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"Research Note"

STREAMFLOW TIME SERIES MODELING OF ZAYANDEHRUD RIVER*

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Abstract– Multiplicative seasonal autoregressive integrated moving average models are appropriate for the monthly stream flow of the Zayandehrud River in western Isfahan province, Iran, through the Box and Jenkins time series modeling approach. Among the selected models interpreted from ACF and PACF, seasonal multiplicative ARIMA (1,1,0) × (0,1,1) satisfied all tests and showed the best performance. Seasonal moving average parameter in the model indicates periodicity, and long memory in the streamflow, while a nonseasonal autoregressive parameter indicates the linearity of the monthly streamflow. The model forecasted streamflow for 24 leading months showed the ability of the model to predict and forecast statistical properties of the streamflow.

Keywords– Hydrologic time series, box-Jenkins approach, seasonality, ARIMA model, Zayandehrud River

1. INTRODUCTION

The most commonly used time series model in hydrologic time series modeling is the Box-Jenkins ARIMA model [1]. The ARIMA model has two general forms: ARIMA (p,d,q) and multiplicative ARIMA(p,d,q)$\times$(P,D,Q) in which p and q are non seasonal autoregressive and moving average, P and Q are seasonal autoregressive and moving average parameters, respectively. The other two parameters, d and D, are required differencing used to make the series stationary.

2. MATERIAL AND METHODS

The Box and Jenkins modeling approach has three steps. Model identification is the first step. The second step is to estimate parameters and diagnostic checking, and the last step is to apply goodness of fit test.

In this step, the model that seems to represent the behavior of the series is searched through autocorrelation and partial autocorrelation functions (ACF and PACF) for further investigation and parameter estimation [2]

After identifying models, we need to obtain efficient estimates of the parameters. Several methods are available for estimating parameters including maximum likelihood (ML), conditional least squares (CLS) and unconditional least squares (ULS). Among these methods, maximum likelihood seems to be the best [e.g., 1- 2- 3]. The parameters should be statistically significant at $\alpha = p\%$ and satisfy two conditions, namely stationary and invertibility for autoregressive and moving average models, respectively [4].

The third step, Goodness-of-fit tests, verifies the validity of the model using some tools. The residuals of the model are usually considered to be time-independent and normally distributed over time. The most

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common tests applied to test time - independence and normality are the Portmanteau lack of fit test, the nonparametric Kolmogrov – Smirnov and Anderson – Darling tests.

3. APPLICATION AND CASE STUDY

The Zayandehrud River basin is located in the central part of Iran at the eastern hillslope of the Zagros Mountain and the western region of Isfahan province. It consists of three main subbasins among which the only subbasin with permanent and uncontrolled streamflow is the Plasjan basin. The Plasjan basin is directly connected to the Zayandehrud reservoir, the largest surface reservoir in the region, which has an important role in supplying water to the region. An average annual flow from the Plasjan subbasin is about 135×10^6 m^3. In this paper, we try to fit an ARIMA model to the main branch of the Zayandehrud river, namely the Plasjan river in western Isfahan province. We use a monthly streamflow time series of this river in the period 1970-1999.

Based on the ACF and PACF of the logarithmic series, two models are examined for further consideration. The first is ARIMA$(1,1,1)\times(0,1,1)$ and the second is ARIMA $(1,1,0)\times(0,1,1)$.

The ARIMA$(1,1.1)\times(0,1,1)$ model is not accepted because the nonseasonal moving average parameter is not significant. Thus, we leave out the parameters and try ARIMA $(1,1,0)\times(0,1,1)$ (Table 1).

Table 1. Result of parameter estimation for the second model

Estimation method	Type (Order) and values of parameters ARIMA$(1,1,0)\times(0,1,1)$	Absolute Value of t	Probability of t	Invertibility and stationary condition
ML	Q(1)=0.87 p(1)=0.76	21.03 22.4	0.0001< 0.0001<	Satisfies invertibility and stationarity
CLS	Q(1)=0.81 p(1)=0.75	23.56 21.24	0.0001< 0.0001<	Satisfies invertibility and stationarity
ULS	Q(1)=0.99 p(1)=0.76	9.13 22.71	0.0001< 0.0001<	Satisfies invertibility and stationarity

ML: Maximum Likelihood CLS: Conditional Least Square ULS: Unconditional Least Square

The results of time independent and normal test of residuals show the adequacy of the second model, estimated by ML. Therefore, the model whose parameters are estimated by ML is the best model. The selected model is written

$$0.76(B)\nabla^1\nabla_{12}^{12}Z_t = -0.87(B^{12})a_t \tag{1}$$

Figure 1 shows the model prediction and observed streamflow of the Plasjan River which match well together. The above model was then used for forecasting streamflow from January, 2000 to December, 2001. Figure 2 illustrates the forecasted monthly streamflow by the selected model and observed streamflow in this period. Comparing the observed and model forecasted streamflow indicates the same monthly variation for both series. This may imply the capability of the multiplicative ARIMA model in forecasting.

4. CONCLUSIONS

A multiplicative ARIMA $(1,1,0)\times(0,1,1)$ was fitted to monthly nonstationary stream flow of the Plasjan River by Box and Jenkins [1] modeling approach. As a result, the procedure can not identify the model easily by first interpretation [e.g., 5-6, among others]. Another important note is the significance of nonseasonal AR (1) and seasonal MA (1) parameters in the selected model that might be the result of long-memory time series [7-9] and linearity of streamflow generating mechanisms such as the snowmelt process of the Zayandehrud basin and the groundwater of the AR(1) process. This conclusion is based on the conceptual watershed model that was originally suggested by Thomas and Fiering [11] and demonstrated by Salas and Smith [10] who noted that an independent precipitation process will result in an ARMA (1,1) for streamflow. The effect of climate variability on rainfall and streamflow periodicity might be the reason of seasonal MA(1), although this traditional ARIMA model building can be improved using Fractional ARIMA models [5-7]. However, this is the subject of ongoing works on Zayandehrud river time series modeling.

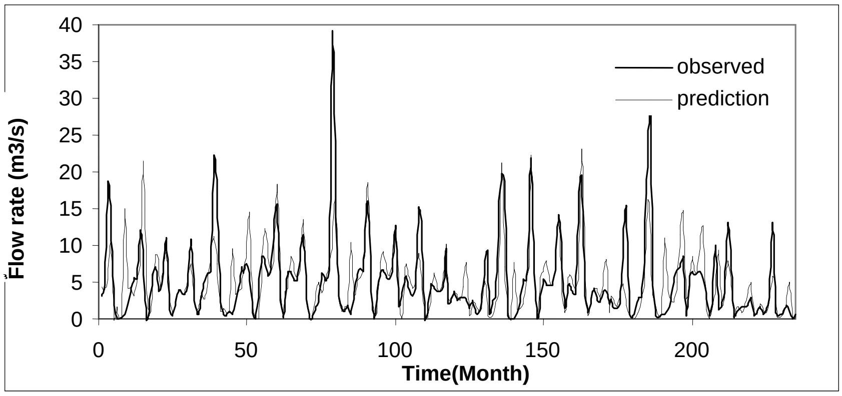

Fig. 1. Observed and model prediction of monthly streamflow of Plasjan River

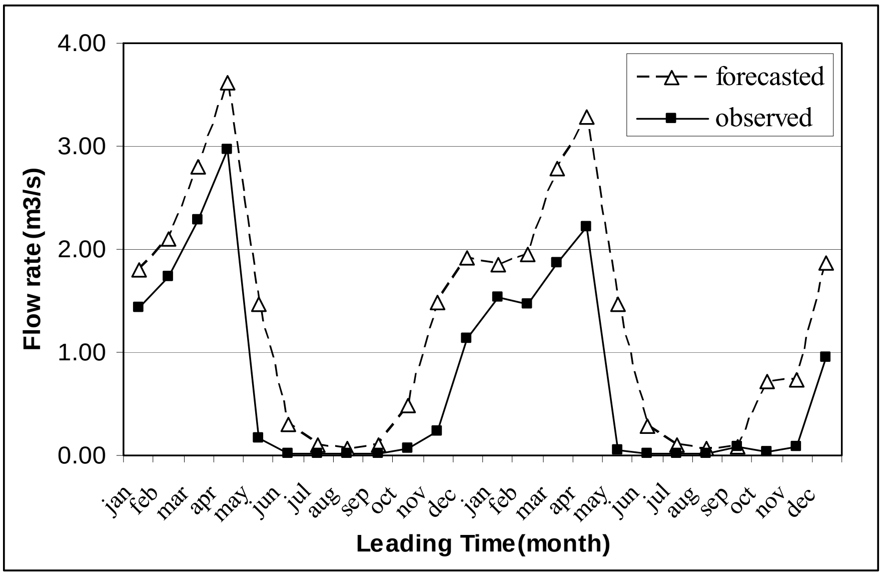

Fig. 2. Observed and forecasted monthly streamflow of Plasjan River

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