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# ON THE PERFORMANCE OF A MODIFIED MULTIPLE-DEME GENETIC ALGORITHM IN LRFD DESIGN OF STEEL FRAMES*

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**Abstract–** This paper investigates the performance of a multiple-deme genetic algorithm (GA) with modified reproduction operators, in optimal design of planar steel frames according to the AISC-LRFD specification. The design objective is to minimise the weight of frame subject to strength, displacement and constructability constraints. A number of new crossover and mutation operators, used alongside the standard operators are utilised in optimum design of a number of steel frames subjected to the constraints of the AISC-LRFD specification, with and without considering the second order effects, as set out by the code requirements. This modified GA (MGA) is shown to have a very fast convergence and to produce relatively high-quality designs. This paper also utilizes the concept of multiple-deme in the GA, as it has been used successfully for other metaheuristic population-based methods. The multiple-deme GA is used alongside the modified GA operators and the algorithm is named the modified multiple-deme GA (MMDGA). The modified GA (MGA) and modified multiple-deme GA (MMDGA) are applied to three benchmark problems and the results are compared to those obtained by other metaheuristic methods. In the majority of cases, the results of comparisons suggest the superiority of the MMDGA in terms of the quality of final design and the total number of performed finite elements analyses.

**Keywords–** Optimum design, multiple-deme genetic algorithm, steel frames, AISC-LRFD

## 1. INTRODUCTION

For more than two decades genetic algorithms (GA) have been successfully utilised in optimal design of structures. Amongst the more recent applications, Kaveh and Kalatjari [1], Toğan and Daloğlu [2, 3], and DEDE et al. [4] used GA for design of planar and space trusses. Also, Camp et al. [5], Pezeshk et al. [6], Hayalioglu [7], Lagaros et al. [8], Sarma and Adeli [9], Prendes Gero et al. [10, 11], Degertekin [12], and Degertekin et al. [13] employed GA for design of planar and space frames. Furthermore, Foley and Schinler [14], Hayalioglu and Degertekin [15, 16], Yun and Kim [17], and Hadj Ali et al. [18] used GA to obtain optimum design of nonlinear steel frames with or without semi-rigid connections.

More recently other metaheuristic methods, such as Simulated Annealing (SA), Tabu Search (TS), Ant Colony Optimization (ACO), Differential Evolution (DE), Particle Swarm Optimization (PSO), Harmony Search (HS), Imperialist Competitive Algorithm (ICA), and Charged System Search Algorithm (CSSA) have been developed and employed for structural design optimisation. Amongst them Li et al. (PSO) [19], Wu and Tseng (DE) [20], Kaveh and Talatahari (ICA) [21], Kaveh and Laknejadi [22] and Kaveh and Malakouti Rad [23] used the metaheuristics for design of planar and space trusses. Also, Camp et al. (ACO) [24], Kargahi et al. (TS) [25], Degertekin (HS) [26], Saka (HS) [27], Kaveh and Talatahari

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(ACO) [28], Kaveh and Talatahari (ICA) [21], Kaveh and Talatahari (CSSA) [29], Hasançebi et al. (SA) [30] and Hasançebi et al. (HS) [31] used the metaheuristics for design of linear planar and space frames and Degertekin (SA) [12] and Degertekin et al. (TS) [13] used the metaheuristics for design of nonlinear space frames.

A major drawback of standard GA is that this algorithm needs a significant number of objective and constraint evaluations during the optimisation process. Recent studies on GAs have had the tendency to focus on reducing the computation time through modification of the algorithm (e.g. see Lagaros [8], Foley and Schinler [14], Prendes Gero et al. [10, 11], Toğan and Daloğlu [3] and Safari and Maheri [32]). These newer versions of GAs are referred to by different names in the literature including the modified, enhanced, improved or intelligent GAs. Recently a multiple-deme genetic algorithm, utilising modified reproduction operators was developed and used by the authors for allowable stress design of steel frames according to AISC specification (AISC-ASD) and its performance was evaluated against standard GA and Tabu Search solutions [32]. The aim of the present study is to investigate the performance of this modified GA algorithm, tailored for optimal LRFD design of steel frames, compared to other metaheuristic optimisation methods such as: ant colony, harmony search, improved ant colony and imperialist competitive algorithms, taken from the current literature.

## 2. OPTIMIZATION PROBLEM FORMULATION

The discrete optimum design problem of steel frames, in which the minimum weight is considered as the objective, can be stated as follows:

$$Minimize\ \ W(x)=\sum_{i=1}^{ng}\gamma_i A_i\sum_{j=1}^{mi}L_j \tag{1}$$

subjected to the strength (or stress) constraints of AISC–LRFD [33] and the displacement and constructability constraints. In Eq. (1), *mi* and *ng* are the total number of members in the *i*-th group and the total number of groups in the frame, respectively. Also, $A_i$ and $\gamma_i$ are cross-sectional area and the material unit weight of *i*-th member group and $L_j$ is the length of *j*-th member. Design variables *(x)* are the element section sizes from the available W-shapes of a standard list.

All constraints are given in normalized forms which are suitable for GA so that the objective function can be arranged in an unconstrained manner. The displacement constraint is:

$$g_i(x)=\frac{\Delta_i}{\Delta_i^u}-1\le 0 \qquad i=1,\dots,ns \tag{2}$$

Where, $\Delta_i$ is relative interstorey drift in storey *i*, while $\Delta_i^u$ is its limit (storey height/300 in the numerical examples) and *ns* is the total number of storeys.

The strength (or stress) constraints are taken from the AISC-LRFD [33]. For members subjected to bending moments and axial force, these constraints are expressed in terms of the following interaction formulas:

$$for\ \left(\frac{P_u}{\phi P_n}\right)_i\ge 0.2;\quad g_i(x)=\left(\frac{P_u}{\phi P_n}\right)_i+\frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}}+\frac{M_{uy}}{\phi_b M_{ny}}\right)_i-1.0\le 0 \quad i=1,\dots,nm \tag{3}$$

$$for\ \left(\frac{P_u}{\phi P_n}\right)_i< 0.2;\quad g_i(x)=\left(\frac{P_u}{2\phi P_n}\right)_i+\left(\frac{M_{ux}}{\phi_b M_{nx}}+\frac{M_{uy}}{\phi_b M_{ny}}\right)_i-1.0\le 0 \quad i=1,\dots,nm \tag{4}$$

Where, *nm* is the total number of members in the frame; $P_u$ is the required axial strength (tension or compression); $P_n$ is the nominal axial strength (tension or compression); $\phi$ is the resistance factor

($\phi = \phi_t = 0.90$ for tension, $\phi = \phi_c = 0.85$ for compression); $M_{ux}$ and $M_{uy}$ are the required flexural strengths about the x and y axes, respectively (for plane frames, $M_{uy} = 0$); $M_{nx}$ and $M_{ny}$ are the nominal flexural strengths about, respectively, the x and y axes and $\phi_b$ is the flexural resistance factor ($\phi_b = 0.90$). The nominal tensile strength of a member is computed as:

$$P_n = A_g F_y \tag{5}$$

and the nominal compressive strength of a member is computed as:

$$P_n = A_g F_{cr} \tag{6}$$

$$for \quad \lambda_c \leq 1.5; \quad F_{cr} = \left(0.658^{\lambda_c^2}\right) F_y \tag{7}$$

$$for \quad \lambda_c > 1.5; \quad F_{cr} = \left(\frac{0.877}{\lambda_c^2}\right) F_y \tag{8}$$

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} \tag{9}$$

In the above relations, $A_g$ is the cross-sectional area of the member; $K$ is the effective length factor; $E$ is the modulus of elasticity; $r$ is the radius of gyration; $L$ is the member length, and $F_y$ is the yield stress of steel. Also, $\lambda_c$ should be calculated about each of the two main axes and the maximum value will be the governing value for calculating $F_{cr}$. The effective length factor $K$, for braced and unbraced frames is calculated from the following approximate equations given by Dumonteil [34]:

For unbraced members:

$$K = \sqrt{\frac{1.6G_A G_B + 4(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \tag{10}$$

For braced members:

$$K = \frac{3G_A G_B + 1.4(G_A + G_B) + 0.64}{3G_A G_B + 2.0(G_A + G_B) + 1.28} \tag{11}$$

where, $G_A$ and $G_B$ refer to stiffness ratio or relative stiffness of a column at its two ends.

The required flexural strength of beams and columns considering second order effects is computed from the following relationship [33]:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{12}$$

where, $M_{nt}$ is the required flexural strength in a member assuming there is no lateral translation (*nt*) in the frame, and $M_{lt}$ is the required flexural strength in a member as a result of lateral translation (*lt*) of the frame only. Also, the term $B_1$ accounts for the amplification of the first-order *nt* moment associated with member curvature effects. $B_1$ is defined by:

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} \tag{13}$$

where, $C_m$ is the equivalent moment factor. For compression members subjected to transverse loading only at their ends this factor is given by $C_m = 0.4 - 0.6(M_1 / M_2)$, where $M_1 / M_2$ is the ratio of the smaller to larger end moments in the non-sway case. Also, $P_u$ is the required axial compressive strength for the member under consideration and $P_{e1}$ is the Euler buckling load [33].

In Eq. 12, $B_2$ accounts for the amplification of the member end moments associated with lateral translation of the storey as expressed by any of the following equations:

$$B_2 = \frac{1}{1 - \sum P_u \left( \frac{\Delta_{0h}}{\sum HL} \right)} \tag{14.1}$$

$$B_2 = \frac{1}{1 - \frac{\sum P_u}{\sum P_{e2}}} \tag{14.2}$$

where, $P_u$ is the required axial strength of all columns in a storey; $\Delta_{0h}$ is the lateral interstorey drift; $H$ is the sum of all storey horizontal forces producing $\Delta_{0h}$ and $L$ is the storey height. Also, $P_{e2} = \frac{\pi^2 EI}{(K_2 L)^2}$, where $K_2$ is in the plane of bending assuming that side-sway is allowed.

When the required axial compressive strength of a column exceeds the Euler load limit, in cases when side-sway is prevented $(P_{e1})$, Eqs. (3) and (4) are no longer valid and the following constraint must be used instead:

$$g_i(x) = (\frac{P_u}{P_{e1}})_i - 1.0 \leq 0 \qquad i = 1,\ldots,nc \tag{15}$$

where, $nc$ is the total number of column members in the frame and the other terms are as before.

The constructability constraint for the requirement that the selected W-shapes decrease in size as the columns extend from the base upwards is given as:

$$g_i(x) = (\frac{d_{cu}}{d_{cl}})_i - 1.0 \leq 0 \qquad i = 1,\ldots,nj \tag{16}$$

where, $nj$ is the total number of column to column joints and $d_{cu}$ and $d_{cl}$ are depths of steel sections selected for the upper and lower floor columns, respectively.

After calculating the objective function and the constraints, the unconstrained (or penalized) objective function $F(x)$ can be written according to the static penalty function method [35] as:

$$F(x) = f^W + \alpha_1 \sum_{i=1}^{nm} C_i^{\sigma} + \alpha_2 \sum_{i=1}^{ns} C_i^{\Delta} + \alpha_3 \sum_{i=1}^{nj} C_i^{cons.} \tag{17}$$

where $f^W$ is the normalized objective function relative to the maximum possible weight of the frame and is defined as $f^W = W(x)/W_{max}$. Also, $\alpha_i$ are the penalty coefficients used to tune the intensity of penalization as a whole. In the numerical examples of the present study these coefficients are set to $\alpha_1 = 1/3$ and $\alpha_2 = \alpha_3 = 1$. Also, $C_i^{\sigma}$, $C_i^{\Delta}$, and $C_i^{cons.}$ are constraint violations for, respectively, the interaction formula (or the Euler load limit, whichever is applicable), interstorey drift, and the constructability requirements. The constraint violation $C_i$ is expressed as [6]:

$$C_i = \begin{cases} 0 & if \ g_i(x) \leq 0 \\ g_i(x) & if \ 0 < g_i(x) \leq 1.0 \\ g_i^2(x) & if \ g_i(x) > 1.0 \end{cases} \tag{18}$$

where, $g_i(x)$ are normalized constraints as defined in this section.

## 3. MODIFIED MULTIPLE-DEME GENETIC ALGORITHM

This section provides a brief description of the main features of the MMDGA. More details on this new method can be found in Reference [32].

- ***Reproduction operators*:** The MMDGA incorporates three special types of crossover operators, referred to as the 'boosted', 'geometric', and 'boosted geometric' as well as a special mutation operator called 'enhancing' mutation, along with the standard crossover, the standard mutation and the sorting mutation of Foley and Schinler [14].

A boosted crossover forms a child with the best genes from two parents. First, for every gene of each of the two parents a penalized fitness value, as given by Eq. (19) is accounted. Then the best gene (the gene with the lowest penalized fitness value) will be the corresponding gene of the offspring chromosome. The penalized fitness value of member *i* is defined as:

$$F_i = f_i^W + \alpha_1 C_i^\sigma \tag{19}$$

where, $f_i^W$ is the normalized weight of the *i*-th member, i.e. the ratio of the element weight relative to the maximum possible weight of that member according to the available profile list. If a gene in a chromosome indicates a group of columns or a group of beams (when section grouping is utilized for reducing the size of a problem), the penalized fitness value of that gene will be the sum of the corresponding values for all members in the group.

A geometric crossover is similar to the classical crossover operator but it exchanges physically meaningful groups of genes between parents. In a chromosome representing a 2D frame, the union of columns on one axis, members of a storey, and members of a bay can form physically meaningful groups of genes.

A boosted geometric crossover is a combination of the geometric and the boosted crossover in which the exchange between two parents is at group level, while for each group of genes a partial fitness has been assigned. The type of members group to be crossed is selected randomly for each pair of parents. For crossover in the *storey level*, for every storey *i* of each of the two parents, a penalized fitness value is determined using Eq. (20) and the winner storey will be the corresponding storey of the offspring chromosome.

$$F_i = \sum_{j=1}^{nm,i} \left( f_j^W + \alpha_1 C_j^\sigma \right) + \alpha_2 C_i^\Delta \tag{20}$$

In Eq. (20), *nm,i* denotes the total number of members in *i*-th storey. Other terms are as described previously.

For crossover in the *axis level*, for every axis *i* of each of the two parents, a penalized fitness value is calculated according to Eq. (21) and the winner axis will be the corresponding axis of the child chromosome, while the other genes of the child chromosome (beam sections) are chosen from the better parent (parent with the better penalized objective function). i.e.:

$$F_i = \sum_{j=1}^{nc,i} \left( f_j^W + \alpha_1 C_j^\sigma \right) + \alpha_3 \sum_{k=1}^{nj,i} C_k^{cons.} \tag{21}$$

where, *nc,i* and *nj,i* denote the total number of columns and the number of column to column joints in *i*-th axis, respectively.

Similarly, for crossover in the *bay level*, for any bay *i* of each of the two parents, a penalized fitness value is calculated ( Eq. 22) and the winner bay will be the corresponding bay of the child chromosome, while the other genes of the child chromosome (column sections) are chosen from the better parent.

$$F_i = \sum_{j=1}^{nb,i} \left( f_j^W + \alpha_1 C_j^\sigma \right) \tag{22}$$

where, *nb,i* denotes the total number of beams in the *i*-th bay.

In a sorting mutation all columns in each axis and all beams in each bay will be sorted from the lowest storey upwards, while an enhancing mutation checks every gene of a parent and improves any defects that may exist and, when possible, lightens the frame by reducing the size of members.

These operators are used in conjunction with the multiple-deme (multi-population) genetic algorithm in which the population is divided into several subpopulations, each of which has its own controlling parameters. The multiple-deme GA increases population diversity and enhances search performance. In this algorithm, after a certain number of generations, the migration is executed by which a number of best individuals from the source subpopulations are sent to replace the worst individuals of the destination subpopulations. The migration mechanism for exchanging beneficial genetic information among the subpopulations is applied to encourage the proliferation of good traits throughout the whole population. Three main parameters of multiple-deme GA in need of appropriate tuning includes: (i) migration rate; (ii) migration interval; and (iii) migration policy. More details on this subject can be found in reference [30].

In this study an integer encoding (a type of real valued encoding) is used instead of binary encoding for chromosome representation. Each chromosome has a length of *nm*, the total number of the frame's members or groups, in which each gene can take an integer number corresponding to the code of section in the profile list. The arrangement of the genes in the chromosome is shown in Fig. 1. In this figure, as an example, it is assumed that the beam sections can be selected from the entire 267 W-shape standard sections of AISC list while column sections can only be selected from 36, W14 standard sections. Also in this study, a rank scaling approach is used for fitness scaling of the penalized objective functions in order to remove the defect of tuning good penalty coefficient in the static penalty function method [32].

Based on the above descriptions an algorithm is coded in MATLAB programming space. The analysis of each frame is conducted using a displacement based finite element method. The pseudo-code presented in Fig. 2 shows the process sequence of the MMDGA with forward migration.

## 4. DESIGN EXAMPLES

Utilizing the developed program, optimal design of three benchmark problems is performed. Each problem is solved in two different cases; one without considering the second order effects, i.e. assuming a linear elastic analysis while $B_1$=$B_2$=1 (case 1), and another conducting a linear elastic analysis followed by enforcing the AISC-LRFD specification requirements in considering the P-δ and P-Δ effects; i.e. taking into account $B_1$ and $B_2$ multipliers (case 2). The GA with modified operators (MGA) and the modified multiple-deme GA (MMDGA) are applied to these problems and the results are compared to solutions carried out by others and reported in literature. Due to the stochastic nature of the optimization methods and also to avoid the effect of initial solutions on the final results, each example is solved 30 times. For every set of these 30 runs, the best, the worst and the average results are reported. Also, the standard deviation and the coefficient of variation of results, showing the robustness of each algorithm, are presented. In all design examples, the in-plane effective length factors of the column members are calculated as $K_x > 0$ while the out-of-plane effective length factor is specified as $K_y = 1.0$. Each column is considered as non-braced along its length while the unbraced length for beam members is specified in each problem separately. Also, in line with other works, the shear deformations are ignored.

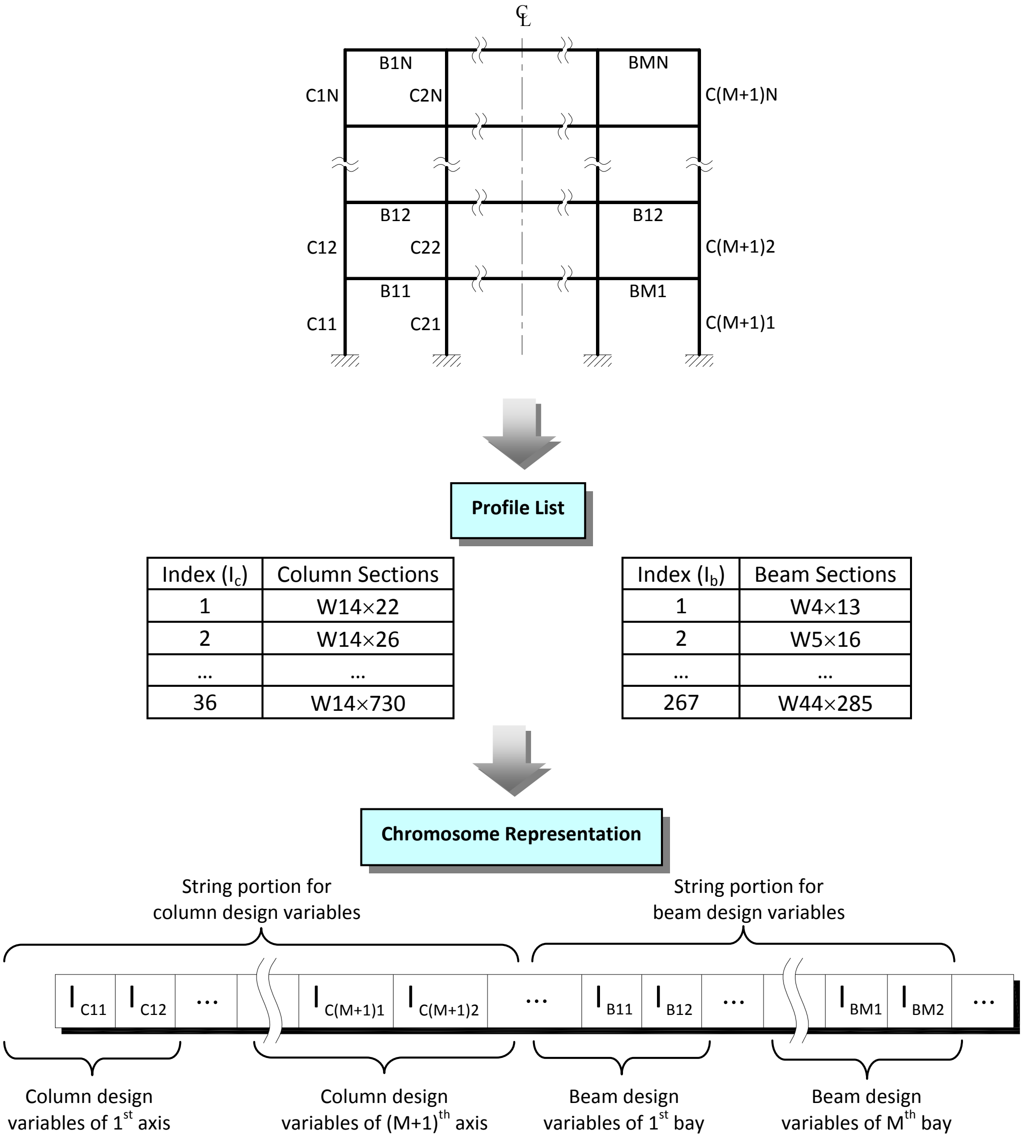


Fig. 1. Constructing a chromosome in the used GA

Initialize $P$ subpopulations of size $n_i$ ($i$=1:$P$)
*Generation* = 1
**for** (max. *Generation*) **do**
    **for** each subpopulation **do in parallel**
        Evaluate unconstrained objective function for each individual from F.E.A.
        Do fitness scaling of individuals
        Select the parents
        **if** *Generation* mod (migration interval) = 0 then
            Send ($k$×$n_i$) best individuals of $i^{th}$ deme to $(i+1)^{th}$ deme
            Replace ($k$×$n_i$) individuals in the $(i+1)^{th}$ deme
        **end if**
        Produce the next generation:
            Send elites to the next generation without any modification
            Produce the crossover children by standard and modified crossover operators
            Produce the mutation children by standard and modified mutation operators
    **end parallel for**
    *Generation* = *Generation* + 1
**end for**

Note: $k$= migration rate; F.E.A.= Finite Element Analysis

Fig. 2. The pseudo-code for MMDGA with forward migration

***a) Two-bay, three-storey frame***

The first benchmark problem originally presented by Wood et al. [36] is a two-bay, three-storey frame undergoing a single-load case as shown in Fig. 3. This frame was optimized by Hall et al. [37] in accordance with the AISC-LRFD specification using an OC method. It was also designed subject to the same specification by Pezeshk et al. [6] using a GA and by Camp et al. [24] using an ACO method. The values of the uniform and the point loads in Fig. 3 are factored loads appropriate for direct application of the strength/stability provisions of the AISC-LRFD specification. Displacement constraints were not imposed for the design. A modulus of elasticity of $E$ = 200 GPa (29,000 ksi), a yield stress of $F_y$ = 248.2 MPa (36 ksi) and a material unit weight of $\gamma$ = 77.08 kN/m$^3$ (2.84×10$^{-4}$ kip/in$^3$) were used. The unbraced length factor for each beam member was specified to be 0.167. Imposed fabrication conditions require that all six beams be of the same W-shape and that all nine columns have identical sections. The beam group section may be chosen from the entire 267 W-shapes of AISC standard list, however, the column group section is limited to W10 sections (18 W-shapes). Therefore, the resulting search space has a size of 4,806 designs.

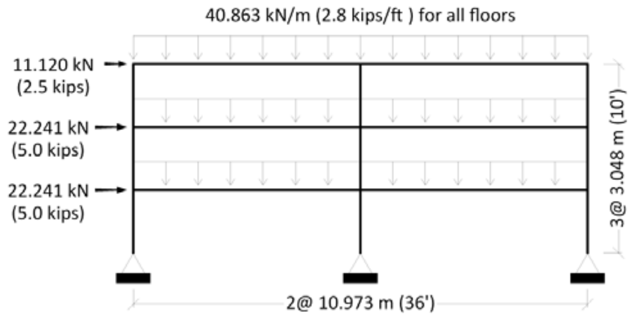


Fig. 3. Two-bay, three-storey problem

This problem is solved here again using the MGA and MMDGA methods. The parameter settings of the MGA and MMDGA for this problem are listed in Table 1. These parameters were determined through numerical experiments through multiple simulation runs. In each generation, the total number of offspring created by crossover and mutation operators is equal to the population size minus the number of elites. In this table the ‘crossover fraction’ denotes the fraction of the offspring from the remaining population, which must be produced by the crossover operator. The rest of the children will be produced by the mutation operator. Also, each crossover and mutation operator type creates a percentage of the operator’s offspring as specified in Table 1. It is noted that because in this example beam sections are identical for all beam elements as are all column elements, the geometric and the boosted geometric crossover operators, as well as the sorting mutation operator, have no meaning; therefore, their percentage values are set to zero. Also, as there are only two design variables (the chromosome length is equal to two), only one point crossover from the three standard crossover types is usable. Moreover, for comparability of the results, a total population size of 40 over 30 generations is used for both MGA and MMDGA methods. All other parameters are kept the same.

The problem was run 30 times for each of the two cases using the two GA-based algorithm totalling 120 runs (2×2×30=120). The following is a summary of the findings.

***Case 1. Ignoring the second order effects***

Table 2 shows the best designs and a statistical report of the results obtained for each of the MGA and MMDGA solutions over 30 runs. In this table, it can be observed that both algorithms produced a best design with W10×60 for columns and W24×62 for beams, corresponding to a weight of 83.591 kN. Hall et al. [37], Pezeshk et al. [6] and Camp et al. [24] all reported the same optimal design for this frame. It is worth mentioning that an exhaustive search has revealed no better solution for this problem [6].

Table 1. GA parameters used for two-bay, three-storey problem

| | MGA | MMDGA |
|---|---|---|
| Population size | 40 | 10 |
| Number of demes | 1 | 4 |
| Number of elites | 2 | 1 |
| Crossover fraction | 60% | 60% |
| Standard crossover percentage | 60% | 60% |
| Geometric crossover percentage | 0% | 0% |
| Boosted crossover percentage | 40% | 40% |
| Boosted geometric crossover percentage | 0% | 0% |
| Standard mutation percentage | 60% | 60% |
| Sorting mutation percentage | 0% | 0% |
| Enhancing mutation percentage | 40% | 40% |
| Standard mutation probability | 0.2 | 0.2 |
| Migration rate | - | 10% |
| Migration interval | - | 5 |
| Migration direction | - | Forward |

Table 2 also compares the results provided by MGA and MMDGA solutions with those obtained by others. The GA and ACO respectively required an average of approximately 1,800 and 3,000 frame analyses to converge and terminate. These are substantially more than the 230 and 220 frame analyses required by the MGA and MMDGA methods, respectively. Thus, our proposed algorithms have improved the standard GA's performance and resulted in a significant reduction in computational effort. This is while the robustness of the GA has also improved. Also, as we can observe in Table 2, in over 30 runs of the MGA and MMDGA solutions, we have reached a coefficient of variation of 3.9% and 2.8% respectively. These are also appreciably better than the values of 26.3% and 8.8%, obtained respectively in GA and ACO solutions, indicating the robustness of the two former algorithms. However, the MGA and MMDGA have found the best solution in 53% and 63% of the runs, which is less than the 84% obtained by ACO. Nevertheless, when we consider the substantial difference between the required number of frame analyses by ACO with those of the MGA and MMDGA, the superiority of the latter algorithms is revealed. Moreover, comparing the results of the MGA and MMDGA methods, Table 2 shows that the MMDGA has produced better results.

Table 2. Design results for two-bay, three-storey frame; case 1

| | Hall et al. [37] (OC) | Pezeshk et al. [6] (GA) | Camp et al. [24] (ACO) | This study | |
|---|---|---|---|---|---|
| | | | | MGA | MMDGA |
| Best design, column section | W10×60 | W10×60 | W10×60 | W10×60 | W10×60 |
| Best design, beam section | W24×62 | W24×62 | W24×62 | W24×62 | W24×62 |
| Best weight (kN) | 83.591 | 83.591 | 83.591 | 83.591 | 83.591 |
| Worst weight (kN) | - | 157.654 | N.R. | 94.774 | 87.253 |
| Average weight (kN) | - | 98.217 | 85.241 | 85.301 | 84.658 |
| Standard deviation (kN) | - | 25.880 | 7.531 | 3.327 | 2.370 |
| Coefficient of variation | - | 26.3% | 8.8% | 3.9% | 2.8% |
| Average no. of F.E.A. | N.R. | 1,800 | 3,000 | 230 | 220 |
| Average no. of F.E.A. for initially best design | N.R. | 900 | 880 | 160 | 175 |
| Percentage of the best design in different runs | - | 20% | 84% | 53 % | 63% |

*Note:* 1 N=0.225 lb; N.R.= Not Reported; F.E.A.= Finite Element Analysis

It is noted that MMDGA is sensitive to the number of designated demes. The problem was run with different number of demes to observe its effect on the quality of solution. The number of demes investigated varied as 2, 4 and 8 with the corresponding subpopulation sizes of 20, 10 and 5, respectively, resulting in a total population size of 40. In all the runs, migration direction was set to 'forward'. For the number of demes equal to 2, the solution reached the frame weight of 85.281 kN with a 3.8% coefficient of variation in the 30 runs. These values are 84.658 kN and 2.8%, respectively, when the number of demes is set equal to 4; and 85.024 kN and 6.5%, respectively, when the number of demes is considered to be 8. Therefore, the number of demes equal to 4 has produced the best results. Also, when the number of demes is very large (say equal to 8 in this problem), the worst results are obtained. This may be attributed to the small size of the subpopulations resulting in a reduced search space and therefore low diversity.

The solution's quality is also sensitive to the direction of migration. The problem was solved again using the previous number of demes, while the migration was allowed in 'both' directions. In this case, only when the number of demes is set equal to 8 has the algorithm produced a better result compared to the forward migration case, while for the other two number of demes (2 and 4), the forward migration direction has produced better results. Therefore, a number of demes equal to 4 and a migration in forward direction are found to be the best parameter settings for MMDGA in this problem.

Figure 4 shows a typical convergence history for a best design in each of the MGA and MMDGA methods. This figure highlights the rapid convergence associated with both proposed algorithms, due to utilizing the modified GA reproduction operators. Also, Fig. 5a shows that all members of the optimum design satisfy the relevant strength constraints. The maximum value of the strength (stress) ratio is obtained as 91.13% for all members.

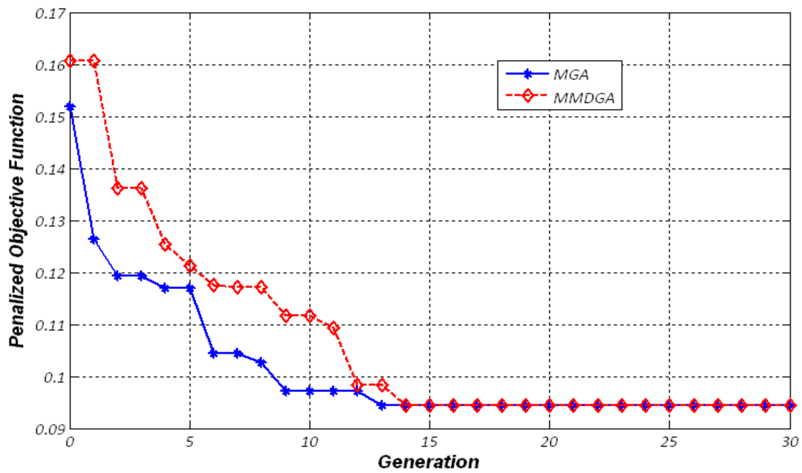


Fig. 4. Convergence history of two-bay, three-storey frame (case 1)

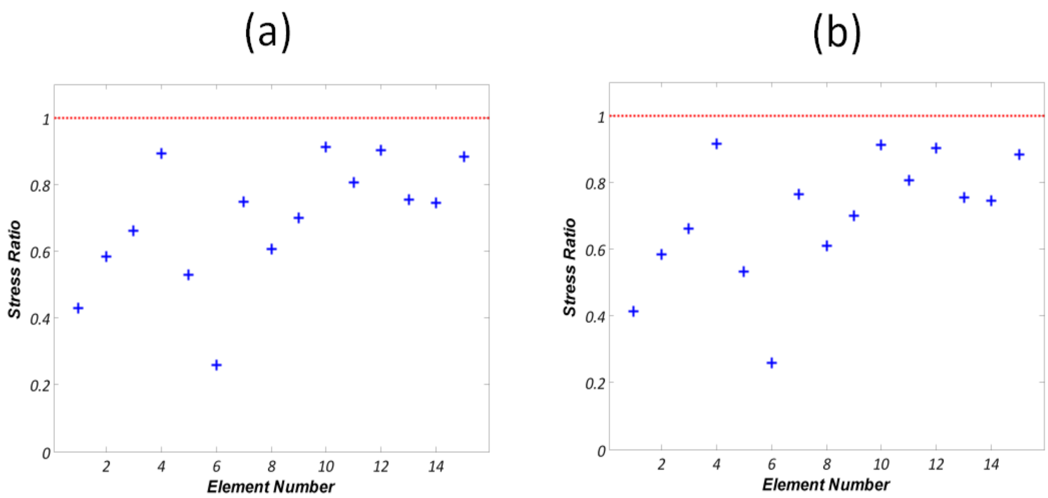


Fig. 5. Stress ratios of two-bay, three-storey frame: (a) Case 1 (b) Case 2

### *Case 2. Considering the second order effects*

In this case the linear analysis was followed by the second order effects considerations of the AISC-LRFD specification. The best designs, also the statistical report of each of the MGA and MMDGA methods over 30 runs are summarized in Table 3. From this table it can be deduced that the optimal design consists of W10×60 sections for columns and W24×62 sections for beams, with a total frame weight of 83.591 kN; a weight similar to the case where the second order effects were ignored. This may be attributed to the low level of gravity loads on the columns resulting in small values for the moment amplification factors. This is deduced from checking the $B_1$ and $B_2$ factors of the best design where $B_1$ equals 1.000 for all columns and $B_2$ has a maximum value of 1.027 for all storeys. Fig. 5b also shows that all members of the optimum design satisfy the strength constraints. The maximum value of the strength (stress) ratio, in all members, is 91.44%. This figure is very close to the corresponding figure obtained for the Case 1 solution, evidently due to the minor effects of the $B_1$ and $B_2$ multipliers in this problem.

Table 3. Design results for two-bay, three-storey frame; case 2

| | Pezeshk et al. | This study | |
|---|---|---|---|
| | [6] (GA) | MGA | MMDGA |
| Best design, beam section | W24×62 | W24×62 | W24×62 |
| Best design, column section | W10×60 | W10×60 | W10×60 |
| Best weight (kN) | 83.591 | 83.591 | 83.591 |
| Worst weight (kN) | 138.998 | 110.340 | 89.379 |
| Average weight (kN) | 89.337 | 88.542 | 83.963 |
| Standard deviation (kN) | 10.262 | 2.910 | 1.478 |
| Coefficient of variation | 11.5% | 2.6% | 1.7% |
| Average no. of F.E.A. | 1,800 | 235 | 250 |
| Average no. of F.E.A. for initially best design | N.R. | 160 | 160 |
| Percentage of the best design in different runs | 43% | 47% | 67% |

*Note:* 1 N=0.225 lb; N.R.= Not Reported; F.E.A.= Finite Element Analysis

Table 3 also compares the MGA and MMDGA results with those obtained by Pezeshk et al. [6] using a standard GA. Both MGA and MMDGA algorithms have produced better results compared to the GA developed by Pezeshk et al. in terms of the worst weight, average weight, coefficient of variation, average number of finite element analysis, as well as the percentage of obtaining the best design in 30 runs; the MMDGA algorithm again fares better than the MGA.

### *b) One-bay, ten-storey frame*

A one-bay, ten-storey frame consisting of 30 members, originally presented by Pezeshk et al. [6], is selected as the second benchmark problem (see Fig. 6). This frame was designed by Pezeshk et al. [6] using a standard GA. The same frame was also designed by Camp et al. [24] using an ACO, by Degertekin [26] using an HS, and by Kaveh and Talatahari [28] using an improved ACO (IACO). The frame was designed according to the AISC-LRFD specification [33] and a displacement constraint; interstorey drift < storey height/300, was imposed. The modulus of elasticity was assumed to be $E$ = 200 GPa and a yield stress of $F_y$ = 248.2 MPa was used. Fabrication conditions requiring the same beam section be used for every three consecutive storeys, beginning at the foundation, as well as the same column section to be used in every two consecutive storeys were implemented. The element grouping resulted in four beam sections and five column sections for a total of nine design variables. Each of the four beam element groups could be chosen from the entire 267 W-shapes of the AISC standard list, and the five column element groups were limited to the W12 and W14 sections (66 W-shapes). Therefore, the resulting search space had a size of approximately 6.36($10^{18}$) designs. For each beam member, the unbraced length was specified to be one-fifth of the span.

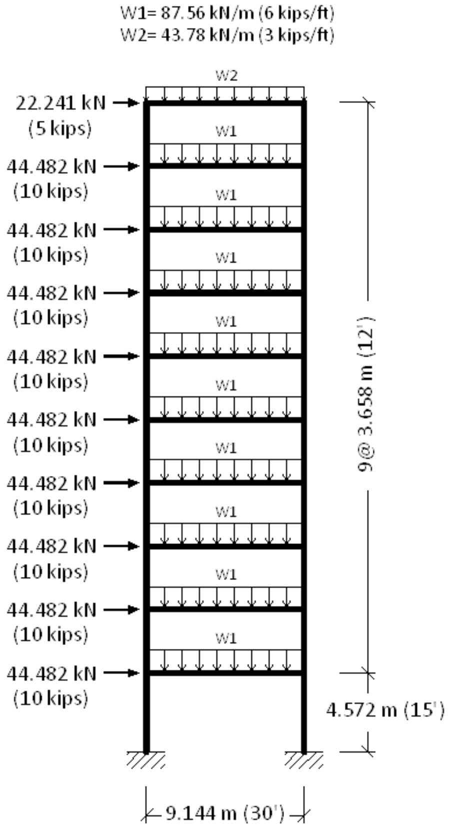


Fig. 6. One-bay, ten-storey problem

Similar to the previous example, 30 runs were conducted for each of the two design procedures and for each of the two cases, totalling 120 runs. Each run was limited to 60 generations. The parameter settings of MGA and MMDGA for this problem are listed in Table 4. It is noted that all the GA parameters are identical for both MGA and MMDGA procedures, except for the number of demes and the population size of each deme. However, the total population size was assumed to be the same for both algorithms so that the results could be compared. Also, because the frame has only one bay, the geometric and boosted geometric crossover in the level of bay would be redundant. The following is a summary of results for this problem.

Table 4. GA parameters used for one-bay, ten-storey problem

| | MGA | MMDGA |
|---|---|---|
| Population size | 60 | 20 |
| Number of demes | 1 | 3 |
| Number of elites | 2 | 2 |
| Crossover fraction | 60% | 60% |
| Standard crossover percentage | 30% | 30% |
| Geometric crossover percentage | 20% | 20% |
| Boosted crossover percentage | 30% | 30% |
| Boosted geometric crossover percentage | 20% | 20% |
| Standard mutation percentage | 30% | 30% |
| Sorting mutation percentage | 10% | 10% |
| Enhancing mutation percentage | 60% | 60% |
| Standard mutation probability | 0.2 | 0.2 |
| Migration rate | - | 10% |
| Migration interval | - | 10 |
| Migration direction | - | Both |

### *Case 1. Ignoring the second order effects*

Table 5 lists the optimum design details developed by the MGA and MMDGA and compares the results with those obtained through other metaheuristic algorithms. The best MGA design yielded a frame 4.1% lighter than the one obtained by the standard GA. It also improved by 0.25% on the optimum solution by ACO. However, the MGA design was heavier than the designs obtained by HS and IACO algorithms. On the other hand, MMDGA showed much better results. The best MMDGA design resulted in a frame that weighs 272.877 kN. This is 5.8% lighter than the design of the standard GA, 2.0% lighter than the design of ACO, 0.84% lighter than the design of HS and 0.76% lighter than the design obtained by IACO. Both modified GA algorithms have improved on the standard GA's performance, indicating the effectiveness of the proposed modifications to the GA's operators in enhancing the results.

Table 5. Design details comparison of one-bay, ten-storey frame; case 1

| Element group | AISC W-shapes | | | | | |
|---|---|---|---|---|---|---|
| | Pezeshk et al. [6] (GA) | Camp et al. [24] (ACO) | Degertekin [26] (HS) | Kaveh & Talatahari [28] (IACO) | This study | |
| | | | | | MGA | MMDGA |
| Beam 1-3S | W33×118 | W30×108 | W33×118 | W33×118 | W36×135 | W33×118 |
| Beam 4-6S | W30×90 | W30×90 | W30×99 | W30×90 | W30×99 | W30×108 |
| Beam 7-9S | W27×84 | W27×84 | W24×76 | W24×76 | W30×90 | W24×76 |
| Beam 10S | W24×55 | W21×44 | W18×46 | W14×30 | W18×40 | W16×40 |
| Column 1-2S | W14×233 | W14×233 | W14×211 | W14×233 | W14×211 | W12×230 |
| Column 3-4S | W14×176 | W14×176 | W14×176 | W14×176 | W14×159 | W14×159 |
| Column 5-6S | W14×159 | W14×145 | W14×145 | W14×145 | W14×132 | W14×120 |
| Column 7-8S | W14×99 | W14×99 | W14×90 | W14×90 | W12×87 | W14×90 |
| Column 9-10S | W12×79 | W12×65 | W14×61 | W12×65 | W12×53 | W12×58 |
| Weight (kN) | 289.739 | 278.503 | 275.185 | 274.990 | 277.819 | 272.877 |

*Note:* 1 N=0.225 lb; S=Storey

Table 6 gives a statistical report on the 30 runs of both the MGA and MMDGA solutions and compares the results with those obtained using other solutions. As can be observed, the heaviest frame weight in the 30 runs for both the MGA and MMDGA solutions is 285.458 kN. This is still lighter than the best design obtained by the standard GA, reported by Pezeshk et al. [6] as weighing 289.739 kN. Also, the MMDGA algorithm has produced better results in terms of average weight, standard deviation, and coefficient of variation compared to not only the standard GA, but also the MGA, ACO, HS and IACO methods. Moreover, in a series of 30 runs, the MMDGA produced its best solution in 43% of the runs which is better than both the ACO and MGA algorithms with corresponding values of 37% and 33%, respectively.

Table 6. Design results for one-bay, ten-storey frame; case 1

| | Pezeshk et al. [6] (GA) | Camp et al. [24] (ACO) | Degertekin [26] (HS) | Kaveh & Talatahari [28] (IACO) | This study | |
|---|---|---|---|---|---|---|
| | | | | | MGA | MMDGA |
| Best weight (kN) | 289.739 | 278.503 | 275.185 | 274.990 | 277.819 | 272.877 |
| Worst weight (kN) | N.R. | N.R. | N.R. | 285.870 | 285.458 | 284.078 |
| Average weight (kN) | N.R. | 281.608 | 279.895 | 278.040 | 280.378 | 279.487 |
| Standard deviation (kN) | N.R. | 3.042 | 7.740 | 2.740 | 3.103 | 2.860 |
| Coefficient of variation | N.R. | 1.1% | 2.8% | 1.0% | 1.1% | 1.0% |
| Average no. of F.E.A. | 3,000 | 8,300 | 3,600 | 2,500 | 2,400 | 2,550 |
| Average no. of F.E.A. for initially best design | 2,400 | 5,100 | 2,600 | N.R. | 1,600 | 1,800 |
| Percentage of the best design in different runs | N.R. | 37% | N.R. | N.R. | 33% | 43% |

*Note:* 1 N=0.225 lb; N.R.= Not Reported; F.E.A.= Finite Element Analysis

Regarding the level of computational effort, the MGA and MMDGA produced their optimum designs with averages of 2,400 and 2,550 frame analyses, respectively. These are not only less than the 3,000 analyses required by the standard GA solution, but are also much less than the 8,300 and 3,600 analyses required by the ACO and HS algorithms, respectively and are in par with the number of analyses required by IACO.

Figure 7 shows a typical convergence history for a best design of the one-bay, ten-storey frame in the MGA and MMDGA solution. In this figure, the vertical axis is in logarithmic scale. A rapid convergence for both the MGA and MMDGA solutions can be seen in this figure. This is attributed to the performance of the proposed modified GA reproduction operators.

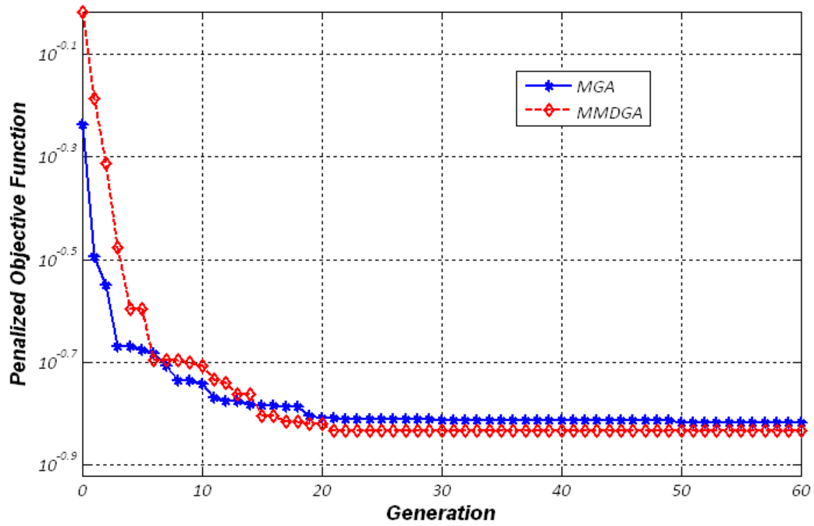


Fig. 7. Convergence history of one-bay, ten-storey frame (case 1)

Moreover, another advantage of the MMDGA algorithm in solving this problem can be seen in the fact that the combined strength constraint has a maximum ratio of 100.00% in all the members and the interstorey drift constraint has a maximum ratio of 98.86% in all the storeys as shown in Figs. 8a and 8b, respectively. Therefore, in this frame both the element strength (stress) ratio and the interstorey drift constraints dominate the optimum design.

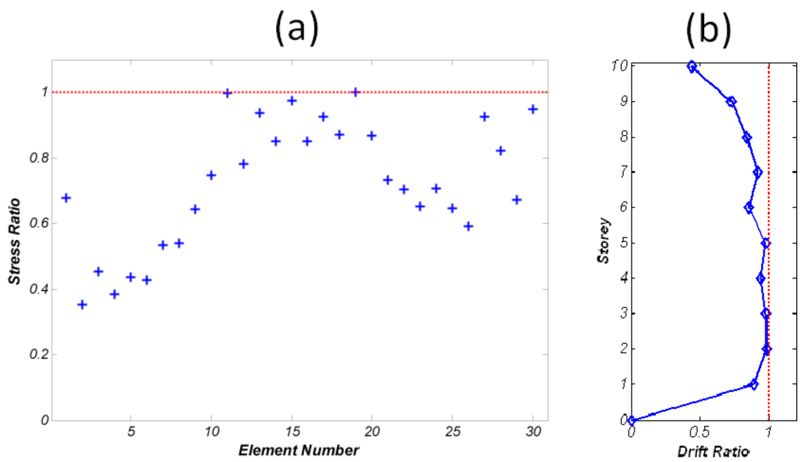


Fig. 8. (a) Stress ratios (b) Drift ratios; of one-bay, ten-storey frame (case 1)

In order that the effects of the number of demes on the quality of solution could be ascertained, similar to the previous example, the problem was run with 2, 3 and 6 demes corresponding to subpopulation sizes of 30, 20 and 10, respectively. It is observed that, although all three cases were capable of producing the optimum solution, the 3 demes solution with migration in both directions resulted in better average weight, standard deviation and coefficient of variation.

### *Case 2. Considering the second order effects*

Table 7 shows details of the best designs reached in the MGA and MMDGA solutions, as well as the results obtained by Pezeshk et al. [6] utilizing a standard GA. The MGA and MMDGA algorithms yielded respectively 7.6% and 8.0%, lighter frames compared to the one obtained by the standard GA. The average weight of the MGA and MMDGA designs are respectively 288.345 kN and 286.227 kN with coefficients of variation of 1.3% and 1.0%. Pezeshk et al [6] have not reported a corresponding value; therefore, comparisons cannot be made. Also, the MGA and MMDGA algorithms produced optimum designs at approximately 1,500 and 1,750 frame analyses, respectively; both less than the 2,400 frame analyses required by the standard GA to reach the first best solution. The MGA and MMDGA algorithms also terminated the search process after around 2,250 and 2,400 frame analyses, respectively. These are, again, less than the 3,000 frame analyses required by the standard GA to terminate the optimization process.

Table 7. Design details comparison of one-bay, ten-storey frame; case 2

| Element group | AISC W-shapes | | |
|---|---|---|---|
| | Pezeshk et al. [6] (GA) | This study | |
| | | MGA | MMDGA |
| Beam 1-3S | W36×150 | W36×135 | W33×118 |
| Beam 4-6S | W30×90 | W30×116 | W30×99 |
| Beam 7-9S | W27×84 | W27×94 | W30×99 |
| Beam 10S | W14×53 | W21×50 | W16×40 |
| Column 1-2S | W14×233 | W14×211 | W14×233 |
| Column 3-4S | W14×211 | W14×145 | W14×159 |
| Column 5-6S | W12×152 | W14×120 | W14×145 |
| Column 7-8S | W12×106 | W14×90 | W12×87 |
| Column 9-10S | W10×68 | W12×58 | W12×53 |
| Weight (kN) | 307.408 | 284.078 | 282.833 |

*Note:* 1 N=0.225 lb; S=Storey

The best optimum design is evidently produced by the MMDGA method and has a maximum combined strength (stress) ratio of 100.20% in all members; this is at the boundary of the feasible region. Also, the interstorey drift constraint has a maximum ratio of 98.55% in all the storeys.

### *c) Three-bay, 24-storey frame*

The last benchmark example is the three-bay, 24-storey steel frame, consisting of 168 members and undergoing a single load case as shown in Fig. 9. This frame was originally designed by Davison and Adams [38]. It was also designed by Camp et al. [24] using an ACO algorithm, by Degertekin [26] using an HS algorithm, by Kaveh and Talatahari [28] using an improved ACO (IACO) algorithm, and again by Kaveh and Talatahari [23] using an imperialist competitive algorithm (ICA). This frame was optimized according to the AISC-LRFD specification [33] subject to a displacement constraint of the form: interstorey drift < storey height/300. A modulus of elasticity of $E$ = 205 GPa (29,732 ksi) and a yield stress of $F_y$ = 230.3 MPa (33.4 ksi) were also used. The fabrication conditions required grouping the members as shown in Fig. 9, and resulted in 16 column sections and 4 beam sections for a total of 20 design variables. Each of the 4 beam element groups could be chosen from all the 267 W-shapes listed in AISC standard list, while the 16 column element groups were limited to W14 sections (37 W-shapes).

Therefore, the size of the resulting search space was approximately $6.27(10^{34})$ designs. All beams and columns were considered unbraced along their lengths.

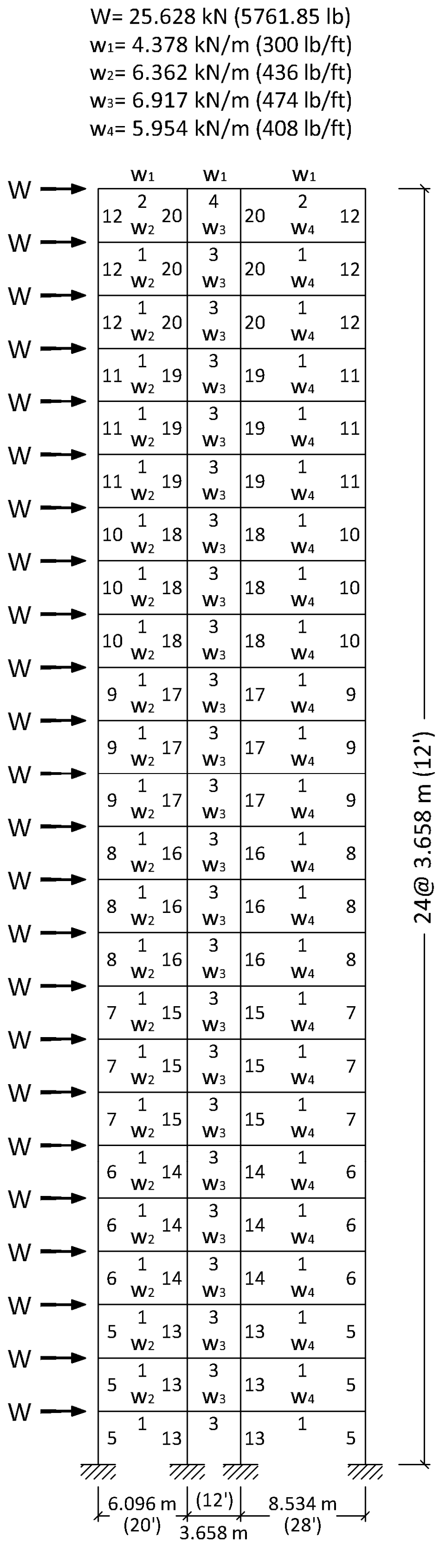


Fig. 9 Three-bay, 24-storey problem

The parameter settings of the MGA and the MMDGA are the same as the previous problem (see Table 4) except that the population size for the MGA is set to 80 and the number of demes is set to 4, each deme having 20 individuals. Also, the runs were limited to 100 generations. The following is a summary of the results.

***Case 1. Ignoring the second order effects***

Table 8 lists design details developed by the MGA, the MMDGA, the ACO [24], the HS [26], the IACO [28] and the ICA [23]. The best MGA design resulted in a frame that weighs 907.769 kN. This is 7.4% lighter than the design obtained by the ACO, 5.0% lighter than the result of the HS, 6.2% lighter than that of the IACO, and 4.1% lighter than the design obtained by ICA. The MMDGA has produced even better results. The best MMDGA design produced a frame that weighs 898.129 kN, which is 8.4% lighter than that of the ACO algorithm, 6.0% lighter than the design of the HS algorithm, 7.2% lighter than the IACO design and 5.1% lighter than the design obtained by the ICA.

Table 8. Design details comparison of three-bay, 24-storey frame; case 1

| Element group no. | AISC W-shapes | | | | | |
|---|---|---|---|---|---|---|
| | Camp et al. [24] (ACO) | Degertekin [26] (HS) | Kaveh & Talatahari [28] (IACO) | Kaveh & Talatahari [23] (ICA) | This study | |
| | | | | | MGA | MMDGA |
| 1 | W30×90 | W30×90 | W30×99 | W30×90 | W30×90 | W30X90 |
| 2 | W8×18 | W10×22 | W16×26 | W21×50 | W8×18 | W8X15 |
| 3 | W24×55 | W18×40 | W18×35 | W24×55 | W21×44 | W24X55 |
| 4 | W8×21 | W12×16 | W14×22 | W8×28 | W6×9 | W10X15 |
| 5 | W14×145 | W14×176 | W14×145 | W14×109 | W14×159 | W14X159 |
| 6 | W14×132 | W14×176 | W14×132 | W14×159 | W14×145 | W14X132 |
| 7 | W14×132 | W14×132 | W14×120 | W14×120 | W14×109 | W14X90 |
| 8 | W14×132 | W14×109 | W14×109 | W14×90 | W14×90 | W14X90 |
| 9 | W14×68 | W14×82 | W14×48 | W14×74 | W14×61 | W14X61 |
| 10 | W14×53 | W14×74 | W14×48 | W14×68 | W14×48 | W14X48 |
| 11 | W14×43 | W14×34 | W14×34 | W14×30 | W14×48 | W14X48 |
| 12 | W14×43 | W14×22 | W14×30 | W14×38 | W14×22 | W14X22 |
| 13 | W14×145 | W14×145 | W14×159 | W14×159 | W14×120 | W14X109 |
| 14 | W14×145 | W14×132 | W14×120 | W14×132 | W14×109 | W14X99 |
| 15 | W14×120 | W14×109 | W14×109 | W14×99 | W14×109 | W14X99 |
| 16 | W14×90 | W14×82 | W14×99 | W14×82 | W14×82 | W14X74 |
| 17 | W14×90 | W14×61 | W14×82 | W14×68 | W14×74 | W14X68 |
| 18 | W14×61 | W14×48 | W14×53 | W14×48 | W14×53 | W14X53 |
| 19 | W14×30 | W14×30 | W14×38 | W14×34 | W14×22 | W14X26 |
| 20 | W14×26 | W14×22 | W14×26 | W14×22 | W14×22 | W14X22 |
| Weight (kN) | 980.677 | 955.745 | 967.330 | 946.250 | 907.769 | 898.129 |

*Note:* 1 N=0.225 lb

The MGA and MMDGA solutions are further compared with those obtained by other metaheuristics in Table 9. It can be seen that both the MGA and MMDGA have produced either better or comparable results in terms of the average weight, standard deviation and coefficient of variation compared to the ACO, HS, IACO and ICA methods. In 30 runs, the MGA and MMDGA methods required approximately 5,000 and 5,150 frame analyses, respectively, to terminate the evolution process; these are significantly less than the 15,500 analyses required by the ACO, the 14,651 analyses required by the HS, and the 7,500 analyses performed by the ICA to termination. However, in this problem, on the number of required analyses the IACO algorithm appeared more efficient than both the MGA and MMDGA algorithms with only 3,500 frame analyses.

Table 9. Design results for three-bay, 24-storey frame; case 1

| | Camp et al. [24] (ACO) | Degertekin [26] (HS) | Kaveh & Talatahari [28] (IACO) | Kaveh & Talatahari [23] (ICA) | This study | |
|---|---|---|---|---|---|---|
| | | | | | MGA | MMDGA |
| Best weight (kN) | 980.677 | 955.745 | 967.330 | 946.250 | 907.769 | 898.129 |
| Worst weight (kN) | N.R. | N.R. | N.R. | N.R. | 968.685 | 954.892 |
| Average weight (kN) | 1,021.111 | 990.263 | 916.900 | N.R. | 922.817 | 919.925 |
| Standard deviation (kN) | 20.288 | 25.800 | 12.59 | N.R. | 18.982 | 15.409 |
| Coefficient of variation | 2.1% | 3.6% | 1.4% | N.R. | 2.1% | 1.7% |
| Average no. of F.E.A. | 15,500 | 14,651 | 3,500 | 7,500 | 5,000 | 5,150 |
| Average no. of F.E.A. for initially best design | 12,500 | 9,924 | N.R. | N.R. | 4,200 | 4,750 |
| Percentage of the best design in different runs | N.R. | N.R. | N.R. | N.R. | 27% | 37% |

*Note:* 1 N=0.225 lb; N.R.= Not Reported; F.E.A.= Finite Element Analysis

Figure 10 shows typical convergence history for one of the best designs of both the MGA and MMDGA. In this figure, a rapid decrease in the MGA optimization process curve can be observed in the initial stages of the evolution. On the other hand, the MMDGA appears to follow a gradual approach towards its best solution, nevertheless it finally reaches a better solution.

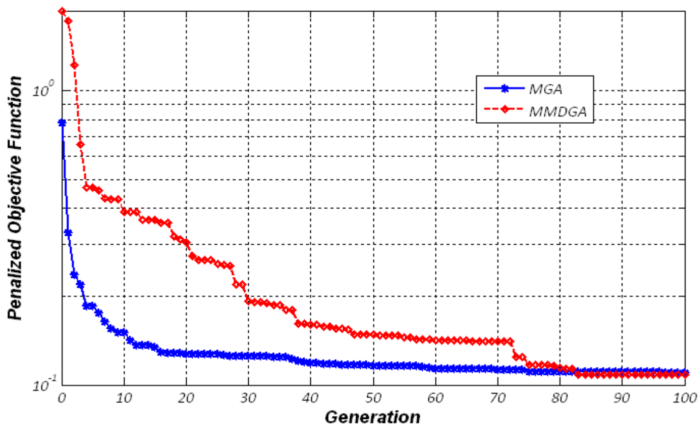


Fig. 10. Convergence history of three-bay, 24-storey frame (case 1)

It should be noted that in the best design, obtained by the MMDGA, the interstorey drift controls the design process; the strength requirements for both the beams and the columns do not appear to be critical to the design. The interstorey drift constraint has a maximum ratio of 100.01% in all the storeys, whereas, the maximum combined strength (stress) ratio has a value of 80.80% as shown in Fig. 11. It is also interesting to note that in the best design the interstorey drift constraint is within 90% of its upper limit in 17 storeys out of 24 as shown in Fig. 11b.

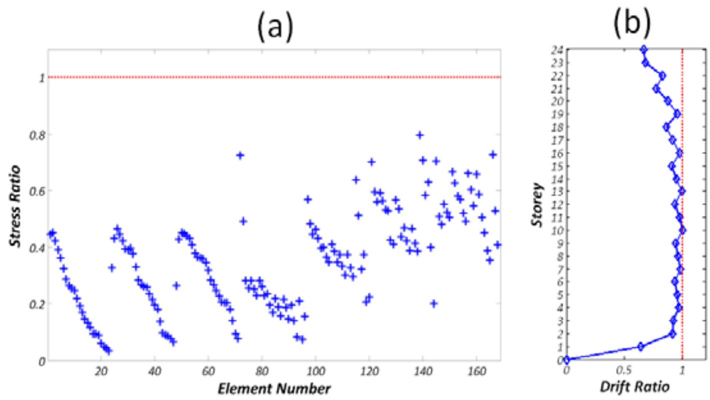


Fig. 11. (a) Stress ratios (b) Drift ratios; of three-bay, 24-storey frame (case 1)

### *Case 2. Considering the second order effects*

For this case, also 30 runs were made in solutions by the MGA and MMDGA. It is interesting to observe that the best design in this case is the same design as that of the case 1. It was obtained when the second order effects were considered. This is because the AISC-LRFD code of practice magnifies the sway and non-sway moments through the $B_1$ and $B_2$ multipliers and the lateral displacement of the frame is not amplified. Also, the best design for this problem was found in the previous case to be governed by the interstorey drift which is not altered in this case.

### *d) On the performance of each reproduction operator*

To investigate the performance of the different GA operators used, the percentage of the 'successful children' relative to the total children created by each operator in the one-bay, ten-storey frame (case 1) as a typical case, is shown in Fig. 12a. Here, the term 'successful' means that the child is better than, or at least has an equal score to that of its parent or parents. Also, Fig. 12b shows the percentage of the 'absolutely successful' children for each operator. An 'absolutely successful' child means that the child has a better score compared to that of its parent or parents. This figure shows that the geometric, boosted and the boosted geometric crossover operators, as well as the sorting and enhancing mutation operators have better success compared to the standard GA operators. Moreover, Fig. 12c gives further information regarding the question of what is the contribution of each type of operator in creating the best or the worst child during the whole evolution process. This figure shows that the boosted crossover has created the highest percentage of best children while the standard mutation operator has created the highest percentage of worst children for this problem and for the selected run. However, this should not be considered as a general conclusion as the other modified operators have performed better in other runs or in other problems.

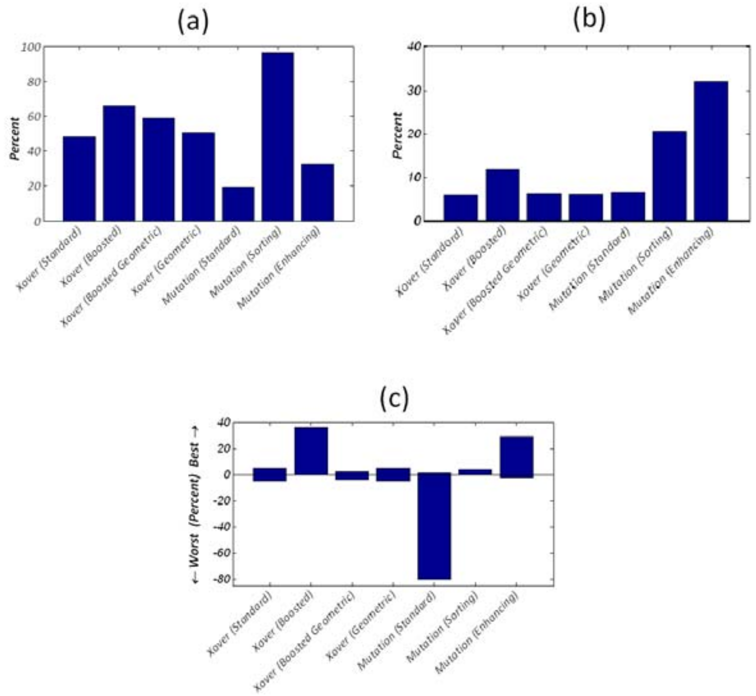


Fig. 12. (a) Successful children (b) Absolutely successful children (c) New best and new worst children; of one-bay, ten-storey frame (case 1)

## 5. CONCLUSION

In this paper a number of new crossover and mutation operators used previously alongside the standard operators for optimum allowable stress design of steel frames were utilised, again, to optimally design a number of steel frames subjected to the constraints of the AISC-LRFD specification, with and without considering the second order effects, as set out by the code requirements. This modified GA (MGA) was shown to have a very fast convergence and to produce relatively high-quality designs. This paper also utilized the concept of multiple-deme in the GA, as it has been used successfully for other metaheuristic population-based methods. The multiple-deme GA was used alongside the modified GA operators and the algorithm is named the modified multiple-deme GA (MMDGA). The MGA and MMDGA were tested on three benchmark steel frame problems. These example problems demonstrated the efficiency and applicability of the modified GA methods to design steel frame structures satisfying AISC-LRFD specification and other constraints. More specific conclusions drawn from the results of this investigation may be summed up as follows:

1) Regarding the optimum design solution, both the MGA and MMDGA produced much lighter designs compared to the standard GA; MMDGA fares better than the MGA. The MMDGA invariably produced lighter designs compared to all other metaheuristics. For the one-bay, ten-storey and the three-bay, 24-storey problems without considering the second order effects, MMDGA produced, respectively, 0.76% and 5.1% lighter frames compared to the previous best designs. In the case of the one-bay, ten-storey problem undergoing second order effects, the improvement on the previous best design increases to 8.0%.

2) Regarding the computational effort needed for analyses, both MGA and MMDGA required much smaller number of frame analyses compared to most other metaheuristic methods. In the two-bay, three-storey problem, the two algorithms required less than one/tenth the number of frame analyses required by the best previously obtained results. For the one-bay, ten-storey problem with and without due consideration for second order effects, the numbers of frame analyses required by the MGA and MMDGA were comparable with that of the IACO but much smaller than that required for the GA, ACO and HS methods. The same is true for the three-bay, 24-storey problem except that the IACO algorithm required less computational effort.

3) For the examples considered, in every run both the MGA and MMDGA produced frames with weights close to the weights of their optimum designs, showing consistency of the solutions. Standard deviations and coefficients of variation were also quite small in all the examples, proving the robustness of the two algorithms.

4) For the examples solved, considering the second order effects as set out by the AISC-LRFD requirements, the optimum design solutions were not considerably changed. However, this may not be true when we carry out an actual geometrical second order analysis rather than the approximate method of the code of practice [6].

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