# STRUCTURAL RELIABILITY ANALYSIS USING CHARGED SYSTEM SEARCH ALGORITHM[*]

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**Abstract–** This paper focuses on the design point and the failure probability of problems with continuous random variables. The charged system search (CSS) algorithm is utilized as the optimization tool to achieve minimum reliability index under limit state function. In order to acquire the optimal solution, random variables such as elastic modulus, loads, and geometric parameters are selected as decision variables of the problem which are optimized by means of the CSS algorithm. This algorithm is inspired by the Coulomb and Gauss's laws of electrostatics from physics. In order to evaluate the accuracy and efficiency of this algorithm, several numerical examples are studied and the results are compared to those of the existing methods.
The proposed method is capable of finding a design point over the failure surface and calculates the reliability index with a reasonable accurately. As the proposed framework enforces low computational time and holds a satisfactory convergence rate, it is a competent methodology to calculate different types of reliability problems.

## 1. INTRODUCTION

The well-known reliability analysis refers to the measurement of structural failure probability calculating the limit state functions to identify satisfaction or dissatisfaction of the imposed limitation. In order to perform the reliability analysis of structures, statements such as safety as well as failure probability, which are the complement of reliability, have to be considered. Firstly, we define a limit state function that is compulsory to the abovementioned definitions. While a structure or part of a structure go out of limitation, structure cannot perform their responsibilities, and this specific section is named "a limit state". In the process of increasing the limit state to an expected value, it will be considered as an unreliable structure.

Generally, the methods of probability analysis for the sake of structural reliability can be divided into three main categories:
- The moment methods, say first and second-order reliability methods [1, 2].
- The sampling methods with the important ones being the Monte Carlo simulation and importance sampling [2, 3].
- The optimization methods which will be computed by the meta-heuristic algorithms [4, 5].

Although the utilization of moment methods are simple, the precision of these methods to compute large issues and small values of failure probability or non-linear limit state functions is unsatisfactory. Besides, the sampling methods enforce excessive computational time to calculate the failure probability. In sampling methods such as Monte Carlo Simulation (MCS), the state-limit function can be calculated

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using the random samples generated via probability distribution function (PDF) of each uncertain variable. Consequently, the failure probability can be acquired by dividing the number of samples which will result in a negative value for limit-state function over the total number of samples. Even though the well-known MCS methodology is able to identify the global optimum solution without demanding the analytical formulation of limit-state function, this solution approach necessitates a large number of simulations which will result in an excessive computational time. Heuristic algorithms have found many applications in optimization problems in the last decade. The essence of these algorithms lies in the fact that their capability to converge to a good solution does not depend on the specific search space to which they are applied. The objective function of these solution techniques is the minimum distance of limit-state function from the origin of standard normalized coordinate system.

Recently, a sampling-based methodology is introduced to identify the optimal structures by Alvarez and Hurtado [6]. In [7], the well-known particle swarm optimization technique is employed to determine the optimal design of a ten-bar truss based on a reliability criterion. Reliability-based Design Optimization using Response Surfaces Method is investigated in [8], and structural uncertainties in dynamic behavior prediction of piping systems are studied in [9]. As a new meta-heuristic approach, this article utilizes charged system search algorithm (CSS) for determining structural reliability. The CSS is a population based meta-heuristic optimization algorithm which has been proposed recently by Kaveh and Talatahari [10]. In the CSS, each solution candidate is considered as a charged sphere called a Charged Particle (CP). The electrical load of a CP is determined considering its fitness. Each CP exerts an electrical force on all the others according to the Coulomb and Gauss laws from electrostatics. Then the new positions of all the CPs are calculated utilizing Newtonian mechanics, based on the acceleration produced by the electrical force, the previous velocity, and the previous position of each CP. Many different structural optimization problems have been successfully solved by the CSS.

In this paper, after a brief presentation of the concepts of structural reliability, a concise background of the CSS is provided. Numerical examples are studied to show the efficiency and accuracy of the proposed method. Finally, the paper is concluded by some remarks.

## 2. STRUCTURAL RELIABILITY

Concept of limit state that is used for definition of the failure in reliability analysis, is the boundary between a suitable and unsuitable performance. If limit state function becomes zero or $g(\mathbf{X}) = 0$, limit-state surface, the boundary between the two domains (safe and failure) is conventionally defined. Also, $g(\mathbf{X}) > 0$ defines the safety domain and $g(\mathbf{X}) < 0$ defines the failure domain.

### a) The Cornell reliability index

In First Order Second Moment Methodology (FOSM), the expectation value (i.e. first moment) and the variance (i.e. second moment) of input parameters can be used to deal with the uncertain nature of reliability calculations. To employ the FOSM methodology efficiently, the total number of basic variables should be limited. Also, the failure or the success of basic variables should be known. The Cornell reliability index can be written as follows [1, 2]:

$$\beta = \frac{\mathrm{E}[g]}{\sqrt{\mathrm{Var}[g]}} \tag{1}$$

where, E[g] and Var[g] stand for the mean value and the variance of safety margin, respectively. In the basic formulation of Cornell, the failure function can be calculated as follows:

$$g(R, S) = R - S \tag{2}$$

Where R represents the resistance of structure and S represents the effect of burden. For uncorrelated R and S, the above mentioned reliability index can be rewritten as follows:

$$\beta = \frac{E[R] - E[S]}{\sqrt{Var[R] + Var[S]}} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad or \quad \beta = \frac{\mu_g}{\sigma_g} \tag{3}$$

### *b) The Hasofer-Lind's reliability index:*

According to Hasofer-Lind's definition, the reliability index is equal to the distance between the origin and the closest point to the origin in the limit-state surface [11]. This point is the most probable failure point (MMP or MPFP) and is called the design point holding the maximum of the probability density. Given the large number of basic variables, Hasofer and Lind propose to convert the original basic variables (i.e., *X*) into the standard normalized variables (i.e., *Z*) following normal probability distribution when the mean value and the standard deviation is equal to zero and one, respectively. In the standard normalized space, the distance between each point of the failure surface and the origin of the coordinate system is the same as the distance between a point over the failure surface and the mean point in the unmapped coordinate system. Consequently, the distance between every point of the failure surface and the origin of standard normalized coordinate system is equal to the reliability index in the direction connecting the arbitrary point to the origin of the system coordinate. As defined by Hasofer and Lind, the minimum distance between the failure surface and the origin of standard normalized coordinate system designated as design point (DP) represents the reliability index of the entire structure. In the method of Hasofer-Lind, the uncertain variables follow the normalized standard probability distribution. Besides, Rackwitz and Fiessler present a well designed framework which is capable of converting the non-normal probability distribution function into the normal variables. By substituting the probability density function and cumulative distribution function with the equivalent normal distribution in the design point, the mean and standard deviation can be expressed as follows [12, 13]:

$$\sigma_{x_i'} = \frac{\varphi\left(\Phi^{-1}\left[F_{x_i}\left(x_i^*\right)\right]\right)}{f_{x_i}\left(x_i^*\right)} \tag{4}$$

$$\mu_{x_i'} = x_i^* - \Phi^{-1}\left[F_{x_i}\left(x_i^*\right)\right]\sigma_{x_i'} \tag{5}$$

Converting the design point on each step, the mean and standard deviation of the equivalent normal standard will also be converted.

Considering the aforementioned definitions, the procedure of calculating the reliability index can be treated as an optimization problem. In other words, the objective function is supposed to determine the coordinates of a specific point on failure surface which is closest to the origin in the U space. The mathematical form is as follows:

$$find \quad u = [u_1, u_2, ..., u_n]$$
$$u_i \in U$$
$$to \ minimize : \ \beta(u) = \left(u^T u\right)^{\frac{1}{2}} \tag{6}$$
$$Subject \ to : \ G(u) = 0$$

where $u_i$ is the decision variables which should be selected from Standard Gaussian Space U.

The probability of failure, $P_f$, is equal to probability of bad performance. When all variables have independent normal distribution and limit state is linearized function of the variables, then the failure probability can be computed from [4]:

$$P_f = \int_{-\infty}^{0} \frac{1}{\sigma_g \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{g-\mu_g}{\sigma_g}\right)^2\right] dg = 1 - \Phi(\beta) = \Phi(-\beta) \tag{7}$$

where $\Phi(\beta)$ is a cumulative distribution function and $\beta$ is the safety index or reliability index.

A formidable task in an n-dimensional reliability theory of structure is to calculate the probabilistic multiple integration as [2, 4]:

$$P_f = \text{Prob}[G(X) \le 0] = \int_{G(X) \le 0} f(X)dX \tag{8}$$

where, $P_f$ stands for the failure probability and G(X) represents the state-limit function. X is a random vector representing the parameters of a stochastic structure and following a joint probability distribution function demonstrated by f(X). Also, the domain of the integration is G(X)<0. As finding the accurate value of $P_f$ is almost impossible, prior investigations have introduced several approaches for calculating the approximate value of $P_f$. This paper presents a new competent approach calculating the failure probability as well as the design point.

## 3. OPTIMIZATION ALGORITHM

The Charged System Search contains a number of Charged Particle (CP) where each one is treated as a charged sphere and can insert an electric force to the others. The magnitude of this force for a CP located inside the sphere is proportional to the separation distance between the CPs, and for a CP located outside the sphere is inversely proportional to the square of the separation distance between the particles. The resultant forces persuade the CPs to move towards new locations according the motion laws of Newtonian mechanics. In the new positions, the magnitude and the direction of the forces are reformed and this successive action is repeated until a terminating condition is satisfied. Many different structural optimization problems have been successfully solved by the CSS [14, 15].  The pseudo-code for the CSS algorithm is summarized in the following:

**Level 1: Initialization**

- **Step 1:** *Initialization*. The magnitude of charge for each CP is defined as

$$q_i = \frac{\text{fit}(i) - \text{fitworst}}{\text{fitbest} - \text{fitworst}} \quad i = 1,2,...,N \tag{9}$$

where fitbest and fitworst are the best and the worst fitness of all the particles; fit(i) represents the fitness of the agent i; and N is the total number of CPs. The separation distance $r_{ij}$ between two charged particles is defined as follows:

$$r_{ij} = \frac{\| X_i - X_j \|}{\| (X_i + X_j)/2 - X_{\text{best}} \| + \varepsilon} \tag{10}$$

where $X_i$ and $X_j$ are the positions of the ith and jth CPs, respectively, $X_{\text{best}}$ is the position of the best current CP, and $\varepsilon$ is a small positive number. The initial positions of CPs are determined randomly.

- **Step 2:** *CP ranking.* Considering the values of the fitness function, sort the CPs in an increasing order.
- **Step 3:** *CM creation.* Store a number of the first CPs and the values of their corresponding fitness functions in the Charged Memory (CM).

**Level 2: Search**

- **Step 1:** *The probability of moving determination.* Determine the probability of moving each CP toward the others using the following probability function:

$$p_{ij} = \begin{cases} 1 & \dfrac{\text{fit}(i) - \text{fitbest}}{\text{fit}(j) - \text{fit}(i)} > \text{rand} \vee \text{fit}(j) > \text{fit}(i) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

- **Step 2:** *Forces determination.* Calculate the resultant force vector for each CP as

$$F_j = q_j \sum_{i, i \neq j} \left( \frac{q_i}{a^3} r_{ij} \cdot i_1 + \frac{q_i}{r_{ij}^2} \cdot i_2 \right) ar_{ij} p_{ij} (X_i - X_j) \;, \; \begin{cases} j = 1, 2, \dots, N \\ i_1 = 1, i_2 = 0 \Leftrightarrow r_{ij} < a \\ i_1 = 0, i_2 = 1 \Leftrightarrow r_{ij} \geq a \end{cases} \tag{12}$$

where $F_j$ is the resultant force acting on the jth CP. $ar_{ij}$ is a new parameter, the so-called kind of force and determines the type of the force, where +1 represents for the attractive force and −1 denotes for the repelling force and is defined as

$$ar_{ij} = \begin{cases} +1 & \text{w.p. } k_t \\ -1 & \text{w.p. } 1 - k_t \end{cases} \tag{13}$$

where *"w.p."* stands for "with the probability".

In this algorithm, each CP is considered as a charged sphere with radius *a*, which has a uniform volume charge density.

- **Step 3:** *Solution construction.* Move each CP to the new position and find the velocities as

$$X_{j,new} = \text{rand}_{j1} \cdot k_a \cdot \frac{F_j}{m_j} \cdot \Delta t^2 + \text{rand}_{j2} \cdot k_v \cdot V_{j,old} \cdot \Delta t + X_{j,old} \tag{14}$$

$$V_{j,new} = \frac{X_{j,new} - X_{j,old}}{\Delta t} \tag{15}$$

where $k_a$ and $k_v$ are the acceleration and the velocity coefficients, respectively which can be obtained as follow; and $\text{rand}_{j1}$ and $\text{rand}_{j2}$ are two random numbers uniformly distributed in the range [0..1].

$$k_a = 0.5(1 + \text{iter}/\text{iter}_{max}) \;, \; k_v = 0.5(1 - \text{iter}/\text{iter}_{max}) \tag{16}$$

- **Step 4:** *CP position correction.* If a CP swerves off the predefined bounds, correct its position using the harmony search-based handling approach as described in Kaveh and Talatahari [16].

- **Step 5:** *CP ranking.* Considering the values of the fitness function, sort the CPs in an ascending order.
- **Step 6:** *CM updating.* Include the better new vectors in the CM and exclude the worst ones from the CM.

**Level 3: Terminating criterion controlling**

Repeat the search level steps until a terminating criterion is satisfied. Figure 1 shows the flowchart of the CSS algorithm.
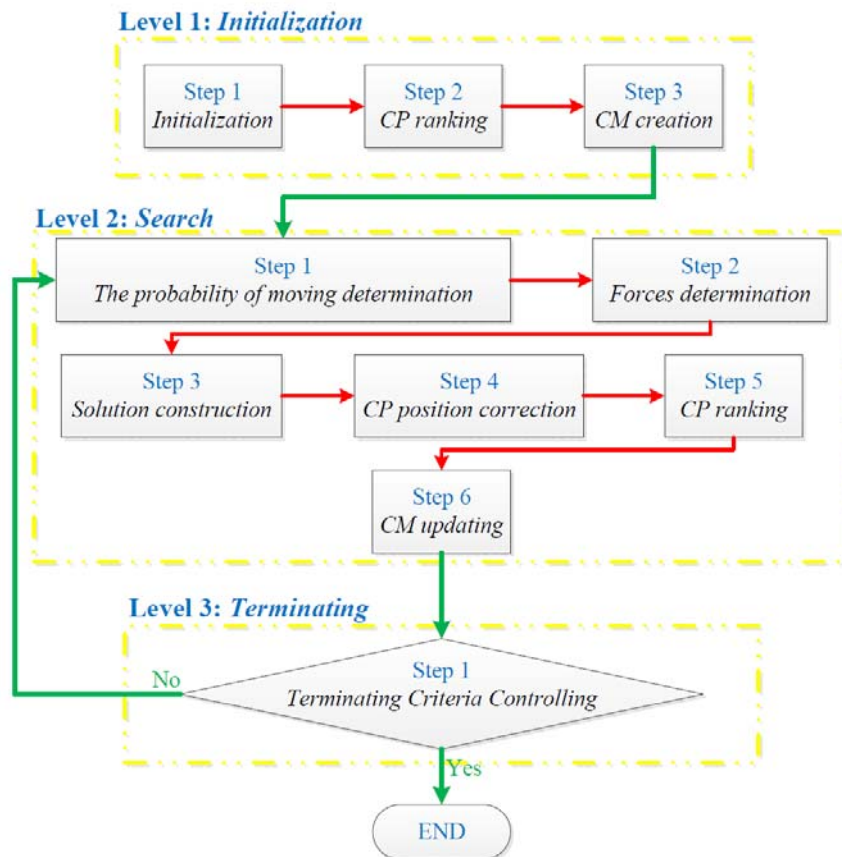


Fig. 1. The flowchart of the CSS algorithm

## 4. NUMERICAL EXAMPLES

In order to show the accuracy and efficiency of the proposed algorithm, in this section several problems are solved from literature and the results are compared. For the examples presented in this paper, the CSS algorithm parameters are set as follows: the number of agents is taken as $4 \times N$ where $N$ is the number of design variables of problem, the maximum number of iterations is set to 200. The algorithms are coded in MATLAB and in order to handle the constraints, a penalty approach is utilized. If the constraints are between allowable limits, the penalty is zero; otherwise the amount of penalty is obtained by dividing the violation of allowable limit to the limit itself.

**Example 1.** In this example, a number of benchmark functions chosen from references are optimized using the CSS and the results are compared. The description of these test problems is provided in Table 1.

Table 1. Comparison of results obtained for the proposed algorithm (using CSS) by different methods

| Limit state function | Variables | Mean | Deviation | Method | Design point in Standard normal space ($U^*$) | Design point in basic space ($X^*$) | β | $P_f$ |
|---|---|---|---|---|---|---|---|---|
| $G(X_1,X_2)=5-0.5(X_1-0.1)^2-X_2$ | $X_1$ $X_2$ | 0 | 1 | Kiureghian and Dakessian [17] | [-2.741,0.965] | - | 2.906 | 0.00183 |
| | | | | PSO-Elegbede [18] | [-2.74163,0.962577] | - | 2.9056 | 0.00183 |
| | | | | CSS (Proposed Method) | [-2.7408,0.9647] | - | 2.906 | 0.00183 |
| $G(X_1,X_2)=exp(0.4(X_1+2)+6.2)-exp(0.3X_2+5)-200$ | $X_1$ $X_2$ | 0 | 1 | Kim and Na [19] | Not Available | - | 2.668 | 0.00382 |
| | | | | MC [19] | Not Available | - | 2.685 | 0.00363 |
| | | | | PSO-Elegbede [18] | [-2.54776, 0.92355] | - | 2.7099 | 0.00337 |
| | | | | CSS (Proposed Method) | [-2.5396, 0.9453] | - | 2.71 | 0.003365 |
| $G(X_1,X_2)=exp(0.2X_1+1.4)-X_2$ | $X_1$ $X_2$ | 0 | 1 | MC [19] | Not Available | - | 3.382 | 0.00038 |
| | | | | CQ2RS [20] | [-1.686, 2.892] | - | 3.47 | 0.000409 |
| | | | | PSO-Elegbede [18] | [-1.688257, 2.893157] | - | 3.4971 | 0.000405 |
| | | | | CSS (Proposed Method) | [-1.6797, 2.8981] | - | 3.35 | 0.000404 |
| $G(X_1,X_2)=18.46154-7.476923\times10^{10}\frac{X_1}{X_2}$ | $X_1$ $X_2$ | 0.001 250 | 0.0002 37.5 | CQ2RS [20] | [0.586, -2.243] | [0.0011172, 165.8875] | 2.318 | 0.010225 |
| | | | | PSO-Elegbede [18] | [0.594605, -2.252998] | [0.001118921, 165.43723] | 2.3309 | 0.009879 |
| | | | | CSS (Proposed Method) | [0.59282,-2.41603] | [0.001,159.39878] | 2.33092 | 0.009878 |
| $G(X_1,X_2,X_3)=X_1-\frac{X_2}{X_3}$ | $X_1$ $X_2$ $X_3$ | 600 1000 2 | 30 33 0.1 | Kim and Na [19] | Not Available | - | 2.232 | 0.01281 |
| | | | | PSO- Elegbede [18] | [-1.483109,0.8773503, -1.47722539] | [555.5067, 1028.95925, 1.8522774] | 2.2697 | 0.01161 |
| | | | | CSS (Proposed Method) | [-1.47971,0.87886,- 1.47971] | [555.60859, 1029.00268, 1.85202] | 2.26969 | 0.01161 |
| $G(X)=1-\frac{\sqrt{3(1-\mu^2)}}{\pi X_1 X_2^2 \cos^2 X_3}\left(\frac{X_6}{2\gamma}+\frac{X_5}{\eta X_4}\right)$ | $X_1$ $X_2$ $X_3$ $X_4$ $X_5$ $X_6$ | 70000 0.0025 0.524 0.9 80000 70000 | 3500 0.000125 0.01048 0.0225 6400 5600 | PSO-Elegbede [18,21] | [-1.7983, -4.1375,0.04372, -0.55845, 1.60866, 0.768705] | [6.3706 ×10¹⁰, 0.0019828, 0.524458159, 0.88743495, 90295.20, 131496.39] | 4.883 | 5.232 × 10⁻⁷ |
| | | | | CSS (Proposed Method) | [-1.76954, -4.04285, 0.39454, -0.65266, 1.57515, 0.75991] | [6.38066×10¹⁰,0.00199, 0.52813, 0.88734, 9.00810×10⁴, 7.42555×10⁴] | 4.79654 | 8.07145×10⁻⁷ |
| $G(X)=2+0.015\sum_{i=1}^{9} x_i^2 - x_{10}$ | $X_1$ $X_2$ $X_3$ $X_4$ $X_5$ $X_6$ $X_7$ $X_8$ $X_9$ $X_{10}$ | 0 | 1 | Der Kiureghian, et. al. [22] | Not Available | - | 2.103 | 5.34×10⁻³ |
| | | | | Grootman [23] | Not Available | - | - | 5.6×10⁻³ |
| | | | | CSS (Proposed Method) | [-1.82916×10⁻⁴, -1.17220×10⁻⁴, 1.09224×10⁻⁴, 9.93216×10⁻⁴, 0.00266, 2.32360×10⁻⁴, -8.21103×10⁻⁴, 4.96147×10⁻⁵, 0.00345, 2.00000] | - | 2 | 2.27498×10⁻² |
| $G(X)=3-x_2+(4x_1)^4$ | $X_1$ $X_2$ | 0 | 1 | Au and Beck [24] | Not Available | - | 2.925 | 1.8×10⁻⁴ |
| | | | | Jorge and Diego [6] | Not Available | - | - | 1.77×10⁻⁴ |
| | | | | Grootman [23] | Not Available | - | - | 2.03×10⁻⁴ |
| | | | | CSS (Proposed Method) | [0.00000,3.00000] | - | 3 | 1.34989×10⁻³ |
| $G(X)=2-x_2-0.1x_1^2+0.06x_1^3$ | $X_1$ $X_2$ | 0 | 1 | Der Kiureghian, et. al. [22] | Not Available | - | 1.996 | 3.47×10⁻² |
| | | | | Grootman [23] | Not Available | - | - | 3.58×10⁻² |
| | | | | CSS (Proposed Method) | [-4.17286×10⁻⁸,2.00000] | - | 2 | 2.27501×10⁻² |
| $G(X)=x_1x_2-146.14$ | $X_1$ $X_2$ | 78064.4 0.0104 | 11709.7 0.0156 | Engelund and Rackwitz [25] | Not Available | - | 5.443 | 1.46×10⁻⁷ |
| | | | | Grootman [23] | Not Available | - | - | 1.11×10⁻⁷ |
| | | | | CSS (Proposed Method) | [-1.56957,-5.09710] | [5.96852×10⁴, 0.00244] | 5.33329 | 4.82228×10⁻⁸ |

**Example 2.** The structure studied in this example is a cantilever beam with rectangular cross-section and is subjected to a concentrated load at the end of beam, as shown in Fig. 2. Limit state function in this example is given by Eq. (17). Variables $F_y$ and $Z$ are normal variables with mean and standard deviations ($F_y$: 0.32, 0.032) kN/mm² and (Z: $1400\times10^3$, $70\times10^3$) mm³, respectively. The variable $P$ is the lognormal variable with mean value equal to 100 kN and standard deviation 40 kN. Length of the beam is constant with L=2 m. In Table 2, the optimal design point obtained by the CSS is compared to those of the previous works.

$$G(X)= ZF_y - PL \tag{17}$$

The state-limit function is relatively linear and the results of our proposed methodology are similar to those of the previous investigators. The convergence history of CSS is illustrated in Fig. 3.

Table 2. Comparison of results obtained for CSS (proposed algorithm) by different methods

| Method | Design point in standard normal space | Design point in basic space | β | $P_f$ |
|---|---|---|---|---|
| [2] Page 200 | $U^*$=(-0.264,-0.126,-0.956) | $x^*$=(0.3149, 1.381×10⁶, 208.2) | 2.192 | 0.0142 |
| **CSS (proposed algorithm)** | **$U^*$=(-0.57728,-0.27578, 2.09560)** | **$x^*$=(0.30152,1.38069×10⁶, 208.15846)** | **2.19109** | **0.01422** |



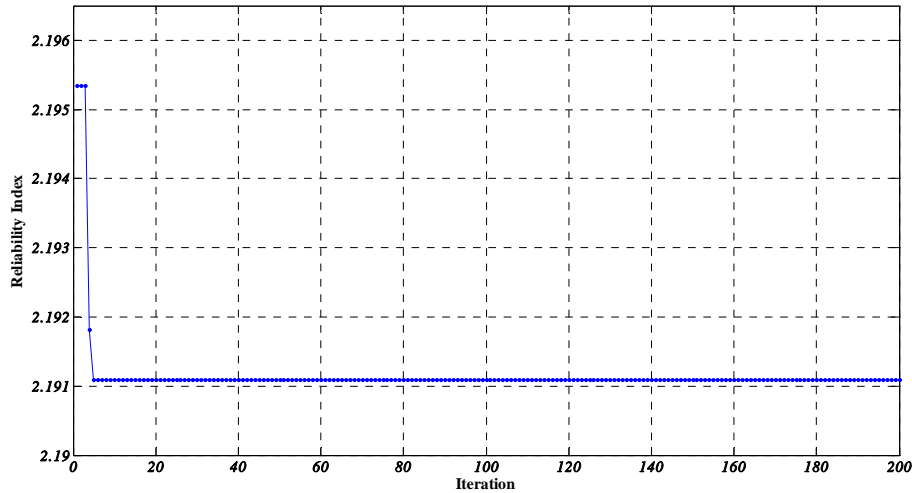Fig. 2. Cantilever beam of Example 2

Fig. 3. Convergence history of the CSS for Example 2

**Example 3.** The structure studied in this example is a beam as shown in Fig. 4. Limit state on the displacement is given by Eq. (18) which is simplified to limit state function (19). Variables $E$ and $I$ are normal variables with mean and standard deviation (E: $2 \times 10^7$, $0.5 \times 10^7$) kN/m$^2$ and (I: $10^{-4}$, $0.2 \times 10^{-4}$) mm$^4$, respectively. Variable $P$ is extreme value distribution type-I variable with mean 4 kN and standard deviation 1 kN. L is deterministic parameter that equals L=5 m. In Table 3, the optimal design point obtained by the CSS is compared to those of the previous works.

$$d = \frac{5PL^3}{48EI} \leq \frac{L}{30} = d_{max} \tag{18}$$

$$G(X) = EI - 78.12P \tag{19}$$

The state-limit function is relatively linear and the results of our proposed methodology are similar to those of the previous investigators. The convergence history of CSS is illustrated in Fig. 5.
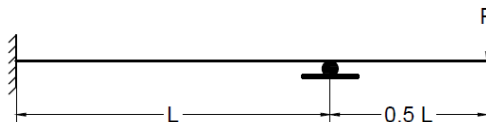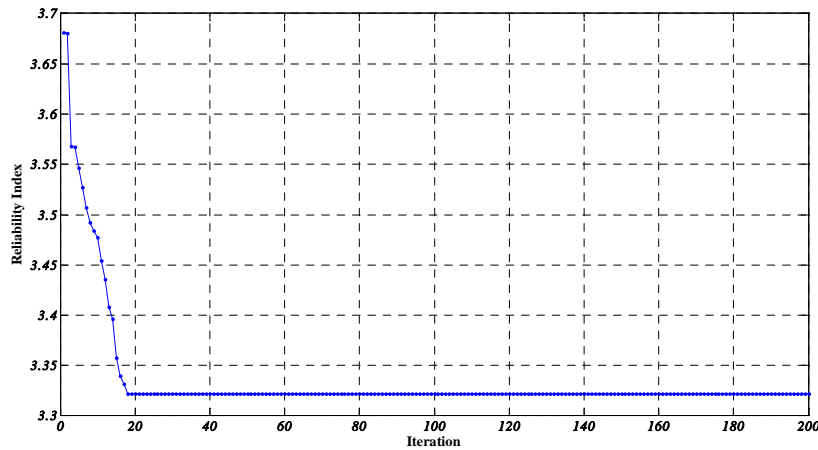


Fig. 4. Beam of Example 3



Fig. 5. Convergence history of the CSS for Example 3

Table 3. Comparison of results obtained for CSS (proposed algorithm) by different methods

| Method | Design point in standard normal space | Design point in basic space | β | $P_f$ |
|---|---|---|---|---|
| [4] Page 104 | U*=(-3.202,-0.5914, 0.659) | x*=($3.9897×10^6$, $0.8817×10^{-4}$, 4.5035) | 3.3222 | $4.4655×10^{-4}$ |
| **CSS (proposed algorithm)** | **U*=(-3.2084,-0.57378, 0.6425)** | **x*=($3.95785×10^6$, $0.88524×10^{-4}$, 4.48497)** | **3.32207** | **$4.46759×10^{-4}$** |

## 5. CONCLUSION

The CSS algorithm is very efficient in solving global optimization problems with continuous variables. Regarding the reliability evaluation of structures, the CSS is capable of finding the optimal design point as well as the failure probability corresponding to the MMP. Simulation results demonstrate the failure probability may be overestimated, meaning a wasteful and inefficient design, when the reliability index is less than its real value. In contrast, the failure probability may be underestimated when the reliability index is more than its real value. In the case of valuable and large structures, a small change in the calculated reliability index will result in a large change in the calculated value of the failure probability. Our proposed approach is capable of finding a design point over the failure surface and calculates the reliability index accurately. As the proposed framework enforces low computational time and holds a satisfactory convergence rate, it is a competent methodology to tackle different types of reliability problems.

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