THREE DIMENSIONAL BEARING CAPACITY ANALYSIS OF GRANULAR SOILS, REINFORCED WITH INNOVATIVE GRID-ANCHOR SYSTEM[*]

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Abstract– Reinforcing soils with biaxial geogrids have been shown to be an effective method for improving the ultimate bearing capacity of granular soils. The pull-out resistance of reinforcing elements is one of the most significant factors in increasing bearing capacity. In this research a new reinforcing element that includes attaching elements (anchors) to ordinary geogrid for increasing the pull-out resistance of reinforcements is introduced. Reinforcement therefore consists of a geogrid and anchors with cubic elements attached to it, named (by the authors) Grid-Anchor. Three-dimensional numerical study was performed to investigate the bearing capacity of square footing on sand reinforced with this system. The effect of depth of the first reinforcement layer, the vertical spacing, the number and width of the reinforcement layers, the angle of anchors, the stiffness of reinforcement and anchors and the distance that anchors are effective were investigated. Three-dimensional finite element analysis by "PLAXIS 3D Tunnel" software, indicated that when a single layer of reinforcement is used there is an optimum reinforcement embedment depth for which the bearing capacity is greatest (u=0.5B). There also appeared to be an optimum vertical spacing of reinforcing layers for multi-layer reinforced sand (h=0.25B). The bearing capacity was also found to increase with increasing the number of reinforcement layers, if the reinforcement was placed within a range of effective depth (d=1.25B). In addition, analysis indicated that increasing reinforcement and anchor stiffness beyond a threshold value does not result in further increase in the bearing capacity (1kN for anchors and 100 kN/m for geogrid). Results show that the Grid-Anchor system of reinforcing can increase the bearing capacity 2.74 times greater than that for ordinary geogrid and 4.43 times greater than for non-reinforced sand.

Keywords– Geosynthetics, numerical analysis, finite element, bearing capacity, square footing, reinforced sand, pull-out resistance, grid-anchor, BCR

1. INTRODUCTION

The use of geosynthetics in civil engineering projects is growing rapidly. One of these applications is the construction of a reinforced soil foundation to increase the bearing capacity of shallow spread footings. More recently, results of several numerical studies, laboratory and large scale tests to determine the ultimate bearing capacity of square shallow foundations supported by sand reinforced with layers of geogrid have been published in the literature [1-11].

In this paper a new reinforcement element to improve the bearing capacity of soils (patented in I.R. of Iran) has been introduced and numerically studied. The main idea behind the new system is adding anchors to ordinary geogrids, and it has been named Grid-Anchor (it is not a trade name yet). Figure 1 shows a schematic arrangement of the system. In this figure a foundation of width B that is supported by the soil reinforced with Grid-Anchor is shown; the anchors are made from 10×10×10 mm cubic elements

mounted on elastic strips that are attached with specific angles to the grids. This dimension can be used both in the laboratory or in the field. Both anchors and grids are made from high density polyethylene (HDPE). There are "N" layers of Grid-Anchor, each having width "b" (Fig. 1). The top layer of the Grid-Anchor is located at a depth "u" below the bottom of the foundation. The distance between consecutive Grid-Anchor layers is equal to "h" and the total depth of reinforcement is equal to "d= u+(N-1)h ". The effective width of anchoring with an angle of "α" is "c".
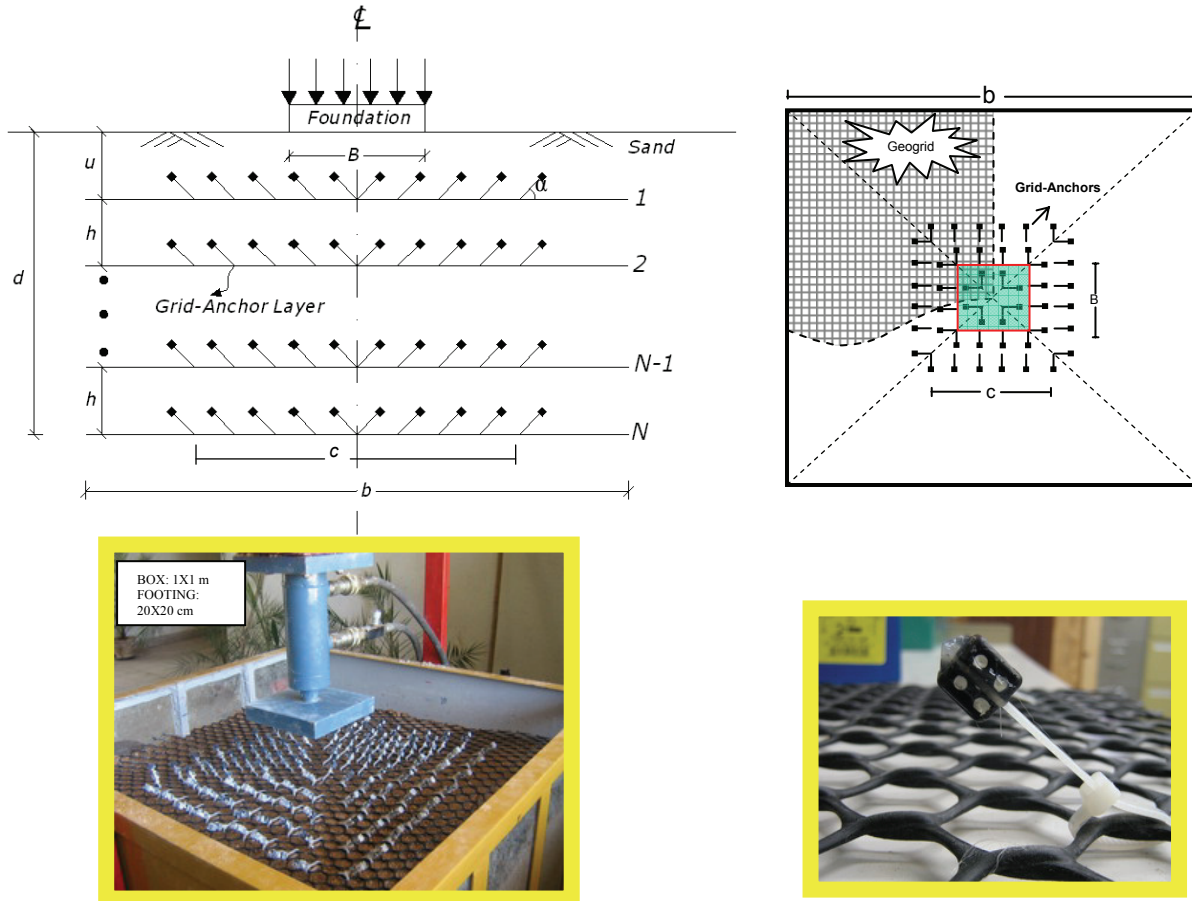

Fig. 1. Schematic model and photos of the square foundation on Grid-Anchor reinforced soil

The increase in the ultimate bearing capacity can be expressed in a non-dimensional form called bearing capacity ratio (*BCR*), defined as:

$$BCR = q_{u(R)}/q_u \tag{1}$$

where q_u = ultimate bearing capacity of un-reinforced soil and $q_{u(R)}$ = ultimate bearing capacity of reinforced soil.

It has been shown in the past that, for given values of b/B, h/B and u/B, there is a critical reinforcement depth ratio, $d/B=(d/B)_{cr}$, at which the bearing capacity ratio practically reaches a maximum value [3,4].

Similarly for given values of d/B, h/B and u/B there is a critical non-dimensional reinforcement width ratio, $b/B=(b/B)_{cr}$, at which the magnitude of BCR also reaches a maximum value. Most of the published studies on this topic have been directed towards evaluation of $(b/B)_{cr}$, $(d/B)_{cr}$, and the variation of BCR with u/B and h/B (for given values of h/B and u/B).

In this paper the results of some 3D numerical finite element analysis on the proposed Grid–Anchor using "PLAXIS 3D Tunnel" software [12], have been presented.

Square footing was selected to minimize the dimensional effects (L/B).

2. BEHAVIOR OF SURFACE SQUARE FOUNDATION ON REINFORCED SOIL

Akinmusuru and Akinbolade [1], Guido et al. [2], Yetimoglu et al. [3], Omar et al. [4], Ismail and Raymond [5], Hataf and Baziar [6], Chung and Cascante [7], Binesh et al. [8], Esmaili and Hataf [9] and Mosallanezhad et al. [10] have performed several experimental comprehensive tests on pilot scale and Adams and Collin [11] have performed such investigation on full scale square footing on Reinforced soil. Different materials such as geotextiles and geogrids were used for reinforcement. In almost all of these researches the depth of the first layer of reinforcement (u), the vertical distance between the layers (h), and the number of reinforcement layers (N) have been changed and the effects of those changes on the bearing capacity of soil have been investigated.

In all the cited studies, the BCR was found to be greater than unity for reinforced soil, if the reinforcement is placed within the effective depth (this ratio variety is between 1.5 in Omar et al. [4] and 2.5 in Yetimoglu et al's. [3] research).

Akinmusuru and Akinbolade [1] conducted their tests on square footing (width=100 mm) and used woven strips as reinforcing elements. They found that for the ratio u/B between 0.5 and 1.0, the increase in BCR was considerable. The optimum value of N was found to be 3.

Guido et al. [2] used square footing (width=310 mm) and geotextile for reinforcing their tests. They found that the optimum value of N was also 3 and the optimum value for the width of reinforcement (b) was equal to 2.5 B, where u/B=0.5. Yetimoglu et al. [3] found that the critical value of u/B, b/B and h/B were equal to 0.25, 4.5 and 0.2 respectively. Omar et al. [4] used Geogrid for Reinforcement and found that the critical value of u/B, b/B and d/B were equal to 1.0, 4.5 and 1.4, respectively. They also found that BCR was between 2.5 and 4.0. Ismail and Raymond [5] used the finite element method (FEM) to determine the load–displacement responses of strip footings on reinforced layered soils. They reported an optimum ratio u/B = 0.31 in the two layer deposit; the reinforcement was more beneficial to the uniform soil deposit than to the two-layer deposit. Adams and Collin [11] reported the critical u/B ratio equal to 0.48. They obtained the critical range for h/B from 0.25 to 1.5 for N=3. Hataf and Baziar [6] used square footing (width=140 mm) and strips from waste tire materials for reinforcement. They found the average critical values of h/B and u/B equal to 0.46 and 0.315, respectively. Chung and Cascante [7] presented results of laboratory testing and numerical simulations on the effect of reinforcement on the low-strain stiffness and bearing capacity of shallow square footing on dry sand. Laboratory tests showed an increase of 100% to 300% not only in bearing capacity, but also in low-strain stiffness. Numerical simulations demonstrated that if reinforcements are placed up to a depth of one footing width (B) below the foundation, better re-distribution of the load to deeper layers was achieved. Numerical simulations and experimental results clearly identify a critical zone between 0.3B and 0.5B, where maximum benefits not only on the bearing capacity but also on the low-strain stiffness of the foundation were obtained.

Mosallanezhad et al. [10] used square footing to investigate the bearing capacity of square footing (width=200mm) on soil reinforced with Grid-Anchor and attempted to verify this system with the ordinary system of reinforcing with geogrid. Test results indicated that the use of Grid-Anchor reinforced sand foundations may increase the ultimate bearing capacity of shallow square footing by a factor of 3.0 rather than un-reinforced sand, and 1.8 rather than ordinary geogrid reinforced sand.

As outlined above, a number of parametric studies have been reported for soil reinforced with traditionally used products for reinforcing. In the present investigation the capability of Grid–Anchor

system as a reinforcing element to increase BCR for square footing is numerically evaluated, presented and compared with that for ordinary geogrid.

3. NUMERICAL ANALYSIS

The finite element program "PLAXIS 3D Tunnel" (version 2.0) was used to model the square footing on reinforced sand. The "PLAXIS 3D Tunnel" program is a geotechnical finite element package specifically intended for three-dimensional analysis of deformation and stability of tunnels, but can generally be used to analyze any geotechnical engineering project.

The simple graphical input procedure enables a quick generation of complex finite element models, and the enhanced output facilities provide a detailed presentation of the computational results. The calculation itself is fully automated and based on robust numerical procedures.

The Mohr–Coulomb model was used for soil and 15-node wedge elements were used for analysis. Reinforcement layers and anchors were modeled using the option already built into the program. They are simulated in "PLAXIS" by the use of special only tension elements with no bending stiffness. When 15-node soil elements are employed, each reinforcement element is defined by 8-Nodes with three degrees of freedom in each node (U_x, U_y, U_z).

The only property in a geogrid dataset is the elastic axial stiffness, EA, entered in units of force per unit width. Geogrid elements in PLAXIS only sustain tension. A fixed-end anchor is a one–node elastic spring element with a constant spring stiffness that is used to model the tying of a single point. They must always be connected to existing geometry lines such as reinforcement, modeled linearly in software. The major anchor property is the axial stiffness, EA; entered in the unit of force. Other parameters used in the analysis are tabulated in Table 1.

Table 1. Parameters used in the analysis

Parameter	Value
Angle of internal friction	40°
Cohesion (kPa)	1
Modulus of elasticity (kPa)	12000
Poisson's ratio	0.3
Unit weight (kN/m^3)	18.6
Soil type	Sand
Soil model	Mohr–Coulomb
Axial stiffness of reinforcement (kN/m)	28
Axial stiffness of anchors (kN)	0.18
Dimension of footing (mm)	200×200
Length of anchors (mm)	50
Model box (m)	1×1×1

The 3D and sectional 2D finite element mesh used for analyses is shown in Fig. 2 and Fig. 3, respectively. A 200×200 mm footing was selected (EI=8500 kNm^2/m, EA=5×10^6 kN/m, thickness=100 mm) for modeling rigid foundation.

For avoiding boundary effects, a model box 1×1×1 m was selected. The model depth was taken 1 m, that is greater than 4B=0.8 m (Prisco et al. [13]) and the width of the model was taken 1 m, i.e. 5B. As in practice in the field, the ends of the geogrid layers are free in x, y and z directions at the boundaries. Figure 4 shows that the displacement has not reached the boundaries in the analysis.

The horizontal part of stress vs. settlement diagram implies that q_u has been reached. To see the mesh refining effect, we refined mesh by trial and found that more refinement had no effect on the results.
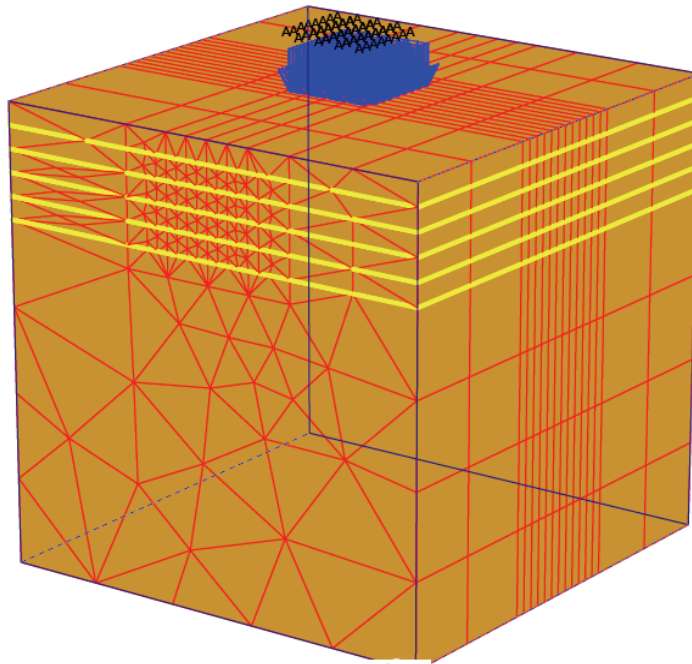

Fig. 2. 3D finite element mesh generation for 5 layer reinforcement
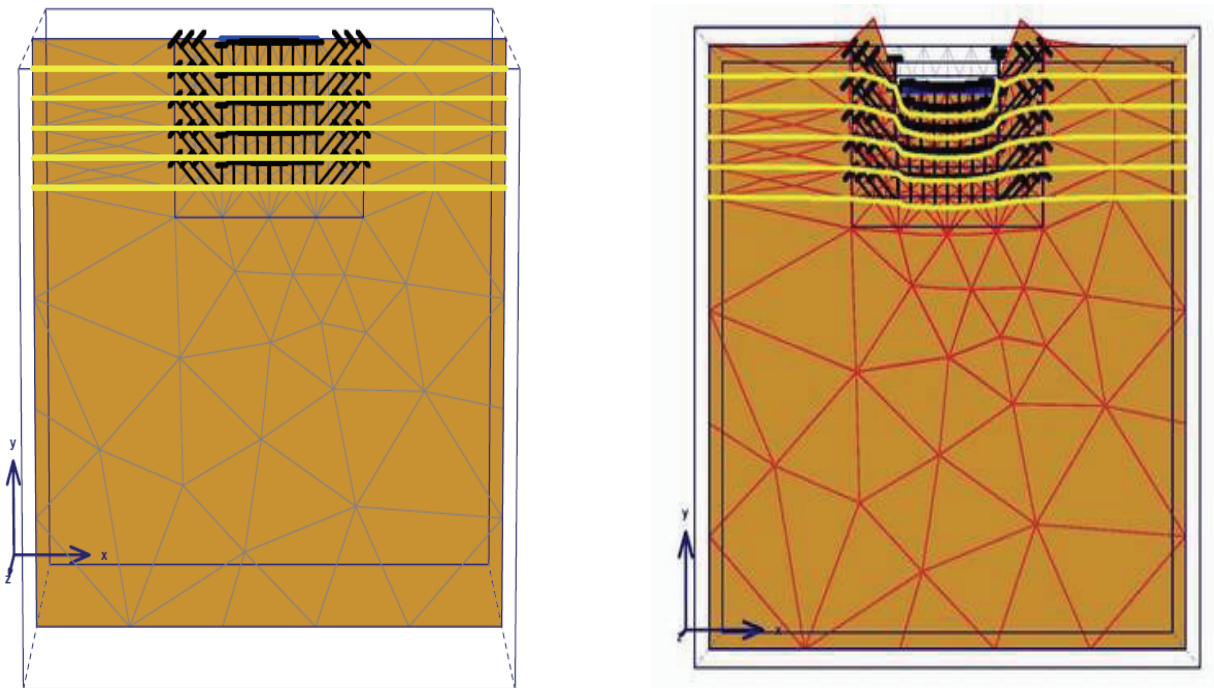with 5472 elements (average element size=13 mm)

Fig. 3. Undeformed and deformed mesh at section plane across
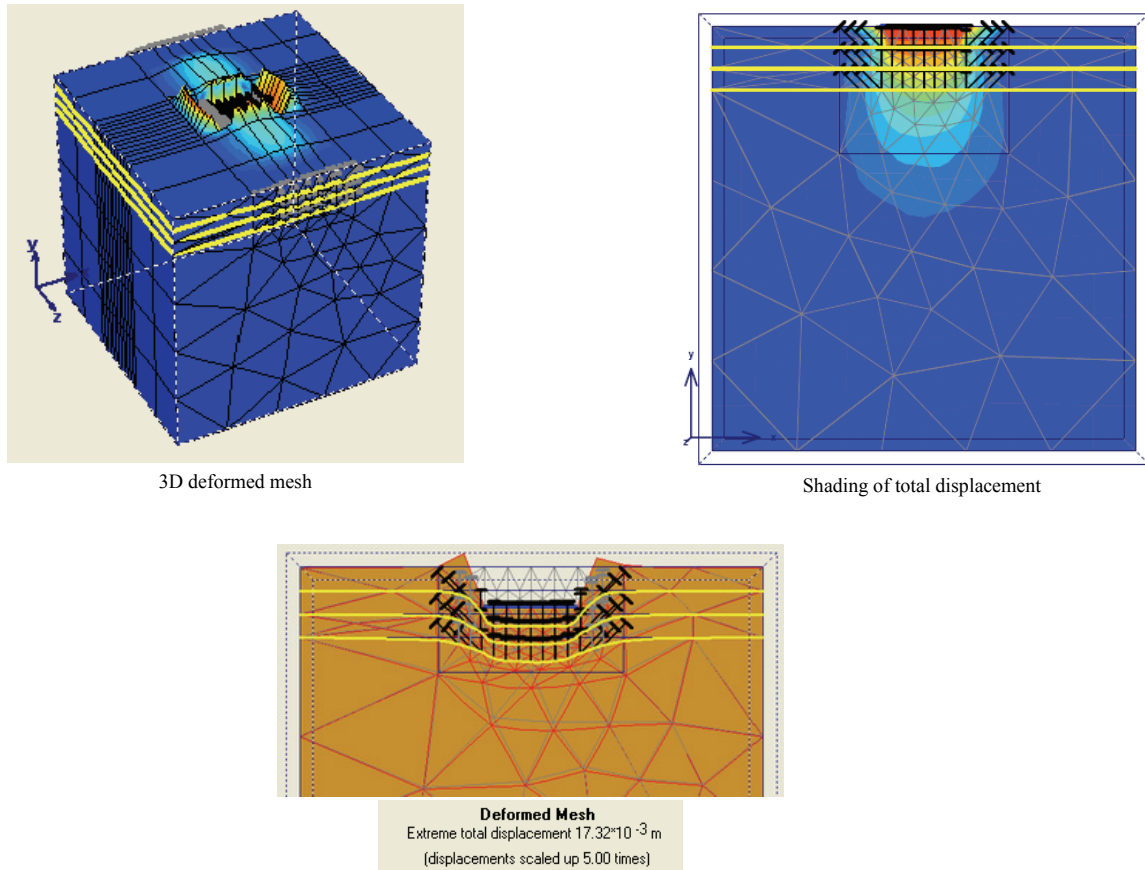the footing center for 5 layer

3D deformed mesh

Shading of total displacement

Deformed Mesh
Extreme total displacement 17.32*10 $^{-3}$ m
(displacements scaled up 5.00 times)

Fig. 4. Selected boundaries and displacement under footing for 3 layer

4. VERIFICATION OF EFFECTIVENESS OF GRID-ANCHOR SYSTEM

To verify the effectiveness of Grid–Anchor system in improving the bearing capacity of soils, the behavior of footings on non-reinforced sand, sand reinforced with ordinary geogrid and sand reinforced with Grid-Anchor under the same conditions (soil properties, N, b/B, h/B, u/B, anchor and reinforcement stiffness) were investigated. Figure 5 shows the comparison between these three statuses. The effectiveness of using Grid-Anchor system as reinforcement elements in increasing the bearing capacity of footings is evident. As can be seen, the value of BCR is almost three times greater than that for ordinary geogrid in this case.
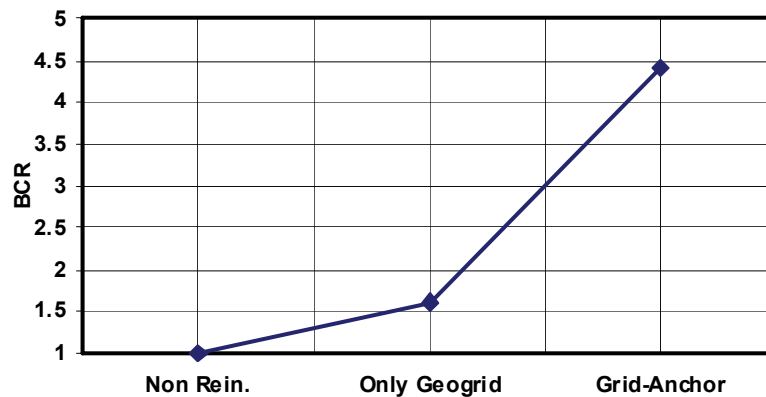

Fig. 5. Variation of BCR with selected reinforcing system
(N=5, u/B=0.25, b/B=5, h/B=0.25)

5. PARAMETRIC STUDIES

In order to determine the effect of different factors on the bearing capacity of sand reinforced with the introduced system, analysis has been performed and the results are presented as follows.

a) Effect of depth to the first reinforcement layer (u)

The depth ratio is defined herein as the ratio between u and B (i.e. u/B). Figure 6 shows the typical relationship between BCR and the depth ratio for single–layer reinforced sand, while Fig. 7 shows it for multi-layer reinforcement. As can be seen, in single–layer reinforcement there was an optimum value of depth ratio at which the BCR was the highest; this optimum value is around 0.5B. The analysis for the multi-layer reinforced sand indicated that the largest BCR values occurred at a depth ratio of around 0.25B.

This is in agreement with the numerical studies which indicated that the effect of depth ratio on the BCR in single–layer reinforced sands was different from that in multi layer reinforced sands [14, 2, 3, 11].
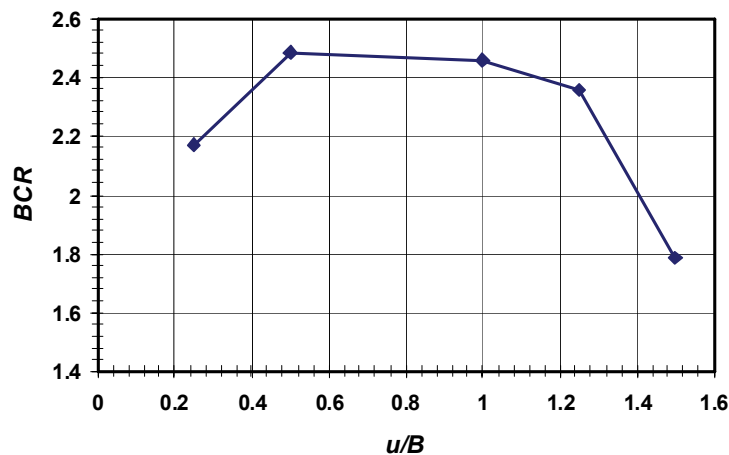

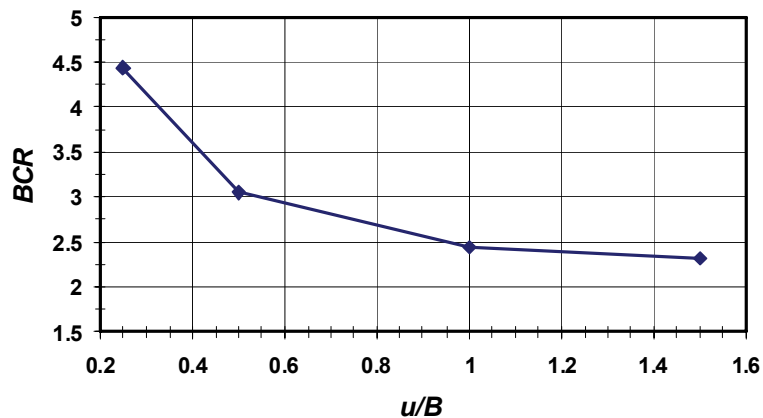Fig. 6. Variation of BCR with depth ratio in single layer reinforced sand (b/B)= 5

Fig. 7. Variation of BCR with depth in multi-layer reinforced sand
(N= 5, h/B =0.25, b/B=5)

b) Effect of number of reinforcement layers (N)

Numerical analysis indicated that the BCR value changes more drastically with the number of reinforcement layers, N, than the other parameters.

The BCR increased with increasing the number of reinforcement layers within a depth of 1.5B and the rate of increase in BCR was less significant beyond this depth; in other words, placing geogrid reinforcement beyond a depth of 1.5B would not significantly increase the bearing capacity.

Figure 8 shows a typical variation of BCR with the number of reinforcement layers. For the results shown, the vertical spacing (h) was kept constant. Several researchers have also observed that increasing the number of reinforcement layers beyond a certain value would not increase the BCR significantly ([1, 3, 4, 15]).
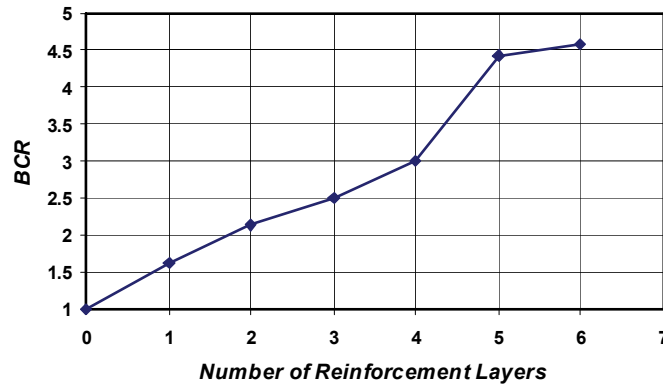

Fig. 8. Variation of BCR with number of reinforcement
layers (u/B=0.25, h/B=0.25, b/B=5.0)

c) Effect of vertical spacing of reinforcement (h)

Numerical studies have indicated that there was an optimum value for the vertical spacing of horizontally placed reinforcement layers. The analysis showed that the optimum vertical spacing was 0.25B for three-layer reinforced sand. Figure 9 shows a typical variation of BCR with the normalized vertical spacing of three-layer reinforced sand.
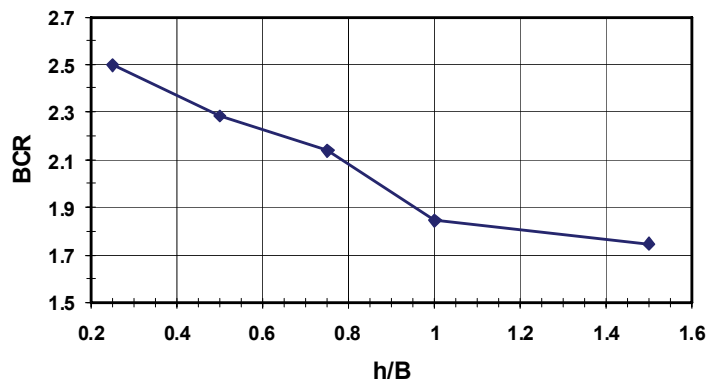

Fig. 9. Typical variation of BCR with vertical spacing of
reinforcement layers (N=3, u/B=0.25, b/B=5)

Similar to these findings, Singh [16] indicated that optimum vertical spacing of reinforcement layers for square footing on geogrid reinforced sands varied between 0.15B and 0.25B. On the other hand, Omar et al. [4], Hataf and Baziar [6] and Yetimoglu et al. [3] found this optimum value as 0.33, 0.46 and 0.2 respectively.

d) Effect of reinforcement width (b)

Numerical studies have shown that the BCR is generally increased with increasing reinforcement width (b). The bearing capacity ratios obtained from these analyses have been plotted with b/B in Fig. 10.

It can be seen from this figure that the BCR attains a maximum at a (b/B)$_{cr}$ of 4.75 for 5 layers and about 3 for 3 layers reinforcement.
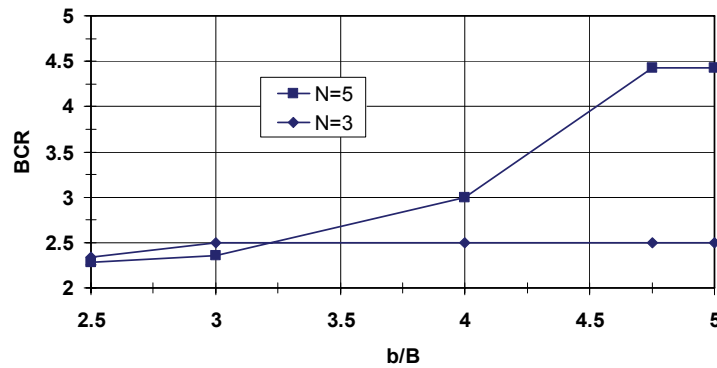

Fig. 10. Variation of BCR with reinforcement width (u/B=0.25, h/B=0.25)

Hence, the investigation on these two specified numbers of layers show that the optimum normalized reinforced width (b/B)$_{cr}$ increases with the increasing number of reinforcement layer (N).

Similarly, Omar et al. [4], based on their study of square footing on geogid-reinforced sand, indicated that the BCR increased with reinforcement width up to an effective value (b/B)$_{cr}$ and remained practically constant thereafter. They reported that the effective b/B ratio was around 4.5 for square foundations.

Several researchers have also indicated that increasing reinforcement width beyond a certain value would not increase the BCR significantly. Guido et al. [15] and Singh [16] advocated that the optimum ratio of b/B was approximately 2 for square footing. Fragaszy and Lawton [17] reported that as reinforcing size increased from 3 to 7 times the footing width, the BCR increased rapidly.

e) Effect of reinforcement stiffness

In the analysis, reinforcement stiffness, "EA" varied from 10 kN/m to 10000 kN/m. Typical variation of BCR with the reinforcement stiffness for one layer reinforcing is shown in Fig. 11.

It is seen that increasing reinforcement stiffness beyond an axial stiffness of approximately 100 kN/m would not result in a significant increase in BCR for single layer reinforcement. This is because of the fact that before the axial stiffness of reinforcement is fully mobilized, geogrid is pulled out and the soil body will slide over the reinforcement.
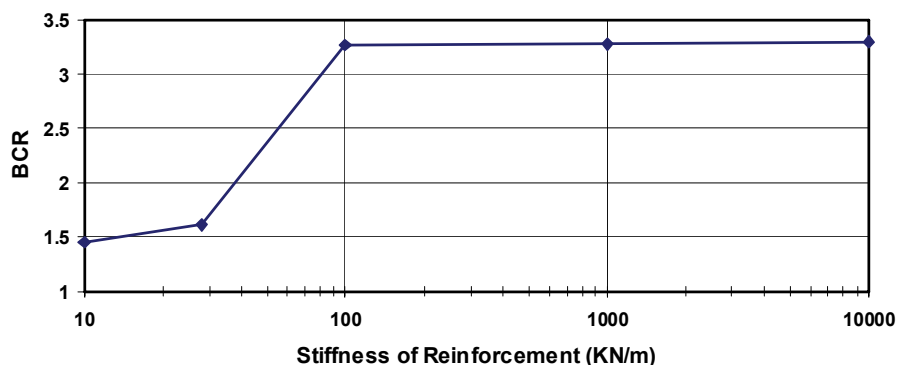

Fig. 11. Variation of BCR with reinforcement stiffness (N=1, u/B=0.25, b/B=5)

f) Effect of anchors stiffness

The fixed-end anchor which is a one-node elastic spring element with a constant spring stiffness tied to a single point was used to model the anchorage system. The major anchor property is the axial stiffness,

"EA". The anchor stiffness was tested and found to be 0.25 kN at most. In the analysis, the axial stiffness of the anchors varied from 0.05 kN to 10 kN.

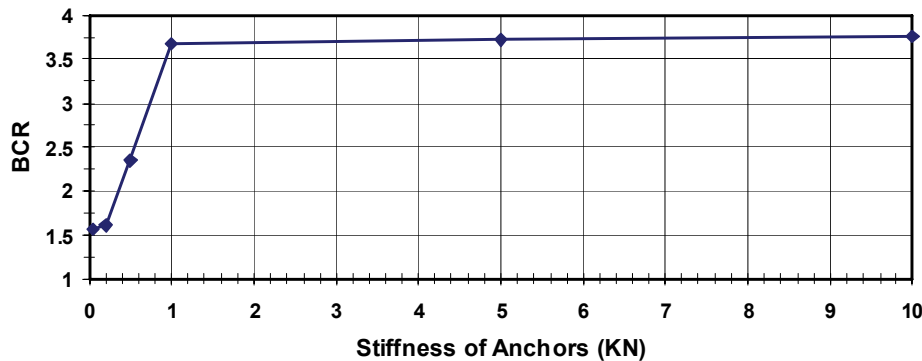Figure 12 shows a typical variation of BCR with the anchor stiffness for single layer reinforcement.

Fig. 12. Variation of BCR with anchor stiffness (N=1, u/B=0.25, b/B=5,
h/B=0.25, geogrid Stiffness =28 kN/m)

Again it can be seen that increasing anchor stiffness beyond a certain value (1kN) would not increase the BCR significantly. This happens because the soil body may collapse before the stiffness of anchors is fully mobilized.

g) Effect of angle of anchors (α)

Figure 13 shows a typical variation of BCR with the angle that the anchors make horizontal for three layer reinforcement. The analysis showed that the BCR generally increased with increasing the angle of anchors to approximately 45 degrees. After this angle, BCR begins to decrease with increasing the angle of anchors to 90 degrees; in this case (i.e. α =90^0) BCR is close to the status of soil being reinforced with ordinary geogrid.
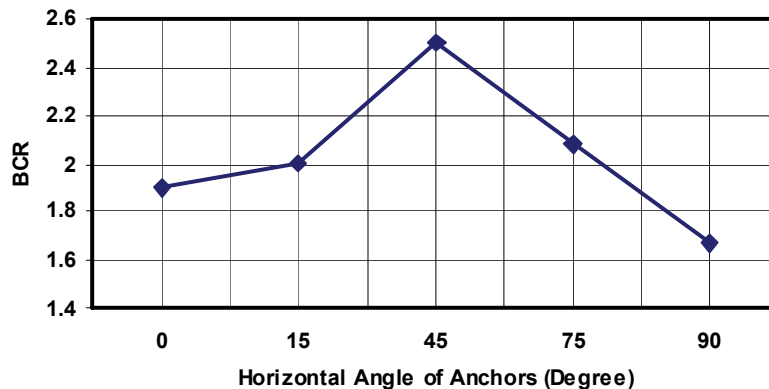

Fig. 13. Variation of BCR with angle of anchors
(N=3, u/B=0.25, b/B=5, h/B=0.25)

h) Effect of anchorage width (c)

The enhanced output facilities of the software provided a detailed presentation of the computational results such as: with clicking on each anchor after calculating the procedure, the user can observe the forces mobilized in them. It has been observed that anchors at the maximum distance of "0.75B" from the center of the foundation have not been mobilized. As shown in Fig. 14, mobilized forces in anchors that stand entirely beneath the foundation are approximately equal; from the edge of footing to 0.5B from the footing center, the forces in the anchors decrease dramatically beyond that distance.
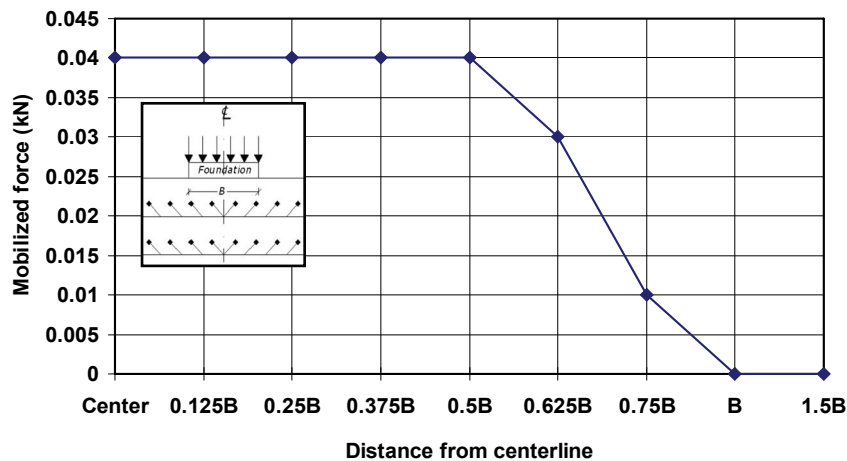
Fig. 14. Typical variation of mobilized force in anchors with distance
from center in 2nd layer (N=5, u/B=0.25, b/B=5, h/B=0.25)

6. SUMMARY AND CONCLUSIONS

A 3 dimensional finite–element study was conducted to investigate the bearing capacity of square footing on Grid-Anchor reinforced sand. The use of innovative Grid-Anchor as reinforcement element to improve bearing capacity of soils compared to using ordinary geogrid was investigated and it is shown that a significant increase in bearing capacity is obtained. The effect of the depth to the first layer of reinforcement, vertical spacing of the reinforcement layer, number of reinforcement layer, size of reinforcement sheet, stiffness of reinforcement and anchors and finally, the angle of anchors on bearing capacity were also investigated. The study brings the following conclusions for Grid-Anchor system:

1. Grid-Anchor system of reinforcing can increase the bearing capacity 2.74 times greater than that for ordinary geogrid and 4.43 times greater than for non-reinforced sand.

2. The value of BCR for square footing could be increased significantly by incorporating Grid-Anchor reinforcement at an optimum distance below the foundation base (i.e. d=1.25B).

3. The BCR for Grid-Anchor system increases significantly with reinforcement layer number within a certain effective zone. In this study the optimum number of reinforcement layer were found as N=5.

4. For multi-layer reinforced sand there is an optimum vertical spacing of reinforcement layer. The optimum spacing for the reinforced sand investigated is 0.25B.

5. For multi-layer reinforced sand, the highest BCR occured at an embedment depth of the first reinforcement layer approximately 0.25B, which was different from single-layer reinforcement with u=0.5B. The maximum depth of placement of the first layer of reinforcement should also be less than about 1.5B to take advantage of reinforcement.

6. Maximum width of reinforcement layers required for mobilization of maximum BCR is about 4.7B.

7. Effective angle of anchors was found to be about 45°.

8. Effective width of anchorage system was determined to be about 1.5B.

9. Increasing the stiffness of reinforcement and anchors beyond a certain value would only result in a small increase in BCR. For the conditions investigated, these values for one layer reinforcing were 100 kN/m for geogrid and 1 kN for anchors.

Since the influence of limiting conditions such as footing size, scale effects, density of soil, etc. on BCR have not been investigated, to establish a more accurate design criteria for Grid-Anchor reinforcing system, further studies are in progress by the authors.

NOMENCLATURES

BCR	Bearing capacity ratio
q_u	Ultimate bearing capacity of un-reinforced soil
$q_{u(R)}$	Ultimate bearing capacity of reinforced soil
B	Footing width
N	Number of reinforcement layers
c	Anchorage width
b	Reinforcement width
u	Depth of first layer of reinforcement below footing base
d	Total reinforced depth
α	Angle of anchors with horizontal
EA	Axial stiffness of reinforcement
EI	Bending stiffness of footing
U_x, U_y, U_z	Three degree of freedom in each node of reinforcement

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