacement (m)	Velocity (m/sec)	Acceleration (m/sec^2)	Max. control force (kN)	Max. base shear reduction (%)
Case J	1.17*	6.66*	47.7	738.01	42.9
Case M	1.49*	8.73*	50.5	669.89	52.4
Case N	1.36*	7.78*	49.9	631.59	52.9

* Relative to the 8th floor responses

ii. Discrete procedure: Based on the procedure outlined in section 4, to achieve a discrete stable weighting matrix (DSWM), a positive semi-definite matrix $\mathbf{I}_0$ is selected and the discrete Riccati matrix Eq. (17), is solved. Finally the bracket in Eq. (18) is determined to insure that the negative semi-definiteness of the difference of the Lyapunov function (13) is achieved.

After extensive analysis to obtain discrete stable weighting matrix, Eq. (23) is recognized as a proper arrangement for the $\mathbf{I}_0$ matrix. Coefficient α and the value of γ are assigned to be 7 and 21, respectively, and coefficient β is assigned to be 0.417 such that the average required control force is equal to 72.68 kN. The resulting responses of this case, named Case N, are shown in Table 3 and Fig. 2. It is clear from Fig. 2 that the displacement responses of the floors in Case N are almost similar to Case M, and the velocity responses are slightly smaller than Case M. But the acceleration responses of Case N are remarkably smaller in higher floors and slightly greater in lower floors in comparison with Case J. The responses of the driver mass, shown in Table 3 are greater than Case J and smaller than Case M. The maximum required control force of Case N is remarkably lower than the other cases.

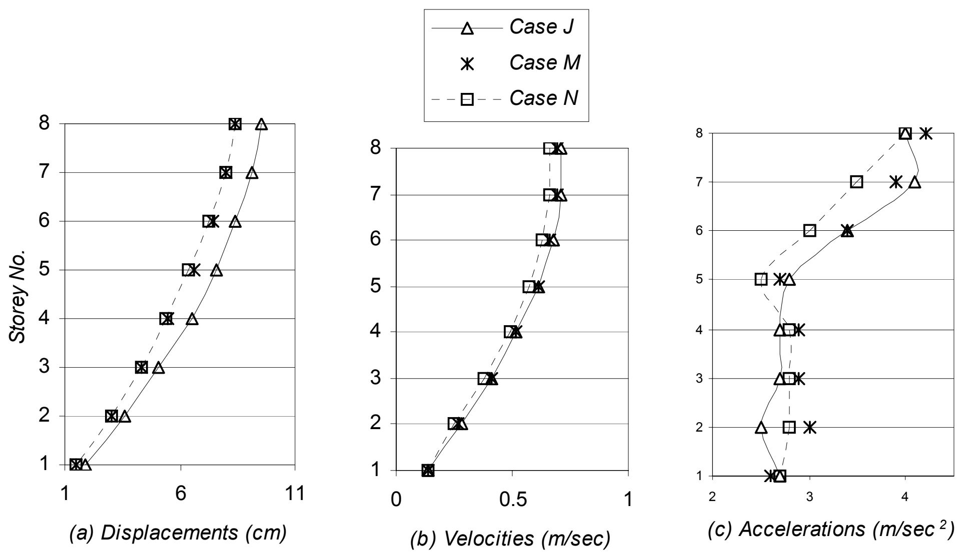

Fig. 2. Comparison of the responses of the different stable procedure using discrete instantaneous optimal control method

Some other cases with different α values are also examined. In these cases, the same above-mentioned procedure for selecting coefficients are carried out. The obtained results show that by increasing coefficient α, the acceleration responses of the floors, the responses of the driver mass, and the maximum required control force are decreased. On the other hand, the velocity responses of the floors increase slightly while the displacement responses increase remarkably.

Conditions of the Lyapunov stability method. To obtain a discrete stable weighting matrix, a routine procedure based on the Lyapunov direct method was presented in section 4. In that procedure, to ensure the Lyapunov stability, necessary and sufficient conditions must be satisfied. First, to ensure positive definiteness of the **Q** matrix, the values of $\mathbf{V}(\mathbf{z}_k)$ in Eq.(13) for all $\mathbf{z}_k$ must be greater than or equal to zero. Second, to ensure negative semi-definiteness of the first direct deference of $\mathbf{V}(\mathbf{z}_k)$, i.e. $\Delta\mathbf{V}(\mathbf{z})$ in Eq. (18), the principal minors of the bracket in Eq. (18) must alternately be negative.

These conditions have been investigated for all cases. Here, the results of these conditions using Case J are presented. The values related to the first condition are shown in Fig. (3). The values of the second condition are sequentially determined as follows; -4.7e6, 1.7e3, -5.4e19, 1.6e26, -4.6e32, 1.2e39, -3.5e45, 1.6e51, -7.5e52, 1.0e55, -1.5e57, 2.2e59, -3.2e61, 4.7e63, -6.8e65, 9.8e67, -1.4e70, and 1.8e71. Since both necessary and sufficient conditions are satisfied, Case J is a discrete stable weighting matrix.

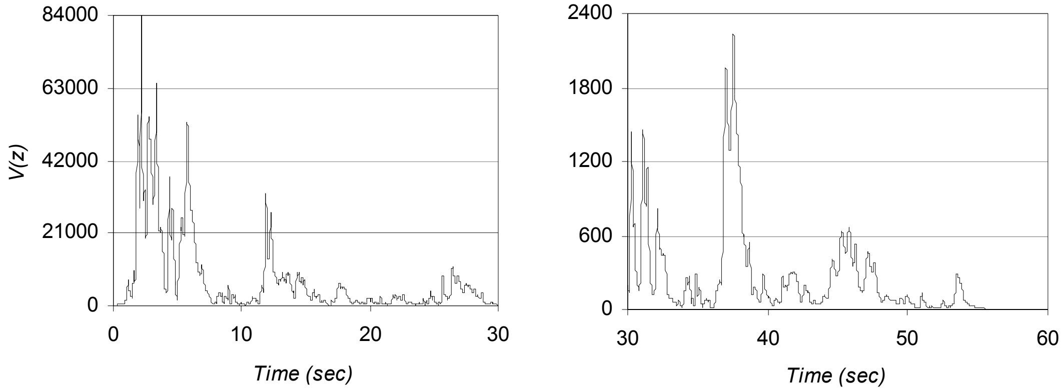

Fig. 3. The values of the Lyapunov stability function, using coefficients of *Case J* and discrete instantaneous optimal control method

Time increment problem. As discussed previously, selection of a proper time increment in an instantaneous optimal algorithm is a major problem. As shown in Table 1, by decreasing time increment, the required control force in every time step will decrease, and consequently the control system will be ineffective. This means that the designer must use a new weighting matrix whenever the time increment changes. This obviously imposes another inconvenience in designing an efficient algorithm.

This problem can only be overcome by using stable weighting matrices, CSWM or DSWM. Since at Δt equal to 0.02 sec, responses of the floors converge, these resulting responses of the building are compared with the resulting responses obtained from two other time increments equal to 0.01 sec, and 0.005 sec. The displacement responses of the floors and the relative displacement of the driver mass using DSWM are completely identical. Some of the other responses of the entire building differ very slightly, for instance, the acceleration responses of the driver mass for these three time increments are 49.9, 50.3, and 50.5 m/sec^2, respectively or the average control forces are 72.68, 72.72, and 72.75 kN, respectively. Since the time length in the ground acceleration record file is 0.02 sec for Δt smaller than 0.02 sec, the program needs to calculate instant ground acceleration by interpolation. It seems that these differences come from these computational efforts of the program.

Brief notes: Analysis shows that a designer can always find a proper usual weighting matrix such as Case J, which produces an admissible efficiency for a control system. But this weighting matrix consumes a large amount of the maximum required control force. A usual weighting matrix is also sensitive to the time increment changes. On the other hand, using a continuous stable weighting matrix such as Case M increases the efficiency of the control system. This weighting matrix significantly increases the responses of the driver mass. Meanwhile, a continuous weighting matrix is also sensitive to the time increment changes.

Finally, using a discrete stable weighting matrix such as Case N is strongly recommended because it remarkably increases the efficiency of the control system. This weighting matrix conserves the required maximum control force. Also, as a very desirable feature, a discrete stable weighting matrix is not sensitive to the time increment changes.

f) Comparison of different optimal control methods

In order to evaluate the precision of the discrete instantaneous optimal control algorithm, Case N is selected for comparing with the other optimal methods, Fig. 4. These optimal methods are the classical closed-loop optimal method, instantaneous optimal method using stable weighting matrix [5], and instantaneous optimal Wilson-θ method [7, 8]. As discussed previously, the control system using the discrete instantaneous optimal control method will produce high efficiency if it is designed with a proper stable weighting matrix. For the considered building, high efficiency has been achieved by using specifications of Case N. In Fig. 4, the resulting responses of the floors controlled by Case N are compared with the results of the other optimal methods. All optimal methods require an identical average control forces 72.68 kN.

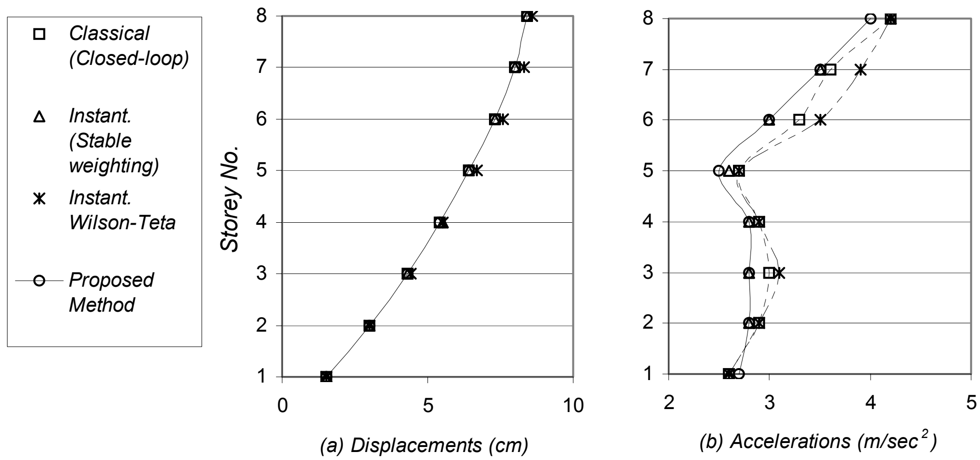

Fig. 4. Comparison of the responses of the building using discrete instantaneous optimal control method and the other optimal methods

As shown in Fig. 4, the displacement responses of the floors using different optimal methods are identical, except that the responses obtained from instantaneous optimal Wilson-θ method are slightly greater than the others. The acceleration responses of the floors using the proposed method are identical to the responses obtained from instantaneous with stable weighting matrix, and are also smaller than other optimal methods. It is remarkable that the efficiency of the control system installed on the building using both instantaneous optimal control method with a continuous stable weighting matrix, and discrete instantaneous optimal method with a discrete stable weighting matrix, are better than classical closed-loop optimal method.

Therefore, based on the above discussion, discrete instantaneous optimal control method employing new discrete stable weighting matrix is proposed as a powerful and reliable optimal algorithm for on-line controlling of structures.

6. STABILITY OF DISCRETE INSTANTANEOUS OPTIMAL CONTROL ALGORITHM

To study the stability of a controlled system, it is sufficient that characteristic values of its closed-loop system matrix are found. In discrete instantaneous optimal control method, closed-loop system matrix is the coefficient of the z_k in Eq. (11), *i.e.* $[\boldsymbol{I} + \boldsymbol{B}_{\mathrm{d}}\boldsymbol{R}^{-1}\boldsymbol{B}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{Q}]^{-1}\boldsymbol{A}_{\mathrm{d}}$. Based on this definition, if all of the characteristic values are inside the unit circle, the controlled system is stable. It is clear that the primary system matrix, properties and location of the driver mass, and the selected weighting matrices directly affect these characteristic values.

In Fig. 5, the characteristic values of the considered building without control are compared with the characteristic values of that building which is controlled with *Case N*. It can be seen that the characteristic values of the lower modes of the controlled building move in a direction toward the inside of the unit circle. Therefore, the considered building equipped with an *AMD* system is more stable than the building without control.

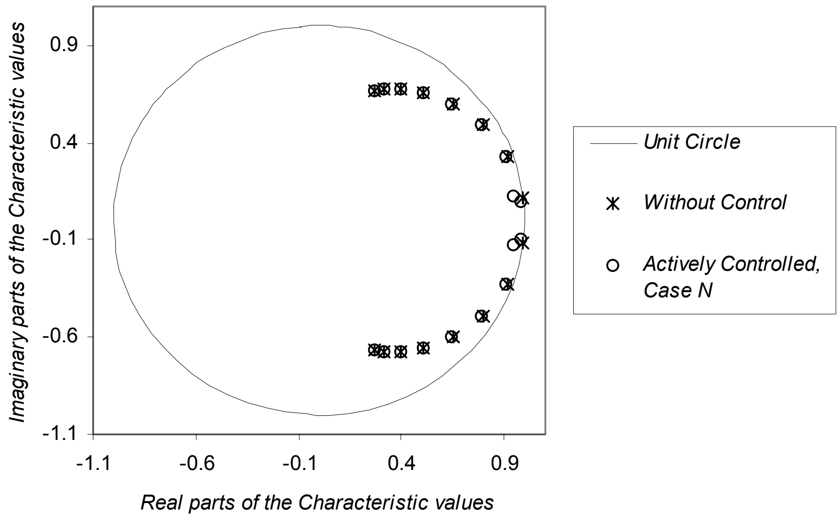

Fig. 5. Comparison of the characteristic values of the building without control system and with a control system using discrete stable weighting matrix, *Case N*

7. CONCLUSIONS

A new algorithm, named *discrete instantaneous optimal control algorithm*, based on the instantaneous optimal control method was presented. The new method employs the discrete state-equation to discretize the dynamical equation of motion and presents a powerful on-line structural algorithm. This algorithm has some distinct features that can be categorized as follows: *(i)* using the entire stiffness and mass matrices of the building with a small modification in the $\boldsymbol{I_0}$ matrix is a proper selection to achieve a discrete weighting matrix, *(ii)* using discrete weighting matrix produces a remarkable stability margin for lower modes of the building, *(iii)* the time increment problem in the class of instantaneous methods is overcome by using a discrete weighting matrix, *(iv)* the efficiency of the control system using this new algorithm with discrete stable weighting matrix is very remarkable, *(v)* the resulting displacement and velocity responses of the controlled building using this new algorithm are identical to the responses obtained from classical closed-loop optimal control method. Furthermore, the acceleration responses of the controlled building using the new method are remarkably smaller than the responses obtained from the classical optimal method.

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