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**"Research Note"**

# NEW CANONICAL FORMS FOR THE ANALYSIS OF SYMMETRIC TRUSS STRUCTURES*

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**Abstract–** For a symmetric structure the degrees of freedom (DOFs) in two sides of the axis of symmetry can be either symmetric or anti-symmetric. If there is no active DOF on the axis of symmetry, then we will have the Form II symmetry for the structural matrices, and alternatively if we have some active DOFs on the axis, we will have Form III symmetry. These forms are already developed and employed in structural dynamics and stability analysis of frame structures. However, for the structures having both symmetric DOFs and anti-symmetric DOFs, simultaneously, we will have different canonical forms, defined in this paper as the Form A and Form B symmetry. Thus the main objective is to develop these forms and explore the governing relationships. The presented method is then applied to the analysis of symmetric structures.

**Keywords–** Canonical forms, symmetry and anti-symmetry, trusses, form A, form B

## 1. INTRODUCTION

Symmetry has been widely used in science and engineering [1-5]. A thorough review can be found in the work of Kangwai et al. [6]. Methods are developed for decomposing and healing the graph models of structures, in order to calculate the eigenvalues of matrices and graph matrices with special patterns [7]. These structures correspond to various matrices such as adjacency, Laplacian, stiffness and mass matrices of special forms, known as *canonical form*. A canonical form, in general, can be defined as a block partitioned matrix which can be transformed into an upper block triangular matrix, thus producing block diagonal entries. The applications of canonical forms to vibrating mass-spring systems and frame structures are developed in [8], (Kaveh and Rahami, 2007). These forms are also applied to the calculation of the buckling load of symmetric frames [9]. These applications are extended to the static analysis of symmetric frames [10].

In classical approaches a symmetric structure is decomposed into substructures and appropriate boundary conditions are then imposed. Naturally for different configuration, loading and boundary conditions, different boundary conditions will be needed.

In general, for a structural system if all the DOFs in the two sides of the axis of symmetry are symmetric or anti-symmetric, depending whether we have some active DOFs on the axis of symmetry or no active DOFs, we will have Form III and Form II symmetry for the structural matrices, respectively. Fig. 1a and Fig. 1b show examples of structures for which the corresponding matrices have these forms. These forms are well documented in the previous papers [7, 8] and briefly discussed in the Appendix A. However, when both symmetric DOFs (vertical components) and anti-symmetric DOFs (horizontal components) are present simultaneously, then we will have different forms called Form A and Form B

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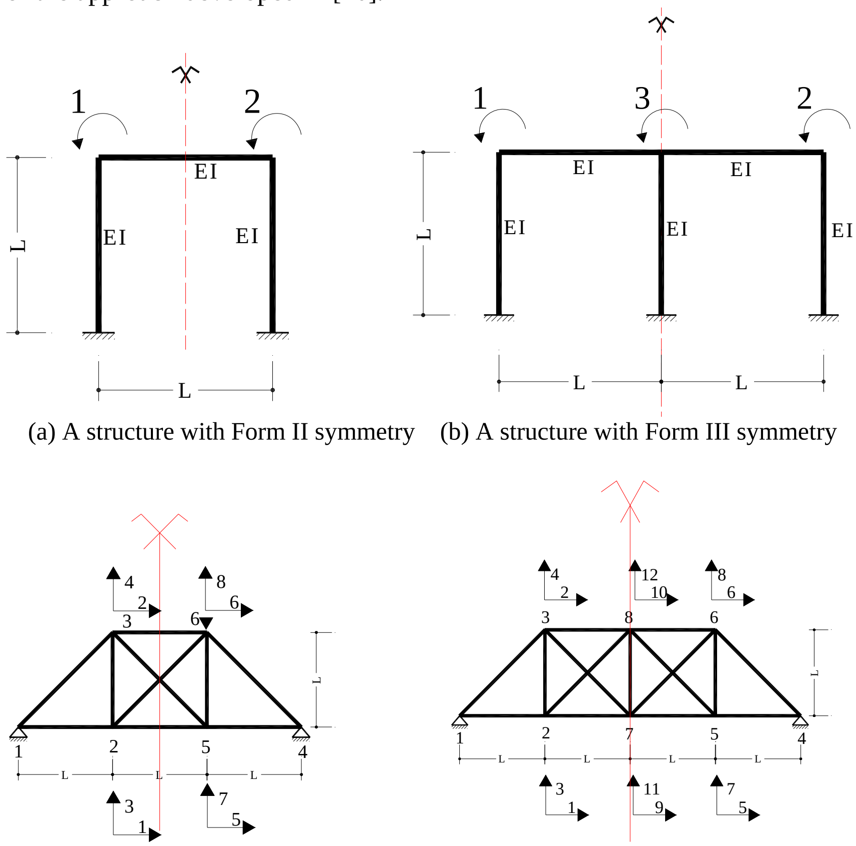
canonical forms, as discussed in this paper. Examples of such structures having these properties are illustrated in Fig. 1c and Fig. 1d, respectively. The method presented in this paper can be considered as the generalization of the approach developed in [10].




(a) A structure with Form II symmetry (b) A structure with Form III symmetry

(c) A structure with Form A symmetry (d) A structure with Form B symmetry

Fig. 1. Structures with matrices corresponding to different canonical forms

In this paper, a general method is presented for the analysis of symmetric structures with their stiffness matrices having Form A and Form B symmetry. For this purpose two submatrices $S$ and $T$ are employed. It is proved that on performing row and column operations on matrices with Form A and Form B, a block diagonal matrix will be obtained which has a similarity property (identical eigenvalues) as that of the original matrices. Having the matrices $T$ and $S$, all the information about the main matrix can be obtained, and it is only sufficient to find the non-singular permutation matrix $P$, i.e. once we know the type of the symmetry we can perform the desired operation on the main matrix, having the two submatrices $S$ and $T$. Here, the displacements of a symmetric structure with Form A and Form B are obtained using $S$ and $T$. For this purpose, first the reverse relationship between the inverse of the main matrix and inverse of $S$ and $T$ is established. Then comparing the displacements of the main system under different types of loading with the product of the inverse of $S$ and $T$ by the load vector, the displacements of the main structure are calculated. Finally, a general loading is expressed in terms of two different symmetry forms and the displacements of the main structure are obtained.

## 2. PRELIMINARY DEFINITIONS AND RESULTS

1. Two matrices $A$ and $B$ are called *similar* if there exists a non-singular permutation matrix $P$ such that

$$B = P^{-1}AP \tag{1}$$

2. The eigenvalues of two similar matrices are identical.

3. *Row operator matrix:* Consider an arbitrary $n\times n$ matrix. *A* matrix which adds the *k*th row to the *l*th row is a $n\times n$ unit matrix, where the entry in the *l*th row and the *k*th column is "1" in place of "0". For reducing the *k*th row from the *l*th row, one should use –1 in place of 1. This matrix is multiplied to the considered matrix from the left-hand side. The inverse of this matrix is itself, with this difference being that the sign of the non-diagonal entry should be changed, i.e. –1 to +1 and +1 to –1.

4. *Column operator matrix*: This is similar to the previous case with the only difference being that the multiplication should be performed from the right-hand side.

5. The inverse of an upper block triangular matrix *R*, in the form of Schur, can be obtained as follows:

$$R=\begin{bmatrix} S & X \\ 0 & T \end{bmatrix} \quad \Rightarrow \quad R^{-1}=\begin{bmatrix} S^{-1} & -S^{-1}XT^{-1} \\ 0 & T^{-1} \end{bmatrix} \tag{2}$$

## 3. ROW AND COLUMN OPERATORS FOR THE FORMS A AND FORM B SYMMETRY

***a) Form A symmetry***

In this case we have no active DOFs on the axis of symmetry and therefore any matrix *R* associated with this case can be expressed in the Form A symmetry as

$$R=\left[\begin{array}{cc|cc} A & C & D & F \\ C^T & B & -F^T & E \\ \hline D & -F & A & -C \\ F^T & E & -C^T & B \end{array}\right] \tag{3}$$

The first and second rows and columns correspond to the left-hand side DOFs and the third and fourth rows and columns belong to the right-hand side of the axis of symmetry. In order to obtain *R*', the necessary permutation matrices *P* and $P^{-1}$ are as follows:

$$P=\left[\begin{array}{cc|cc} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ \hline I & 0 & I & 0 \\ 0 & -I & 0 & I \end{array}\right] \quad \Rightarrow \quad P^{-1}=\left[\begin{array}{cc|cc} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ \hline -I & 0 & I & 0 \\ 0 & I & 0 & I \end{array}\right] \tag{4}$$

$R'$ Leads to

$$R'=P^{-1}RP=\left[\begin{array}{cc|cc} A+D & C-F & D & F \\ C^T-F^T & B-E & -F^T & E \\ \hline 0 & 0 & A-D & -C-F \\ 0 & 0 & -C^T-F^T & B+E \end{array}\right] \tag{5}$$

Resulting in

$$S_R=\begin{bmatrix} A+D & C-F \\ C^T-F^T & B-E \end{bmatrix} \quad \text{and} \quad T_R=\begin{bmatrix} A-D & -C-F \\ -C^T-F^T & B+E \end{bmatrix} \tag{6}$$

and

$$R^{-1}=\left[\begin{array}{cc|cc} A' & C' & D' & F' \\ C'^T & B' & -F'^T & E' \\ \hline D' & -F' & A' & -C' \\ F'^T & E' & -C'^T & B' \end{array}\right] \tag{7}$$

leading to

$$S_{R^{-1}}=\begin{bmatrix} A'+D' & C'-F' \\ C'^T-F'^T & B'-E' \end{bmatrix}=[S_R]^{-1} \qquad T_{R^{-1}}=\begin{bmatrix} A'-D' & -C'-F' \\ -C'^T-F'^T & B'+E' \end{bmatrix}=[T_R]^{-1} \tag{8}$$

Addition and subtraction of the Eqs. (8) result in

$$\begin{bmatrix} A' & -F' \\ -F'^T & B' \end{bmatrix}=\frac{1}{2}\left(S_{R^{-1}}+T_{R^{-1}}\right)=\frac{1}{2}\left([S_R]^{-1}+[T_R]^{-1}\right) \tag{9}$$

and

$$\begin{bmatrix} D' & C' \\ C'^T & -E' \end{bmatrix}=\frac{1}{2}\left(S_{R^{-1}}-T_{R^{-1}}\right)=\frac{1}{2}\left([S_R]^{-1}-[T_R]^{-1}\right) \tag{10}$$

Hence, instead of finding the inverse of a matrix like *R*, one can find the inverse of smaller matrices *S* and *T*, and using Eqs. (9) and (10), the submatrices $A'$, $B'$, $C'$, $D'$ and $E'$ can be constructed. Then the inverse of the matrix *R* can be formed by assembling these submatrices.

***b) Form B symmetry***

In this case we have active DOFs on the axis of symmetry and therefore any matrix *R* associated with this case can be expressed in Form B symmetry as

$$R=\begin{bmatrix} A & C & D & F & G & N \\ C^T & B & -F^T & E & J & H \\ D & -F & A & -C & G & -N \\ F^T & E & -C^T & B & -J & H \\ G^T & J^T & G^T & -J^T & K & 0 \\ N^T & H^T & -N^T & H^T & 0 & L \end{bmatrix} \tag{11}$$

The first and second rows and columns correspond to the left-hand side DOFs and the third and fourth rows and columns belong to the DOFs on the axis of symmetry and the fifth and sixth rows and columns correspond to the right-hand side of the axis of symmetry. In order to obtain $R'$, the necessary permutation matrices *P* and $P^{-1}$ for obtaining $R'$ are as follows:

$$P=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & I & 0 \\ 0 & -I & 0 & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \end{bmatrix} \text{ and } \quad P^{-1}=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ -I & 0 & I & 0 & 0 & 0 \\ 0 & I & 0 & I & 0 & 0 \end{bmatrix} \tag{12}$$

the matrix $R'$ will be obtained as

$$R' = \left[\begin{array}{ccc|ccc} A+D & C-F & G & N & D & F \\ C^T-F^T & B-E & J & H & -F^T & E \\ 2G^T & 2J^T & K & 0 & G^T & -J^T \\ \hline 0 & 0 & 0 & L & -N^T & H^T \\ 0 & 0 & 0 & -2N & A-D & -C-F \\ 0 & 0 & 0 & 2H & -C^T-F^T & B+E \end{array}\right] \quad (13)$$

Corresponding to

$$S_R = \begin{bmatrix} A+D & C-F & G \\ C^T-F^T & B-E & J \\ 2G^T & 2J^T & K \end{bmatrix} \quad \text{and} \quad T_R = \begin{bmatrix} A-D & -C-F & -2N \\ -C^T-F^T & B+E & 2H \\ -N^T & H^T & L \end{bmatrix} \quad (14)$$

and

$$R^{-1} = \left[\begin{array}{ccc|ccc} A' & C' & D' & F' & G' & N' \\ C'^T & B' & -F'^T & E' & J' & H' \\ D' & -F' & A' & -C' & G' & -N' \\ \hline F'^T & E' & -C'^T & B' & -J' & H' \\ G'^T & J'^T & G^T & -J'^T & K' & 0 \\ N'^T & H'^T & -N'^T & H'^T & 0 & L' \end{array}\right] \quad (15)$$

Therefore

$$S_{R^{-1}} = \begin{bmatrix} A'+D' & C'-F' & G' \\ C'^T-F'^T & B'-E' & J' \\ 2G'^T & 2J'^T & K' \end{bmatrix} = [S_R]^{-1} \qquad T_{R^{-1}} = \begin{bmatrix} A'-D' & -C'-F' & -2N' \\ -C'^T-F'^T & B'+E' & 2H' \\ -N'^T & H'^T & L' \end{bmatrix} = [T_R]^{-1} \quad (16)$$

Adding and subtracting the Eqs. (16) leads to

$$\begin{bmatrix} A' & -F' & \frac{1}{2}(G'-2N') \\ -F'^T & B' & \frac{1}{2}(J'+2H') \\ \frac{1}{2}(2G'^T-N'^T) & \frac{1}{2}(2J'^T-H'^T) & \frac{1}{2}(K'-L') \end{bmatrix} = \frac{1}{2}\left([S_R]^{-1}+[T_R]^{-1}\right) \quad (17)$$

and

$$\begin{bmatrix} D' & C' & \frac{1}{2}(G'+2J') \\ C'^T & -E' & \frac{1}{2}(J'-2H') \\ \frac{1}{2}(2G'^T+N'^T) & \frac{1}{2}(2J'^T-H'^T) & \frac{1}{2}(K'-L') \end{bmatrix} = \frac{1}{2}\left([S_R]^{-1}-[T_R]^{-1}\right) \quad (18)$$

Therefore, instead of calculating the inverse of *R* one can use the inverse of smaller matrices *S* and *T*. Here, the submatrices $G'$, $H'$, $N'$, $J'$, $K'$ and $L'$ can be calculated directly from the inverse of the *S* and *T* matrices, and the submatrices $A'$, $B'$, $C'$, $D'$, and $E'$ can be extracted from Eq. (17) and Eq. (18). The inverse of the matrix *R* can then be obtained by assembling of these submatrices.

## 4. CALCULATION OF DISPLACEMENTS FOR FORMS A AND FORM B SYMMETRY

The displacements $\Delta$ of structure under the external loading *P* can be obtained from the force-displacement relationship given by

$$\Delta = K^{-1}P \tag{19}$$

***a) Calculation of the displacements for the form A symmetry***

**Case 1 of Form A**: For the load vector $\overline{P} = [P \quad Q \mid P \quad Q]^T$ we have

$$\begin{pmatrix}\delta_1\\ \delta_2\end{pmatrix} = S^{-1}\begin{pmatrix}P\\ 0\end{pmatrix} - (TI')^{-1}\begin{pmatrix}0\\ Q\end{pmatrix} \qquad \begin{pmatrix}\delta_3\\ \delta_4\end{pmatrix} = (SI')^{-1}\begin{pmatrix}P\\ 0\end{pmatrix} + T^{-1}\begin{pmatrix}0\\ Q\end{pmatrix} \tag{20}$$

**Case 2 of Form A**: For the load vector $\overline{P} = [P \quad Q \mid -P \quad Q]^T$ we have

$$\begin{pmatrix}-\delta_1\\ \delta_2\end{pmatrix} = \begin{pmatrix}\delta_3\\ \delta_4\end{pmatrix} = -[T]^{-1}\begin{pmatrix}P\\ -Q\end{pmatrix} = T^{-1}\begin{pmatrix}-P\\ Q\end{pmatrix} \tag{21}$$

**Case 3 of Form A**: For the load vector $\overline{P} = [P \quad Q \mid P \quad -Q]^T$ we have

$$\begin{pmatrix}\delta_1\\ \delta_2\end{pmatrix} = \begin{pmatrix}\delta_3\\ -\delta_4\end{pmatrix} = S^{-1}\begin{pmatrix}P\\ Q\end{pmatrix} \tag{22}$$

**Case 4 of Form A**: For the load vector $\overline{P} = [P \quad Q \mid -P \quad -Q]^T$ we have

$$\begin{pmatrix}\delta_1\\ \delta_2\end{pmatrix} = S^{-1}\begin{pmatrix}0\\ Q\end{pmatrix} + (TI')^{-1}\begin{pmatrix}P\\ 0\end{pmatrix} \qquad \begin{pmatrix}\delta_3\\ \delta_4\end{pmatrix} = -[T]^{-1}\begin{pmatrix}P\\ 0\end{pmatrix} + (SI')^{-1}\begin{pmatrix}0\\ Q\end{pmatrix} \tag{23}$$

**General loading case of Form A**: We obtain the general loading by combining the above four cases using two methods: $\overline{P} = [W \quad X \mid Y \quad Z]^T$
This is the combination of Case 1 and Case 4.

$$\begin{pmatrix}\delta_1\\ \delta_2\end{pmatrix} = \frac{1}{2}S^{-1}\begin{bmatrix}W+Y\\ X-Z\end{bmatrix} + \frac{1}{2}(TI')^{-1}\begin{bmatrix}W-Y\\ -X-Z\end{bmatrix} \qquad \begin{pmatrix}\delta_3\\ \delta_4\end{pmatrix} = \frac{1}{2}(SI')^{-1}\begin{bmatrix}W+Y\\ X-Z\end{bmatrix} + \frac{1}{2}T^{-1}\begin{bmatrix}-W+Y\\ X+Z\end{bmatrix} \tag{24}$$

***b) Calculation of displacements for the Form B symmetry***

**Case 1 of Form B**: For the load case $\overline{P} = [P \quad Q \quad P \quad Q \quad P' \quad Q']^T$ form Eq. (66) we have

$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix} = S^{-1}\begin{Bmatrix}P\\ 0\\ P'\end{Bmatrix} - I'T^{-1}\begin{Bmatrix}0\\ Q\\ \frac{Q'}{2}\end{Bmatrix} \qquad \begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix} = I'S^{-1}\begin{Bmatrix}P\\ 0\\ P'\end{Bmatrix} + I''T^{-1}\begin{Bmatrix}0\\ Q\\ \frac{Q'}{2}\end{Bmatrix} \tag{25}$$

**Case 2 of Form B**: For the load case $\overline{P}=[P \quad Q \quad P \quad -Q \quad P' \quad Q']^T$ from Eq. (26) we have

$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix}=S^{-1}\begin{Bmatrix}P\\ Q\\ P'\end{Bmatrix}-I'T^{-1}\begin{Bmatrix}0\\ 0\\ \frac{Q'}{2}\end{Bmatrix} \qquad \begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix}=I'S^{-1}\begin{Bmatrix}P\\ Q\\ P'\end{Bmatrix}+I''T^{-1}\begin{Bmatrix}0\\ 0\\ \frac{Q'}{2}\end{Bmatrix} \tag{27}$$

In the special case with $Q'=0$ (anti-symmetric loading), the relationship holds between the above vectors.

$$\begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix}=\begin{bmatrix}I & 0 & 0\\ 0 & -I & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix} \qquad \delta_3=\delta_1, \delta_4=-\delta_2 \text{ and } \delta_6=0 \tag{28}$$

**Case 3 of Form B**: For the load case $\overline{P}=[P \quad Q \mid -P \quad Q \mid P' \quad Q']^T$ from Eq. (29) we have

$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix}=I'T^{-1}\begin{Bmatrix}P\\ -Q\\ -Q'/2\end{Bmatrix}+S^{-1}\begin{Bmatrix}0\\ 0\\ P'\end{Bmatrix} \qquad \begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix}=I''T^{-1}\begin{Bmatrix}-P\\ Q\\ Q'/2\end{Bmatrix}+I'S^{-1}\begin{Bmatrix}0\\ 0\\ P'\end{Bmatrix} \tag{30}$$

In the special case with $P'=0$ (symmetric loading), the relationship holds between the above vectors.

$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix}=-I'\begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix} \qquad \delta_3=-\delta_1,\ \delta_4=\delta_2 \text{ and } \delta_5=0 \tag{31}$$

**Case 4 of Form B**: For the load case $\overline{P}=[P \quad Q \mid -P \quad -Q \mid P' \quad Q']^T$ we have

$$\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix}=I'T^{-1}\begin{Bmatrix}P\\ 0\\ -Q'/2\end{Bmatrix}+S^{-1}\begin{Bmatrix}0\\ Q\\ P'\end{Bmatrix} \qquad \begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix}=I''T^{-1}\begin{Bmatrix}-P\\ 0\\ Q'/2\end{Bmatrix}+I'S^{-1}\begin{Bmatrix}0\\ Q\\ P'\end{Bmatrix} \tag{32}$$

**General loading case of Form B**: This case is obtained by different combinations of the four cases discussed previously.

This is a combination of Case 1 and Case 4. $\overline{P}=[U \quad V \mid W \quad X \mid Y \quad Z]^T$

$$\begin{gathered}\begin{Bmatrix}\delta_1\\ \delta_2\\ \delta_5\end{Bmatrix}=\frac{1}{2}S^{-1}\begin{Bmatrix}U+W\\ V-X\\ 2Y\end{Bmatrix}+\frac{1}{2}I'T^{-1}\begin{Bmatrix}U-W\\ -V-X\\ -Z\end{Bmatrix}\\ \begin{Bmatrix}\delta_3\\ \delta_4\\ \delta_6\end{Bmatrix}=\frac{1}{2}I'S^{-1}\begin{Bmatrix}U+W\\ V-X\\ 2Y\end{Bmatrix}+\frac{1}{2}I''T^{-1}\begin{Bmatrix}-U+W\\ V+X\\ Z\end{Bmatrix}\end{gathered} \tag{33}$$

## 5. NUMERICAL EXAMPLES

*Example with three cases:* Consider the planar truss as shown in Fig. 2. Under the applied load, the displacements of the nodes 2, 3, 5 and 6 are required. For this truss we have $E = 2.07 \times 10^8\ KN/m^2$, $I = 100\,cm^4$, $A = 10\,cm^2$ and $L$=100*cm*.

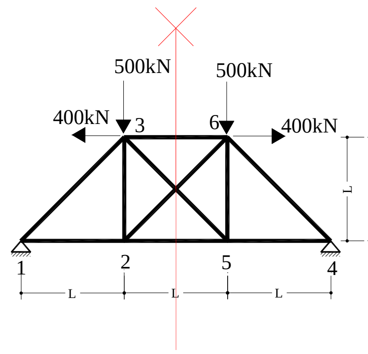



Fig. 2. A planar truss with symmetric loading

(a) For this symmetric structure if the load vector is

$$\overline{P} = \begin{bmatrix} P_{2x} & P_{3x} & P_{2y} & P_{3y} & P_{5x} & P_{6x} & P_{5y} & P_{6y} \end{bmatrix}^T$$

$$\overline{\mathrm{P}} = \begin{bmatrix} 0 & -400 & 0 & -500 & 0 & 400 & 0 & -500 \end{bmatrix}^{\mathrm{T}}$$

Then we have the Case 2 loading with Form A symmetry. Thus using Eq. (21) we obtain the displacements as

$$\overline{\delta} = \begin{bmatrix} \delta_{2x} & \delta_{3x} & \delta_{2y} & \delta_{3y} & \delta_{5x} & \delta_{6x} & \delta_{5y} & \delta_{6y} \end{bmatrix}^T$$

$$\overline{\delta} = \begin{bmatrix} 0 & 2 & -70 & -70 & 0 & -2 & -70 & -70 \end{bmatrix}^T \times 10^{-4}\,cm$$

(b) When the same structure is loaded by

$$\overline{\mathrm{P}} = \begin{bmatrix} 400 & 0 & 0 & -500 & 400 & 0 & 0 & 500 \end{bmatrix}^{\mathrm{T}}$$

then we will have the Case 3 loading with Form A symmetry. Thus using Eq. (22) we obtain the displacements as

$$\overline{\delta} = \begin{bmatrix} 27 & 4 & -19 & -27 & 27 & 4 & 19 & 27 \end{bmatrix}^T \times 10^{-4}\,cm$$

(c) Now consider the same structure under the following general loading:

$$\overline{\mathrm{P}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -900 & 0 \end{bmatrix}^{\mathrm{T}}$$

This example is an instance for general loading case of symmetry form A. Hence using Eq. (24) we obtain the displacements as

$$\overline{\delta} = \begin{bmatrix} -6 & 21 & -67 & -62 & -8 & -3 & -118 & -84 \end{bmatrix}^T \times 10^{-4}\,cm$$

## 7. CONCLUSION

In this article the governing relationships for planar trusses are established for the Form A and Form B canonical forms. Using a similar approach, canonical forms can easily be derived for other types of structures. Here, formulation obtained for calculating the deformation of the structures containing these types of symmetry is established. The displacements of a structure are obtained in terms of two submatrices $T$ and $S$ with dimensions much smaller than that of the stiffness matrix of the entire structure. Since in static analysis, the main computational time belongs to the solution of the equations corresponding to the stiffness matrix, the present method is more economical, because of using smaller number of simultaneous equations. Finally, it should be mentioned that the present method can be employed for the analysis of symmetric trusses with any general loading.

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