

## DETERMINATION OF ULTIMATE LOAD CAPACITY OF CONICAL AND PYRAMIDAL SHELL FOUNDATIONS USING DIMENSIONAL ANALYSIS\*

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**Abstract**– Surveys of the literature indicate that shell foundations are economical structural elements which can be considered as the alternatives of flat foundations. However, the advantage of shell elements in geotechnical engineering has not been explored yet, and these foundations are still being treated as flat footings. The objective of this study is to investigate the geotechnical behavior of two types of shell foundations under axial loading and present a comprehensive formulation for bearing capacity of such foundations. For this purpose, a series of laboratory tests were carried out on six types of shell foundations, namely conical and pyramidal shell foundations. Different shell foundation geometries and Buckingham-Pi theorem were employed to formulate the ultimate load capacity. Experimental results from previous investigations on shell footings were used to verify the proposed formulations. Results of the present laboratory tests have indicated that the pyramidal shell foundations show higher bearing capacities compared to their corresponding conical ones and as the thickness of foundation increases, bearing capacity decreases. Also, load bearing capacity equations of shell foundations determined from dimensional analysis have shown a reasonably good agreement with experimental results.

**Keywords**– Shell foundation, Buckingham-Pi theorem, non-dimensional parameter, laboratory test, ultimate load capacity, experimental investigation

### 1. INTRODUCTION

#### a) *Buckingham-Pi theorem*

Buckingham-Pi theorem explains how for a physical problem including “n” quantities with “m” main dimensions, the quantities can be arranged in the form of “n-m” independent non-dimensional parameters. Assume  $A_1, A_2, A_3, \dots, A_n$  are the given quantities of a problem and are recognized as influential factors in the response of the problem. Equation (1) shows the possible relationship between all quantities.

$$F(A_1, A_2, A_3, \dots, A_n) = 0 \quad (1)$$

If  $\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}$  are defined as non-dimensional parameters, Eq. (1) can be restated in Eq. (2):

$$F(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0 \quad (2)$$

In order to attain non-dimensional parameters, the “m” numbers of quantities with different dimensions are selected and considered as repeating variables. Then every  $\pi$  can be written as a result of

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\*Received by the editors August 11, 2011; Accepted November 5, 2013.

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multiplication of every repeating variable with every other quantity suppose that if quantities  $A_1, A_2, A_3$  collectively include the main dimensions of  $M, L, T$  then  $\pi_i (i = 1 : n - m)$  will be defined:

$$\begin{aligned}\pi_1 &= A_1^{x_1} . A_2^{y_1} . A_3^{z_1} . A_4 \\ \pi_2 &= A_1^{x_2} . A_2^{y_2} . A_3^{z_2} . A_5 \\ &\cdot \\ &\cdot \\ \pi_{n-m} &= A_1^{x_{n-m}} . A_2^{y_{n-m}} . A_3^{z_{n-m}} . A_n\end{aligned}\tag{3}$$

The values of  $x_i, y_i, z_i (i = 1 : n - m)$  are defined so that  $\pi_i (i = 1 : n - m)$  become non-dimensional parameters.

Phatak and Dhonde [1] discussed an experiment that provided a new method for engineers and encouraged the use of this method. They discussed using dimensional analysis to formulate the load and settlement equation for spread footing in sand. The authors also performed a dimensional analysis method [2] to formulate the ultimate torsional strength of reinforced concrete beams using only two experimental results. They demonstrated a unique trial and error procedure that generates the universal dimensional analysis formulation, wherein one of the experimental data sets called the control point, generated the dimensional analysis equation, while in the other data set, the check point was used to validate the already formed dimensional analysis equation. The results of dimensional analysis were then compared with the available experimental results. Results indicated that dimensional analysis could be used to predict results reasonably well and was shown to be an easy and sufficiently accurate method of analysis.

Corrado and Carpinteri [3] applied Dimensional Analysis to a numerical approach based on Nonlinear Fracture Mechanics in order to obtain a synthetic description of the rotational capacity of reinforced concrete beams in bending, otherwise impossible to achieve due to the presence of numerous variables and mechanical nonlinearities. They showed that although the proposed model relied on several mechanical properties of concrete and steel and on the beam size, only two non-dimensional parameters, NP and NC, were responsible for the beam ductility.

### b) Shell foundations

Shell foundations are structural elements with various geometrical shapes such as, pyramidal, conical, triangular and hyper. They are economic alternatives to flat shallow foundations where heavy super structural loads are to be transmitted to weaker soils.

Due to their many advantages, shell foundations have attracted many researchers since the 1970s worldwide. Iyer and Rao [4] conducted a series of experimental tests to investigate the bearing capacity of shell foundations and compared the results with their plain counterparts. The results indicated that the bearing capacity of shell foundations is more than that for flat foundations. This difference was related to the stiffness and geometry of shell elements. Kurian and Jeyachandran [5] conducted experimental tests on various shell foundations and their plain counterparts to investigate the effect of footing configuration on the bearing capacity. Agarwal and Gupta [6] performed tests on conical, hyper and their plain counterpart's foundation under axial loading on sand. The results indicated that an increase in the bearing capacity of shell foundations is related to the difference in footing configuration and interface within footing and soil. Hanna and Abdel-Rahman [7] investigated the behavior of shell foundations in terms of bearing capacity and settlement. They performed their tests on conical, triangular and pyramidal shell foundations and circular, strip and square flat foundations. They noted that shell foundations performance is better than flat foundations and failure surfaces in the former are deeper than the latter. Kurian and

Varghese [8], Kurian and Mohan [9], and Kurian [10] reported on the bearing capacity and distribution of the contact pressure of shell foundations. Esmaili and Hataf [11] carried out a series of loading tests to investigate the influence of shell configuration on ultimate bearing capacity of shell foundations on reinforced and unreinforced sand. They concluded that while shell foundations behavior is more similar to flat footings, ultimate bearing capacity in both reinforced and unreinforced sand decreases.

According to the published articles, it is obvious that few investigations have been done from a geotechnical viewpoint on the shell foundations. Therefore, in order to introduce shell foundations as a reliable structural element in engineering and to find out new relationships in this field, it is necessary to have further research. In this study, a series of laboratory tests were performed on six conical and pyramidal shell foundations in order to generate bearing capacity formulation for conical and pyramidal shell foundations. Then, accuracy of the equations was verified with the experimental results of Hanna and Abdel Rahman [7].

## 2. LABORATORY TESTS

### a) Sand characteristics

Particle size distribution curve of the soil used in this study is shown in Fig. 1. According to the Unified Soil Classification System (USCS), the soil was classified as well graded sand (SW). Table 1 also shows shear strength parameters obtained from direct shear test, unit weight and relative density of the sand.

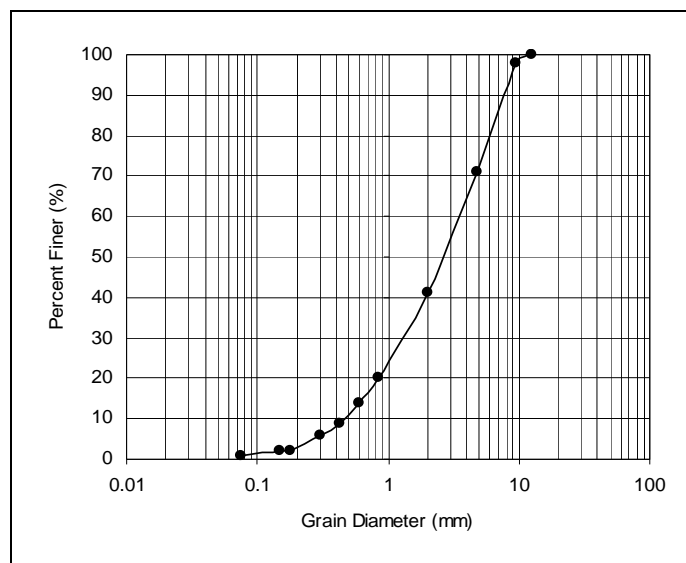


Fig. 1. Soil gradation curve from sieve test

Table 1. Characteristics of the sand tested

| Soil parameters | Angle of shearing resistance ( $\phi^{\circ}$ ) | Dry unit weight $\gamma_d \left( \frac{kN}{m^3} \right)$ | Cohesion $c$ (kPa) | Relative density $Dr$ (%) |
|-----------------|---|--|--------------------|---------------------------|
| Value           | 37  | 15   | 0                  | 36                        |

### b) Footing models and test apparatus

Two types of shell foundations (i.e. conical and pyramidal) made up of cast iron were used in this study to represent the axisymmetric and three-dimensional conditions, respectively (Fig. 2).



Fig. 2. Conical and pyramidal shell foundations constructed with cast iron for axisymmetric and three-dimensional test conditions

To examine the effect of the shell geometry on the ultimate load capacity, three types of conical and pyramidal model shell foundations have been made and tested. Figure 3 and Table 2 show the geometrical configuration and dimensions of these models, respectively.

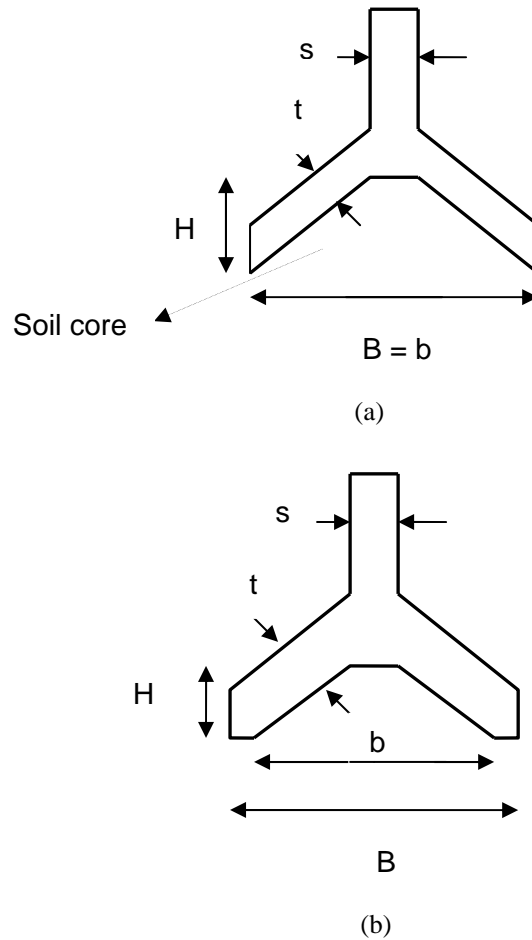


Fig. 3. Geometrical configuration of the foundation models, (a) Conical and pyramidal shell models, type I, (b) Conical and pyramidal shell models, types II and III

Table 2. Dimensions of footing models

| Dimensions                | H<br>(mm) | B<br>(mm) | s<br>(mm) | t<br>(mm) | b<br>(mm) |
|---------------------------|-----------|-----------|-----------|-----------|-----------|
| Shell foundation type I   | 80        | 160       | 40        | 25        | 160       |
| Shell foundation type II  | 64        | 160       | 40        | 35        | 136       |
| Shell foundation type III | 39        | 160       | 40        | 50        | 98        |

A cylindrical tank of the tests was made of steel with the diameter/height of 1 meter and thickness of 4 mm to simulate axisymmetric loading condition. The load was applied on the foundations using the system of a simple lever and the foundation settlement was measured using two dial gauges mounted on two opposite sides of footings. To model three-dimensional condition a box with the dimensions of 1.00×1.00×1.00 meter was built for testing the pyramidal model shell foundations. The loading equipment for these tests had a hydraulic cylinder with a high precision and the settlements were measured in the same way as axisymmetric tests. Figs. 4 and 5 show the unit test boxes utilized for testing conical and pyramidal shell foundations, respectively.



Fig. 4. The unit test box utilized for testing conical shell foundations



Fig. 5. The unit test box utilized for testing pyramidal shell foundations

### 3. TEST PROCEDURE

A total of 12 loading tests were performed on the cited shallow footing models. In order to maintain constant relative density while setting up the test boxes, volume control method was employed in which the inside of the test tanks was marked and soil was compacted in every 50 mm-lift. To obtain a uniform compaction, each layer was tamped using a wood plate with a diameter of 300 mm, dropping from a 150 mm height to get dry unit weight and relative density values about  $15 \text{ kN/m}^3$  and 36%, respectively. After filling the box, the foundation was located on the sand surface at the center of the box while sand paper

was fixed to the internal area and base of foundation to achieve a rough interface at the soil-foundation interface area. The soil core under the shell foundation model was prepared by filling the void space up with sand at the same dry unit weight as the soil in the box. Then, a thin steel plate was placed at the bottom of the shell model before placing it on the sand surface in box. The steel plate was then pulled out from underneath the shell, slowly. The footing was loaded statically using dead weight and hydraulic jack system in axisymmetric and three-dimensional conditions, respectively. In every test, each loading increment was applied on the foundation as long as the settlement reached less than 0.01 mm/min. Then, the loads increased at the same increments until sand failure was observed under foundation. After readings, the load-settlement curve was plotted and the ultimate load was calculated. Since local shear failure was observed in all tests, the ultimate load was obtained by two tangents plotted along the initial and latter portion of the load-settlement curve and the load corresponding to the intersection point of these two lines is taken as ultimate load of the foundations.

**4. TEST RESULTS**

Figure 6 shows load-settlement curves for tests on pyramidal shell foundation. The ultimate load capacities obtained from test results are also illustrated in Table 3.

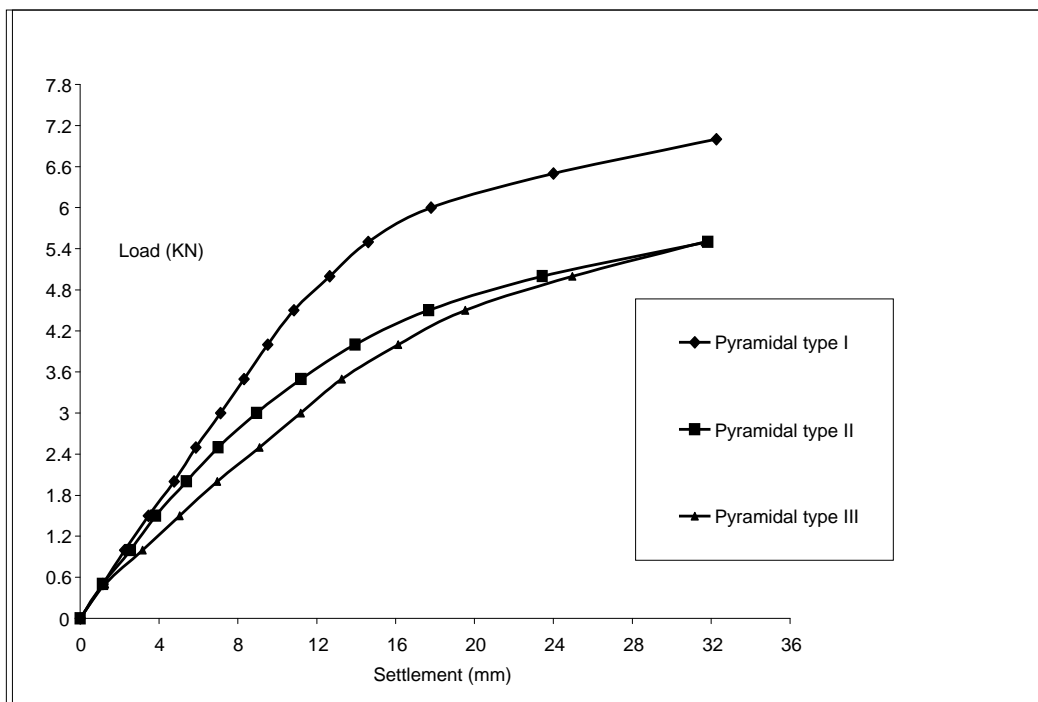


Fig. 6. Load-settlement curves for pyramidal foundations

Table 3. Ultimate load for axisymmetrical and three dimensional tests

| Foundation types       | Type I | Type II | Type III |
|------------------------|--------|---------|----------|
| Ultimate load (KN) for |        |         |          |
| Conical                | 4.75   | 3.8     | 3.4      |
| Pyramidal              | 5.85   | 4.2     | 3.85     |

**5. FORMULATION**

In this paper, soil properties and influential geometrical parameters of laboratory footing models on the ultimate load capacity of conical and pyramidal shell foundations were gathered in the set “V” (Eq. (4)).

$$V = \{F, \gamma_d, H, b, t, \phi, D_r\} \quad (4)$$

Where:

F: Ultimate load capacity of shell foundations

$\gamma_d$ : Dry unit weight of sand

$\phi$ : Angle of shearing resistance of sand

$D_r$ : Relative Density of sand

H, b: Height and dimension of soil core

t: Thickness of shell models

If the dimensions of parameters in Eq. set (4) are defined as below:

$$F = MLT^{-2}$$

$$\gamma_d = ML^{-2}T^{-2}$$

$$H = L$$

$$b = L$$

$$t = L$$

and  $D_r$  = non-dimension

Then, Eq. set (5) can be proposed as follows (Eq. (5)):

$$V = \left\{ MLT^{-2}, ML^{-2}T^{-2}, L, L, L, none, none \right\} \quad (5)$$

Since the term  $MLT^{-2}$  is a force dimension, then:

$$V = \left\{ F, FL^{-3}, L, L, L, non, non \right\} \quad (6)$$

According to the Buckingham-Pi theorem, the numbers of non-dimensional parameters are  $7-2 = 5$ , and non-dimensional groups can be expressed in the following form:

$$\pi_1 = \{F, \gamma_d, H\}$$

$$\pi_2 = \{F, \gamma_d, b\}$$

$$\pi_3 = \{F, \gamma_d, t\}$$

$$\pi_4 = \{F, \gamma_d, \phi\}$$

$$\pi_5 = \{F, \gamma_d, D_r\}$$

Now, if  $\pi_1 = F^a \cdot \gamma_d^d \cdot H$

The values of "a" and "d" can be calculated in the following form:

$$(MLT^{-2})^a \cdot (ML^{-2}T^{-2})^d \cdot L = (M^0 L^0 T^0)$$

$$\begin{cases} a + d = 0 \\ a - 2d + 1 = 0 \end{cases} \Rightarrow a = -1/3, d = 1/3$$

Therefore, the first non-dimensional parameter is written as shown in Eq. (7).

$$\pi_1 = F^{-1/3} \cdot \gamma_d^{1/3} \cdot H \tag{7}$$

In line with this method the values of  $\pi_2, \pi_3$  were also calculated:

$$\begin{aligned} \pi_2 &= F^{-1/3} \cdot \gamma_d^{1/3} \cdot b \\ \pi_3 &= F^{-1/3} \cdot \gamma_d^{1/3} \cdot t \end{aligned}$$

Also, if  $\pi_4 = F^y \cdot \gamma_d^z \cdot \phi$

The values of “y” and “z” can be calculated as below:

$$\begin{aligned} (MLT^{-2})^y \cdot (ML^{-2}T^{-2})^z \cdot (M^0L^0T^0) &= (M^0L^0T^0) \\ \begin{cases} y + z = 0 \\ y - 2z = 0 \end{cases} &\Rightarrow y = z = 0 \end{aligned}$$

Therefore,  $\pi_4 = \phi$  and  $\pi_5 = D_r$ .

Now, if  $\pi_3$  is considered as the result of the multiplication of  $\pi_1, \pi_2, \pi_4, \pi_5$ , Eq. (8) is attained:

$$F^{-1/3} \cdot \gamma_d^{1/3} \cdot t = \beta_1 \cdot (F^{-1/3} \cdot \gamma_d^{1/3} \cdot H)^{\beta_2} \cdot (F^{-1/3} \cdot \gamma_d^{1/3} \cdot b)^{\beta_3} \cdot \phi^{\beta_4} \cdot D_r^{\beta_5} \tag{8}$$

The simplification of Eq. (8) leads to:

$$F = \left[ \beta_1 \cdot \gamma_d^{1/3(\beta_2 + \beta_3 - 1)} \cdot t^{-1} \cdot H^{\beta_2} \cdot b^{\beta_3} \cdot \phi^{\beta_4} \cdot D_r^{\beta_5} \right]^{1/3(\beta_2 + \beta_3 - 1)} \tag{9}$$

Equation (9) shows the general form of ultimate load capacity of conical and pyramidal shell foundations.

To attain the constants  $\beta_i (i = 1 : 5)$ , the parameters and results of loading tests on the different types of conical and pyramidal shell models as shown in Table 4 were used. In addition to the data in Table 4, because of the multiplicity of constants, some of the test results attained by Hanna and Abdel Rahman [7] were also used (Table 5).

Table 4. Parameters and results of laboratory tests (summarized from Tables 1, 2 and 3)

| Type of foundation | H (m) | b(m)  | t(m)  | $\gamma_d (KN / m^3)$ | $\phi(^{\circ})$ | $D_r (%)$ | F (KN) |
|--------------------|-------|-------|-------|-----------------------|------------------|-----------|--------|
| Conical type I     | 0.08  | 0.16  | 0.025 | 15                    | 37               | 36        | 4.75   |
| Conical type II    | 0.064 | 0.136 | 0.035 | 15                    | 37               | 36        | 3.8    |
| Conical type III   | 0.039 | 0.098 | 0.05  | 15                    | 37               | 36        | 3.4    |
| Pyramidal type I   | 0.08  | 0.16  | 0.025 | 15                    | 37               | 36        | 5.85   |
| Pyramidal type II  | 0.064 | 0.136 | 0.035 | 15                    | 37               | 36        | 4.2    |
| Pyramidal type III | 0.039 | 0.098 | 0.05  | 15                    | 37               | 36        | 3.85   |

Table 5. Parameters and results of experimental investigation by Hanna and Abdel Rahman [7]

| Type of foundation | H (m) | b (m) | t(m)  | $\gamma_d (KN / m^3)$ | $\phi(^{\circ})$ | $D_r (%)$ | F (KN) |
|--------------------|-------|-------|-------|-----------------------|------------------|-----------|--------|
| Conical            | 0.04  | 0.16  | 0.03  | 18.5                  | 41               | 79        | 7.73   |
|                    | 0.08  | 0.16  | 0.025 | 17.7                  | 38               | 57        | 4.816  |
| Pyramidal          | 0.04  | 0.16  | 0.03  | 17.7                  | 38               | 57        | 5.686  |
|                    | 0.08  | 0.16  | 0.025 | 18.5                  | 41               | 79        | 11.111 |



Considering Eq. (9) and Tables 4 and 5, nonlinear Eqs. set (10) for conical shell foundations are defined as:

$$\left\{ \begin{aligned} & \left[ \beta_1 \cdot (0.08)^{\beta_2} \cdot (15)^{1/3(\beta_2+\beta_3-1)} \cdot (0.16)^{\beta_3} \cdot (0.025)^{-1} \cdot (37)^{\beta_4} \cdot (0.36)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 4.75 \\ & \left[ \beta_1 \cdot (0.064)^{\beta_2} \cdot (15)^{1/3(\beta_2+\beta_3-1)} \cdot (0.136)^{\beta_3} \cdot (0.035)^{-1} \cdot (37)^{\beta_4} \cdot (0.36)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 3.8 \\ & \left[ \beta_1 \cdot (0.039)^{\beta_2} \cdot (15)^{1/3(\beta_2+\beta_3-1)} \cdot (0.098)^{\beta_3} \cdot (0.05)^{-1} \cdot (37)^{\beta_4} \cdot (0.36)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 3.4 \\ & \left[ \beta_1 \cdot (0.04)^{\beta_2} \cdot (18.5)^{1/3(\beta_2+\beta_3-1)} \cdot (0.16)^{\beta_3} \cdot (0.03)^{-1} \cdot (41)^{\beta_4} \cdot (0.79)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 7.73 \\ & \left[ \beta_1 \cdot (0.08)^{\beta_2} \cdot (17.7)^{1/3(\beta_2+\beta_3-1)} \cdot (0.16)^{\beta_3} \cdot (0.025)^{-1} \cdot (38)^{\beta_4} \cdot (0.57)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 4.816 \end{aligned} \right. \quad (10)$$

Then, the solution of Eq. set (10) leads to the following values:

$$\left\{ \begin{aligned} \beta_1 &= 0.3966 \\ \beta_2 &= -0.3104 \\ \beta_3 &= -1.3771 \\ \beta_4 &= -1.4486 \\ \beta_5 &= -0.3701 \end{aligned} \right.$$

Finally, based on the values of  $\beta$  and Eq. (6), ultimate load formulation of conical shell foundations is in the form of Eq. (11):

$$F = 2.81 \times \gamma_d \times t^{1.12} \times H^{0.348} \times b^{1.54} \times \phi^{1.62} \times D_r^{0.41} \quad (11)$$

For pyramidal shell foundations, nonlinear Eqs. set (12) are defined as:

$$\left\{ \begin{aligned} & \left[ \beta_1 \cdot (0.08)^{\beta_2} \cdot (15)^{1/3(\beta_2+\beta_3-1)} \cdot (0.16)^{\beta_3} \cdot (0.025)^{-1} \cdot (37)^{\beta_4} \cdot (0.36)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 5.85 \\ & \left[ \beta_1 \cdot (0.064)^{\beta_2} \cdot (15)^{1/3(\beta_2+\beta_3-1)} \cdot (0.136)^{\beta_3} \cdot (0.035)^{-1} \cdot (37)^{\beta_4} \cdot (0.36)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 4.2 \\ & \left[ \beta_1 \cdot (0.039)^{\beta_2} \cdot (15)^{1/3(\beta_2+\beta_3-1)} \cdot (0.098)^{\beta_3} \cdot (0.05)^{-1} \cdot (37)^{\beta_4} \cdot (0.36)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 3.85 \\ & \left[ \beta_1 \cdot (0.04)^{\beta_2} \cdot (17.7)^{1/3(\beta_2+\beta_3-1)} \cdot (0.16)^{\beta_3} \cdot (0.03)^{-1} \cdot (38)^{\beta_4} \cdot (0.57)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 5.686 \\ & \left[ \beta_1 \cdot (0.08)^{\beta_2} \cdot (18.5)^{1/3(\beta_2+\beta_3-1)} \cdot (0.16)^{\beta_3} \cdot (0.025)^{-1} \cdot (41)^{\beta_4} \cdot (0.79)^{\beta_5} \right]^{1/3(\beta_2+\beta_3-1)} = 11.111 \end{aligned} \right. \quad (12)$$

Then, the solution of nonlinear Eqs. set (12) leads to the following values:

$$\left\{ \begin{aligned} \beta_1 &= 185.2542 \\ \beta_2 &= -0.6848 \\ \beta_3 &= -1.2375 \\ \beta_4 &= -3.3578 \\ \beta_5 &= -0.2357 \end{aligned} \right.$$

Now, based on the values of  $\beta$  and Eq. (9), ultimate load formulation of pyramidal shell foundations is in the form of Eq. (13):

$$F = 4.6 \times 10^{-3} \times \gamma_d \times t^{1.03} \times H^{0.7} \times b^{1.27} \times \phi^{3.46} \times D_r^{0.24} \tag{13}$$

Figures 7 and 8 show that the proposed formulation error (formulations 11, 13) is acceptable.

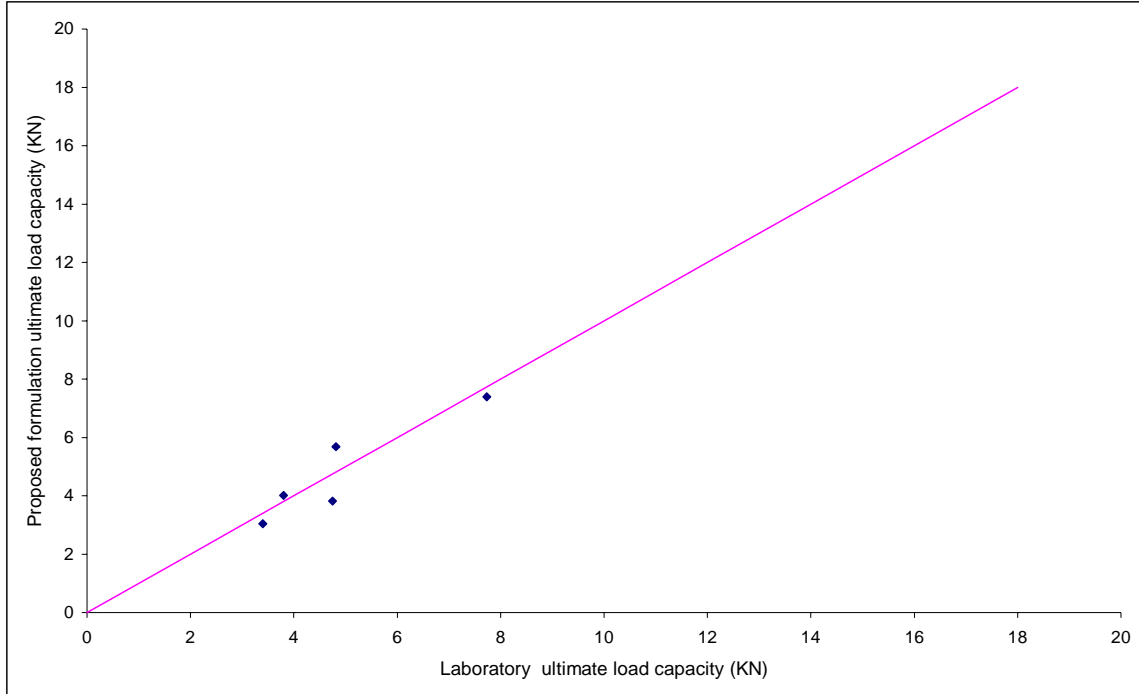


Fig.7. Comparison between proposed formulation of ultimate load capacity and laboratory ultimate load capacity for conical shell foundation

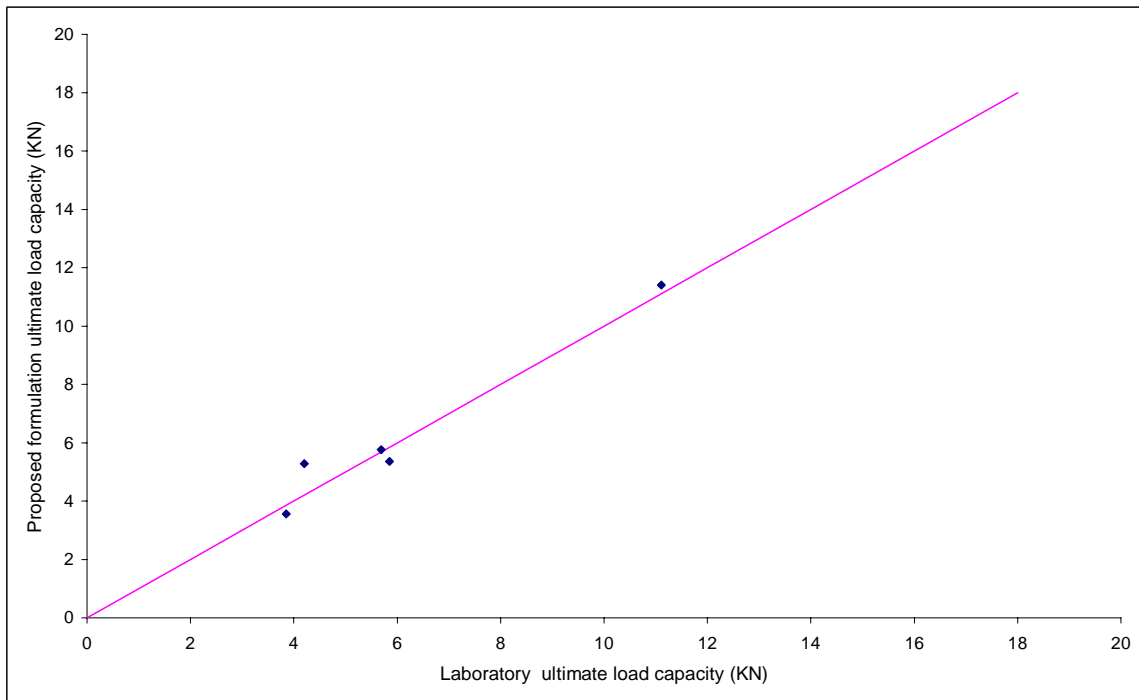


Fig. 8. Comparison between proposed formulation of ultimate load capacity and laboratory ultimate load capacity for pyramidal shell foundation

The following examples show that the ultimate load capacity obtained by the proposed formulations for conical and pyramidal shell foundations are in overall agreement with the experimental test results on shell footings conducted by Hanna and Abdel Rahman [7] and Fernando et al. [12] which were not used in the solution of nonlinear Eqs. sets (10) and (12).

**Example 1.** For conical shell foundation:

Table 6. Reported tests results for conical shell foundations by Hanna and Abdel Rahman [7]

| H(m) | t(m)  | b(m) | $\gamma_d (KN / m^3)$ | $D_r (%)$ | $\phi(^{\circ})$ | F(KN) |
|------|-------|------|-----------------------|-----------|------------------|-------|
| 0.08 | 0.025 | 0.16 | 16.5                  | 22        | 34               | 2.457 |

According to Table 6 and Eq. (11):

$$F = 2.81 \times 16.5 \times 0.025^{1.12} \times 0.08^{0.348} \times 0.16^{1.54} \times 34^{1.62} \times 0.22^{0.41} = 2.99 \text{KN}$$

**Compare with 2.457 KN**

**Example 2.** For pyramidal shell foundation:

Table 7. Reported tests results for pyramidal shell foundations by Hanna and Abdel Rahman [7]

| H(m) | t(m) | b(m) | $\gamma_d (KN / m^3)$ | $D_r (%)$ | $\phi(^{\circ})$ | F(KN) |
|------|------|------|-----------------------|-----------|------------------|-------|
| 0.04 | 0.03 | 0.16 | 16.5                  | 22        | 34               | 2.854 |

According to Table 7 and Eq. (13):

$$F = 4.6 \times 10^{-3} \times 16.5 \times 0.03^{1.03} \times 0.04^{0.7} \times 0.16^{1.27} \times 34^{3.46} \times 0.22^{0.24} = 2.91 \text{KN}$$

**Compare with 2.854 KN**

**Example 3.** For conical shell foundation:

Table 8. Reported tests results for conical shell foundations by Fernando et al. [12]

| H(m)  | t(m)  | b(m) | $\gamma_d (KN / m^3)$ | $D_r (%)$ | $\phi(^{\circ})$ | F(KN) |
|-------|-------|------|-----------------------|-----------|------------------|-------|
| 0.028 | 0.043 | 0.05 | 16.3                  | 61.35     | 43               | 1.496 |

According to Table 8 and Eq. (11):

$$F = 2.81 \times 16.3 \times 0.043^{1.12} \times 0.028^{0.348} \times 0.05^{1.54} \times 43^{1.62} \times 0.6135^{0.41} = 1.398 \text{KN}$$

**Compare with 1.496 KN**

**Example 4.** For pyramidal shell foundation:

Table 9. Reported tests results for pyramidal shell foundations by Fernando et al. [12]

| H(m)  | t(m)  | b(m) | $\gamma_d (KN / m^3)$ | $D_r (%)$ | $\phi(^{\circ})$ | F(KN) |
|-------|-------|------|-----------------------|-----------|------------------|-------|
| 0.025 | 0.043 | 0.04 | 16.3                  | 61.35     | 43               | 0.868 |

According to Table 9 and Eq. (13):

$$F = 4.6 \times 10^{-3} \times 16.3 \times 0.043^{1.03} \times 0.025^{0.7} \times 0.04^{1.27} \times 43^{3.46} \times 0.6135^{0.24} = 1.484 \text{KN}$$

**Compare with 0.868 KN**

## 6. CONCLUSION

In this paper, with the help of Buckingham-Pi theorem, general equations for the ultimate load of conical and pyramidal shell foundations were attained. In order to calculate the constants values, the loading tests on the conical and pyramidal shell models were performed. Finally, two comprehensive equations have been offered for the above mentioned foundations.

The equations confirm that:

- By increasing the dry unit weight ( $\gamma_d$ ), angle of shearing resistance ( $\phi$ ) and relative density ( $D_r$ ) of sand, ultimate load capacity of shell foundations is also increased.
- The influence of angle of shearing resistance on the ultimate load values is greater than other parameters. Additionally, the increase of angle of shearing resistance has much more influence on ultimate load of pyramidal shell foundation in proportion to the same dimension conical ones. The reason is that by increasing the angle of shearing resistance friction, the force between soil core and shell foundation is also increased. Because the internal surface of pyramidal model in contact with soil core is greater than that of conical ones, the bigger friction force was created as a resistance force against settlement, therefore, settlement is decreased and finally ultimate load is increased.
- The increase of height and dimension of soil core (b, H) leads to the increase of ultimate load values of pyramidal and conical shell foundations. This behavior has two reasons: First, the influence of increasing friction force between soil core and shell foundation and the second, the increase of soil core size.

In shell foundations, the soil failure surfaces under foundation are not created until soil core is integrated with shell footing. This integration happens when in the loading process, soil core is compacted as much as possible and thereafter, soil core acts as a part of shell foundation. By increase of “b” and “H”, the soil core volume is also increased and therefore bigger load for integration of shell and core behavior is needed, so ultimate load is increased.

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