Iranian Journal of Science & Technology, Transaction B, Vol. 28, No. B1
Printed in Islamic Republic of Iran, 2004

A COUPLED SURFACE WATER AND GROUNDWATER FLOW MODEL*

H. R., SAFAVI, ** A. AFSHAR, [1]M. A., MARINO[2] AND A. GHAHERI[1]

[1]Dept. of Civil Engineering, Iran University of Science and Technology, Tehran, I. R. of IRAN
E-mail: hasafavi@cc.iut.ac.ir
[2]Dept. of Civil and Environmental Engineering, University of California, Davis, California 95616, USA

Abstract – In hydraulically well-connected groundwater and surface water systems, stream-aquifer interaction has been simulated using a MODFLOW model developed by USGS, which couples hydraulic behavior of groundwater and surface water subsystems. It assumes a constant stream-stage during each stress period, employing a stream flow routing module which is limited to steady flow in rectangular, prismatic channels. One of the challenges in understanding the interaction of surface water and groundwater systems lies in their different time scales. In this paper, the INTRACT model is developed and incorporated into MODFLOW. INTRACT simulates unsteady, nonuniform flow by solving St. Venant equations. Terms that describe leakage between stream and aquifer as a function of streambed conductance and differences in water table and stream stage were incorporated into the continuity equation. INTRACT calculates new stream stages for each time step in a transient simulation based on upstream boundary conditions, stream properties, and estimated head distribution. Next, MODFLOW calculates head distribution using aquifer properties, stresses, and stream stages calculated by INTRACT. This process is repeated until convergence criteria are met for aquifer head distribution and stream stages. Because the time steps used in groundwater modeling can be much longer than time intervals used in surface water simulation, a provision has been made for handling multiple INTRACT time intervals within one MODFLOW time step. Performance of the coupled model was validated using an analytical solution from the previous studies.

Keywords – Groundwater, surface water, modeling, interaction, unsteady flow

1. INTRODUCTION

Surface and groundwater systems are in continuous dynamic interaction. In hydraulically well-connected groundwater and surface water systems, stream-aquifer interaction may be simulated using deterministic responses of sub-systems coupled at the stream-aquifer interface. The processes and simulation of groundwater and surface water interactions have interested researchers for many years. Pinder and Sauer [1] coupled the unsteady river equations with the two dimensional groundwater flow equations to study bank storage effects. Zitta and Wiggert [2] and Morel-Seytoux [3] incorporated bank storage into continuous stream flow simulation. Hall and Moench [4] and Land [5] used the convolution integral to account for river losses to bank storage. Detailed aspects of complex systems and scaling with the encompassing hierarchy theory and its applications are described in [6-8]. The MODFLOW model developed by USGS [9] couples the hydraulic behavior of ground and surface water systems. It employs a simple stream flow routing module, which is limited to steady flow in rectangular, prismatic channels. Time scales for surface and subsurface flow modeling were investigated by Yen and Riggins [10] considering the physical characteristics of stream-aquifer systems. Analytical solutions are developed by solving the linearized Boussinesq groundwater flow equation being subjected to fluctuating stream stages by Govindaraju and Koelliker [11] and Zlotnik and Huang [12].

The Danish Hydraulic Institute developed the MIKE-SHE model [13] with an aquifer-river exchange component that calculates the flow exchange between the aquifer and the river network. Hantush *et al*. [14]

---

*Received by the editors October 12, 2002 and in final revised form September 9, 2003
**Corresponding author

developed analytical solutions for routing stream flow, lateral stream-aquifer interaction, and aquifer storage. The analysis is based on one-dimensional lateral groundwater flow in semi-infinite homogeneous unconfined aquifers, which are in contact with streams through semi-pervious bed sediments.

This paper presents an improved coupled surface and groundwater flow model, and demonstrates its capabilities for practical purposes.

2. INTERACTION BETWEEN SURFACE WATER AND GROUNDWATER

Successful management of water resources involves managing the two main components of a region's water resources, namely groundwater and surface water. Surface water and groundwater are often managed separately; the fact that they are known to exchange water creates a strong incentive for the conjunctive management of these two resources. However, before the conjunctive management of surface water and groundwater can occur, it is imperative to determine how these two systems interact.

One of the challenges in understanding the interaction of surface and groundwater systems lies in their different time scales. Rivers, as a subset of surface water systems, have a much shorter residence time for water than do aquifers. Aquifers have much slower flow velocities, and consequently may show slower changes in the hydraulic head over time.

In modeling, the inherent difference in their time scales makes choosing an appropriate time-step difficult. A long time-step, which would be appropriate for modeling a groundwater system alone, might cause a loss of accuracy by over-averaging the river stage values. A short time-step, while good for modeling river system, would substantially increase the computation time and render the process inefficient for projecting water availability in the distant future. In this paper, a dynamic link was created between a surface water and groundwater model to help assess the role of the time step in modeling the two systems.

3. MATHEMATICAL FORMULATION

Although mathematically exact, analytical models generally can be applied only to simple one-dimensional problems because of rigid boundary conditions and simplifying assumptions. However, for many studies, analysis of one-dimensional flow is not adequate. Complex systems do not lend themselves to analytical solutions, particularly if the types of stresses acting on the systems change with time. Numerical models allow for the approximation of more complex equations and can be applied to more complicated problems without the many simplifying assumptions that are required for analytical solutions. Computer simulation of the interrelationships between surface water and groundwater systems requires the mathematical description of transient effects on potentially complex water table configurations.

From the groundwater perspective, a common simplifying assumption made to ease numerical simulation is that simulation of unsaturated flow. The leakage from surface water to an aquifer is assumed to be instantaneous; i.e., no head loss occurs in the unsaturated zone. This assumption is usually reasonable in a common situation where the thickness of the unsaturated zone between the stream and aquifer is not much.

The interaction between surface water and the underlying aquifer can be represented by the partial differential equation of groundwater flow

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right)-W=S_s\frac{\partial h}{\partial t} \tag{1}$$

where $S_s =$ specific storage[L^{-1}], $K_i =$ principal components of the hydraulic conductivity tensor [LT^{-1}], $h =$ hydraulic head [L], $W =$ source strength [Volume/(Volume T)]

Most, but not all interaction between groundwater and surface water is lumped into the "W" term. This interaction is based on Darcy's Law where the flow rate of water between the river and aquifer is directly proportional to the hydraulic head between the two. The exact form of Darcy's Law used by McDonald and Harbaugh [9], which describes the river/aquifer interaction, is

$$W_{i,j,k} = C'_{i,j,k}(Z - h_{i,j,k}), \text{ for } h_{i,j,k} > H_{BOT} \quad \textit{and} \quad W_{i,j,k} = C'_{i,j,k}(Z - H_{BOT}), \text{ for } h_{i,j,k} < H_{BOT} \tag{2}$$

where, $W_{i,j,k}$ = aquifer recharge rate [L^3T^{1}], $C'_{i,j,k}$ = riverbed conductance [L^2T^1], Z = stage in the river [L], H_{BOT} = elevation of the riverbed bottom [L], $h_{i,j,k}$ = hydraulic head in the aquifer [L], i, j, k refers to raw i, column j, layer k.

It will be assumed that if the aquifer head is below the river bottom, the value of $h_{i,j,k}$ in the stream flow equation will be replaced by H_{BOT}. The conductance term, $C'_{i,j,k}$, is a function of the physical parameters of the river and is defined

$$C'_{i,j,k} = \frac{BKL}{M} = BLC_{i,j,k} \tag{3}$$

where, B = width of river [L], L = length of river in cell i, j, k [L], M = riverbed thickness[L]
K = hydraulic conductivity of the riverbed [LT^1], $C_{i,j,k} = \frac{K}{M}$ [T^{-1}].

If reliable field measurements of stream seepage are available, they may be used to calculate riverbed conductance. Otherwise, a conductance value must be chosen more or less arbitrarily and adjusted during model calibration.

An open-channel flow model has also been coupled to the groundwater model. The surface flow model simulates flow in networks of open channels by solving the one-dimensional equations of continuity and momentum for river flow [15] as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = \pm\, q \tag{4}$$

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{\partial(\frac{Q^2}{A})}{A\partial x} + g\frac{\partial y}{\partial x} - g(S_0 - S_f) \pm q\frac{u}{A} = 0 \tag{5}$$

where, A = cross-sectional area of channel [L^2], Q = flow rate of water in channel [L^3T^1], x = length along the river centerline [L], y = water depth in river [L], u = average velocity [LT^1], q = inflow (+) or outflow (-) into/ from the river [L^2T^1], S_f = friction slope, S_0 = channel slope, g = acceleration of gravity [LT^2].

In these equations, it is assumed that the flow is one-dimensional. The water flow is varied gradually along the channel so that the hydrostatic pressure prevails and vertical acceleration can be neglected. Furthermore, the bottom slop of the channel is small and the channel bed is fixed; that is, the effects of scour and deposition are negligible. These equations are appropriate for unsteady and nonuniform conditions in the channel.

4. NUMERICAL SOLUTION

When Eq. (2) is included in Eq. (4), the resulting continuity equation can be put in finite-difference form [16]

$$\begin{aligned}&\overline{B}\left[\frac{Z_{i+1}^{j+1} + Z_i^{j+1}}{2\Delta t} - \frac{Z_{i+1}^{j} + Z_i^{j}}{2\Delta t}\right] + \Theta\frac{Q_{i+1}^{J+1} - Q_i^{j+1}}{\Delta x_i} + (1-\Theta)\frac{Q_{i+1}^{j} - Q_i^{j}}{\Delta x_i} + \frac{\chi}{2}\Big[C_{i+1}B_{i+1}^{j+1}\left(Z_{i+1}^{j+1} - h^{j+1}\right) \\ &+ C_iB_i^{j+1}\left(Z_i^{j+1} - h^{j+1}\right)\Big] + \frac{(1-\chi)}{2}\Big[C_{i+1}B_{i+1}^{j}\left(Z_{i+1}^{j} - h^{j}\right) + C_iB_i^{j}\left(Z_i^{j} - h^{j}\right)\Big] = 0\end{aligned} \tag{6}$$

where Δx_i is the length of channel segment from point i to $i+1$, Θ is weighting factor for spatial derivatives, χ is a weighting factor for averaged quantities, $\overline{B}$ is average channel width from the previous time interval

$$\overline{B} = \chi\frac{B_{i+1}^{j} + B_i^{j}}{2} + (1-\chi)\frac{B_{i+1}^{j-1} + B_i^{j-1}}{2} \tag{7}$$

The subscripts indicate location in space, and superscripts indicate the time of occurrence. Equation (6) is solved simultaneously for all nodes with the finite-difference form of the momentum equation in form [16]

$$Q_{i+1}^{j+1} + \gamma z_{i+1}^{j+1} - Q_i^{j+1} + \alpha z_i^{j+1} = \delta \tag{8}$$

where

$$\gamma = \frac{\overline{B}\Delta x_i}{2\theta\Delta t} + \frac{\chi C_{i+1} B_{i+1}^{j+1}\Delta x_i}{2\theta}, \qquad \alpha = \frac{\overline{B}\Delta x_i}{2\theta\Delta t} + \frac{\chi C_i B_i^{j+1}\Delta x_i}{2\theta} \tag{8a}$$

and

$$\delta = -\frac{1-\theta}{\theta}(Q_{i+1}^j - Q_i^j) + \left[\frac{\overline{B}\Delta x_i}{2\theta\Delta t} - (1-\chi)(C_{i+1}B_{i+1}^j)\frac{\Delta x_i}{2\theta}\right]z_{i+1}^j + \left[\frac{\overline{B}\Delta x_i}{2\theta\Delta t} - (1-\chi)(C_i B_i^j)\frac{\Delta x_i}{2\theta}\right]z_i^j$$
$$+\frac{\Delta x_i}{2\theta}\left[\chi(C_{i+1}B_{i+1}^{j+1}h^{j+1} + C_i B_i^{j+1}h^{j+1})\right] + (1-\chi)(C_{i+1}B_{i+1}^j h^j + C_i B_i^j h^j) \tag{8b}$$

This provides a similar form of matrix of the flow equations in the i th segment

$$\begin{bmatrix} 1 & \zeta \\ \gamma & 1 \end{bmatrix}\begin{bmatrix} Z_{i+1}^{j+1} \\ Q_{i+1}^{j+1} \end{bmatrix} - \begin{bmatrix} 1 & -\omega \\ -\alpha & 1 \end{bmatrix}\begin{bmatrix} Z_i^{j+1} \\ Q_i^{j+1} \end{bmatrix} = \begin{bmatrix} \varepsilon \\ \delta \end{bmatrix} \tag{9}$$

where ζ, ω and ε are coefficients in the momentum equation such that

$$\lambda = \frac{\Delta x_i}{2\theta\Delta t g A}, \qquad \mu = \frac{2\beta\overline{Q}}{g\overline{A^2}}, \qquad \sigma = \frac{\chi\Delta x_i \; n^2\left|\overline{Q}\right|}{2\theta\overline{A}^2 R^{-4/3}} \tag{9a}$$

$$\varepsilon = (\lambda - \sigma\frac{(1-\chi)}{\chi})(Q_{i+1} + Q_i^j) + \mu\frac{(1-\theta)}{\theta}\left[Q_{i+1}^j - Q_i^j\right] - \frac{1-\theta}{\theta}\left[Z_{i+1}^j - Z_i^j\right] + \frac{\beta\overline{Q}^2}{\theta g\overline{A}^3}\left[A_{i+1}^{j+1} - A_i^{j+1}\right] \tag{9b}$$

$$\zeta = \lambda + \sigma + \mu, \qquad \omega = \lambda + \sigma - \mu \tag{9c}$$

where n is Manning's roughness coefficient.

Solution of the flow equations requires specification of boundary conditions throughout the duration of the simulation at the physical extremities of the network. Boundary conditions can consist of a zero discharge, known discharge as a function of time, known stage as a function of time, or a known unique stage-discharge relationship.

In order to initiate a solution of the system of equations with the specific boundary conditions, initial values of the unknown quantities are required. These values may be obtained from measurements, computed from some other source, or computed from previous simulations. Successful convergence of the computation to the correct solution requires that the initial values be reasonably accurate; the less accurate the initial values, the longer the computation takes to dissipate the initialization error and converge to the true solution.

In a report by McDonald and Harbaugh [9], the derivation of the finite-difference form of Eq. (1) that is used in MODFLOW follows

$$V_{i,j,k-1/2} - h_{i,j,k-1}^m + U_{i-1/2,j,k}h_{i-1,j,k}^m + R_{i,j-1/2,k}h_{i,j,-1,k}^m + (-V_{i,j,k-1/2} - U_{i-1/2,j,k} - R_{i,j-1/2,k} - R_{i,j+1/2,k}$$
$$-U_{i+1/2,j,k} - V_{i,j,k+1/2} + H_{COEFi,j,k)})h_{i,j,k}^m + R_{i,j+1/2,k}h_{i,j+1,k}^m + U_{i+1/2,j,k}h_{i+1,j,k}^m + V_{i,j,k+1/2}h_{i,j,k+1}^m = RHS_{i,j,k} \tag{10}$$

where i, j, and k are row, column and layer indices, m is time level

$$V_{i,j,k} = \frac{Kz_{i,j,k}\Delta x_i \Delta y_j}{\Delta z_k}\ ,\ U_{i,j,k} = \frac{Kx_{i,j,k}\Delta y_j \Delta z_k}{\Delta x_i},\ \ R_{i,j,k} = \frac{Ky_{i,j,k}\Delta x_i \Delta z_k}{\Delta y_j},$$

$$H_{COEFi,j,k} = P_{ijk} - \frac{S_{Si,j,k}\Delta x_i \Delta y_j \Delta z_k}{t^m - t^{m-1}},\quad RHS_{i,j,k} = -F_{ijk} - \frac{S_{Si,j,k} h_{i,j,k}^{m-1} \Delta x_i \Delta y_j \Delta z_k}{t^m - t^{m-1}}$$

$P_{i,j,k}$ is a head-dependent inflow term, and $F_{i,j,k}$ is the inflow term. The term $F_{i,j,k}$ is the flow rate (L^3T^{-1}) from an external source to the aquifer model cell i, j, k.

The INTRACT model [16] is developed to incorporate channel-bed leakage to and from the aquifer. The only variable in the computation scheme upon which leakage depends is the stage Z. The only input needed from the groundwater model is the aquifer heads h, which are fixed values for the solution of Eq. (9). The feedback of leakage quantity occurring in INTRACT is returned to MODFLOW so it can calculate new values of h (Fig.1). For any MODFLOW time step, the leakage quantities for every time level in INTRACT must be calculated and averaged [16] as follows:

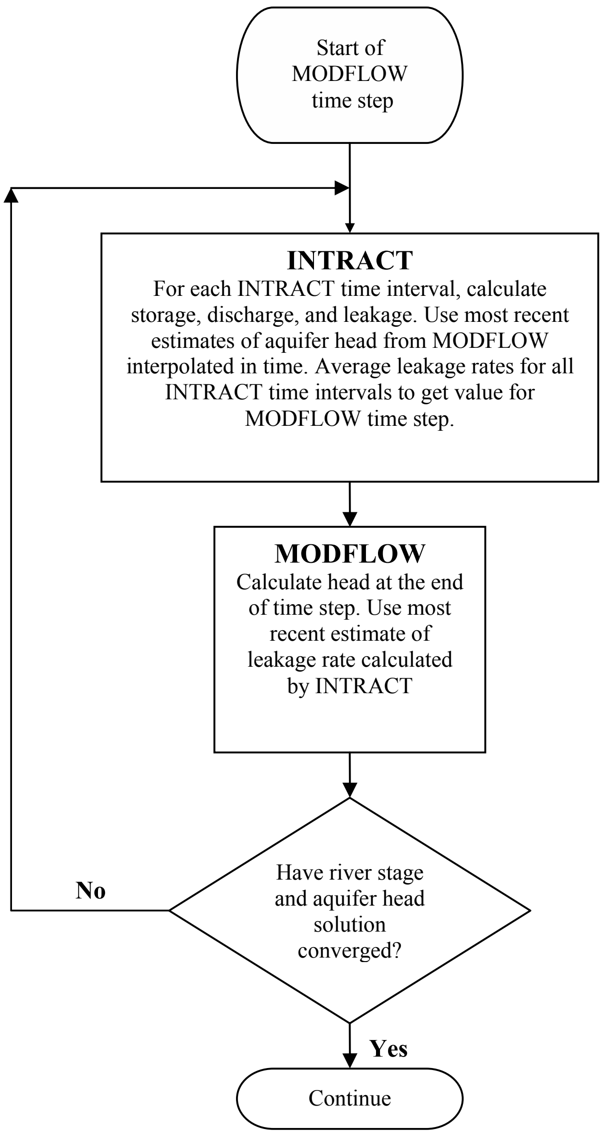

Fig. 1. Iteration procedure between MODFLOW and INTRACT

$$\overline{q\Delta x_i} = \frac{\Delta x_i}{NT}\sum_{j=Ts}^{Ts+NT}\frac{\chi}{2}\left[(C_{i+j}B_{i+1}^{j+1}(Z_{i+1}^{j+1} - h^{j+1}) + C_i B_i^{j+1}(Z_i^{j+1} - h^{j+1})\right] + \frac{1-\chi}{2}\left[(C_{i+1}B_{i+1}^{j}(Z_{i+1}^{j} - h^{j}) + C_i B_i^{j}(Z_i^{j} - h^{j})\right] \quad (11)$$

where T_s is the starting time interval when INTRACT is entered from MODFLOW, and NT is the number of INTRACT time intervals in one MODFLOW time step. Thus, the $\overline{q\Delta x_i}$ term calculated in INTRACT by Eq. (11) for a specific river segment can be passed to MODFLOW and added to the $F_{i,j,k}$ term in Eq. (10) for the aquifer model cell containing the river segment. Values of h are passed from MODFLOW to INTRACT for solving the channel flow Eq. (6), along with the momentum equation for values of Z and Q. After this solution is made iteratively, the leakage rate Eq. (11) is used to determine $\overline{q\Delta x_i}$ for all river segments for the number of INTRACT time intervals within one MODFLOW time step. If multiple INTRACT time intervals occur in one MODFLOW time step, these $\overline{q\Delta x_i}$ values are passed back to MODFLOW and used as the $F_{i,j,k}$ inflow value in the groundwater flow Eq. (10) for the cells interacting with the river segment. Solution to Eq. (10) provides revised values of h to be passed back to INTRACT and the process is repeated. This process is continued until the values of h and Z show no significant change from iteration to iteration, thus signaling the completion of a MODFLOW time step.

The format for entering data into a coupled model is nearly the same as using the MODFLOW model with some modifications. The original input and output instructions were described by McDonald and Harbaugh [9] and the modified format described by Safavi [16].

5. MODEL VALIDATION

In order to validate the INTRACT solution scheme, INTRACT results were compared with results from an analytical solution. Hunt [17] presented an analytical solution for transient drawdown in an infinite uniform aquifer with no flow boundaries along the sides. A line source at distance l from a pumping well such as Fig. 2 exists. The analytical solution of the drawdown as a function of time and space, Φ(*x, y, t*) is given by Hunt as

$$\phi(x,y,t)=\frac{Q_w}{4\pi T}\left\{W\left[\frac{(l-x)^2+y^2}{4Tt/s}\right]-\int_o^\infty e^{-\theta}W\left[\frac{(l+|x|+2T\theta/\lambda)^2+y^2}{4Tt/s}\right]d\theta\right\} \tag{12}$$

where $\lambda[LT^{-1}]$ is a constant of proportionality between the seepage flow rate per unit distance through the streambed and the difference between river and groundwater levels, W is the well function and Θ is the integration variable.

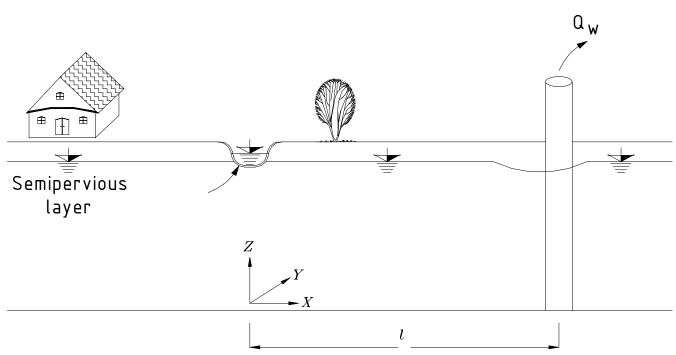

Fig. 2. Definition sketch for the problem considered by Hunt [17]

The set up of the analytical and INTRACT test case used the following parameters:

$$Q_w = 150{,}000\ m^3/yr,\ S = 0.2,\ l = 100\ m,\ T = 0.001\ m^2/s,\ and\ t = 23\ days$$

For a cross section through the well, the drawdown after 23 days is shown in Fig. 3. In Fig. 4, the drawdown is calculated at an observation well halfway between the stream and the well.

The analytical results and the INTRACT results are nearly identical.

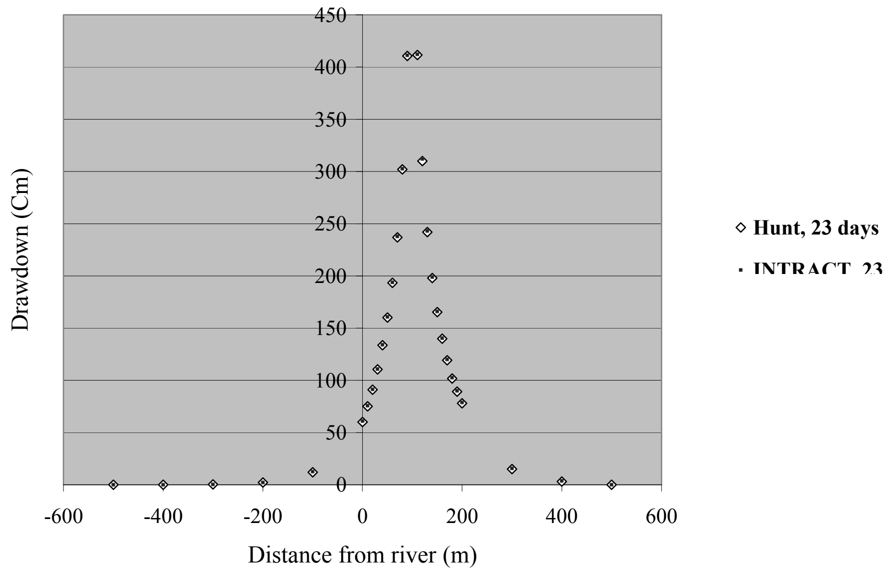

Fig. 3. Comparison of the Hunt analytical drawdown versus INTRACT

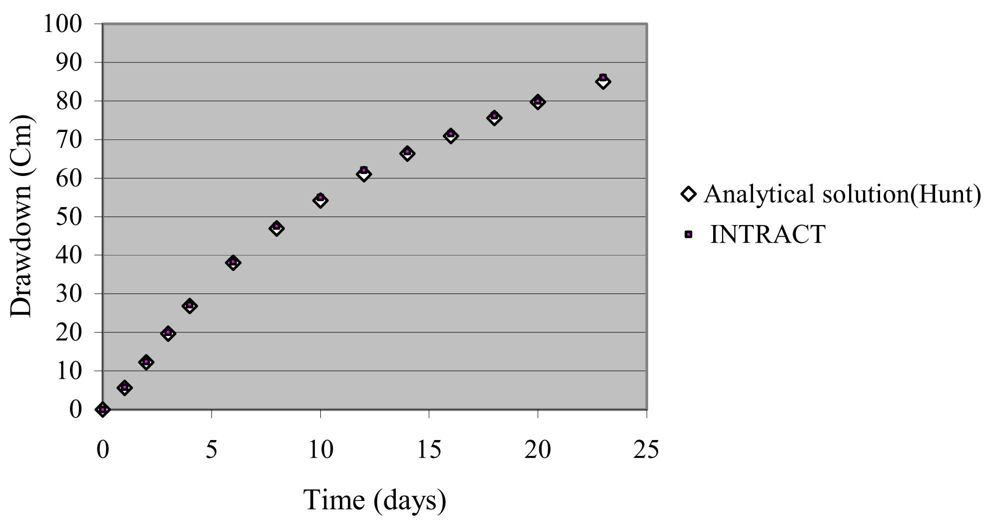

Fig. 4. Transient comparison between analytical solution and INTRACT, half way between the well and the stream

6. CONCLUSIONS

A new coupled surface water and groundwater flow model has been developed by combining MODFLOW and INTRACT to allow the simulation of surface water and groundwater interaction. MODFLOW was originally written with River Package, which calculates leakage between the aquifer and stream, assuming that the stream's stage remains constant during the model stress period. A simple stream flow routing model has been added to MODFLOW, but is limited to steady flow in rectangular, prismatic channels. To overcome these limitations, the INTRACT model, which simulates unsteady, nonuniform flow by solving the St. Venant equations was incorporated into MODFLOW. The new coupled model is most applicable when rapid stream and aquifer changes are modeled in a well-connected system.

Results of the coupled model were compared to the results of previous studies for validating the coupled model. Results of the model are nearly identical to the analytical solutions.

Acknowledgments- The authors wish to thank the deputy of research of the Ministry of Energy (Department of Water Resources), deputy of research of Iran University of Science and Technology, and Department of LAWR at UCDavis for their support.

REFERENCES

1. Pinder, G. F. & Sauer, S. P. (1971). Numerical simulation of flood wave modification due to bank storage effects. *Water Resources Research, 7*(1), 63-70.
2. Zitta, V. L. & Wiggert, J. M. (1971). Flood routing in channels with bank seepage. *Water Resources Research, 7*(5), 1341-1345.
3. Morel-Seytoux, H. J. (1975). Numerical model of flow in a stream-aquifer system: Colorado State University. *Hydrology Paper* No. 74, 73.
4. Hall, F. R., Moench, A. F. (1972). Application of the convolution equation to stream system, Dade County. Florida, U.S. *Geological Survey Water-Resources Investigation Report* 90-4108, 50.
5. Land, L. F. (1977). Computer program documentation user manual-stream flow routing with losses to bank storage or wells: Bay St. Louis Station. *U.S. Survey Computer Contribution Series J*-349, 117.
6. Allen, T. F. H. & Star, T. B. (1982). *Hierarchy perspective for ecological complexity*. University of Chicago Press, Chicago.
7. Kelems, S. V. (1983). Conceptualization and scale in hydrology. *Journal of Hydrology*, 65, 1-23.
8. O'Neill, R. V., DeAngelis, D. L., Waide, J. B. & Allen, T. F. H. (1986). *A hierarchical concept of ecosystems*. Princeton University Press, Princeton.
9. McDonald, M. G. & Harbaugh, A. W. (1988). *A modular three-dimensional finite-difference groundwater flow model*. U. S. Geological Survey Techniques of Water-Resources Investigation Report, Book 6, Chap. A1, 576.
10. Yen, B. C. & Riggins, R. (1991). Time scale for surface-subsurface flow modeling. *Proceedings of the 1991 National Conference of the Irrigation and Drainage Division*, American Society of Civil Engineering, 351-358.
11. Govindaraju, R. S. & Koelliker, J. K. (1994). Applicability of linearized Boussinesq equation for modeling bank storage under uncertain aquifer parameters. *Journal of Hydrology,* Amsterdam, *157*, 349-366.
12. Zlotnik, V. A. & Huang, H. (1999). Effect of shallow penetration and streambed sediments on aquifer response to stream stage fluctuation. *Ground Water, 37*(4), 599-605.
13. Danish Hydraulic Institute. (2001). *MIKE-SHE–An integrated hydrological modeling system*, DHI Inc.
14. Hantush, M. M., Harada, M. & Marino, M. A. (2002). Hydraulics of stream flow routing with bank storage. *Journal of Hydrologic Engineering*, *7*(1) 76-89.
15. French, R. H. (1986). *Open-channel hydraulics*. McGraw-Hill Book Co.
16. Safavi, H. R. (2003). Quality-quantity simulation model for groundwater and surface water interaction. PhD Thesis, Department of Civil Engineering, Iran University of Science and Technology, Tehran, I. R. of Iran.
17. Hunt, B. (1999). Unsteady stream depletion from groundwater pumping. *Ground Water 37*(1), 98-102.