

“Research Note”

**DUCTILITY OF HIGH STRENGTH CONCRETE IN BENDING
 WITH OR WITHOUT CONFINEMENT***

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Abstract– One of the important principals of RC structural design in earthquake regions is ductility considerations, which need a different design approach. In this paper, the parameters affecting high strength concrete HSC are considered and the tools (tables and curves) are prepared for ductile design of reinforced HSC bending sections. Using these tools, it is possible to easily and safely design for ductility of singly and doubly reinforced flexural confined and unconfined HSC sections. The worked examples are also presented.

Keywords– Ductility, flexural section, HSC, confined or unconfined concrete

1. INTRODUCTION

Current knowledge of HSC shows that there are definite advantages in using HSC in structures. Greater strength, increased modules of elasticity, reduced shrinkage, and creep are some of these advantages. However, the nature of failure of the HSC is brittle and not even ductile as normal strength concrete (NSC). Therefore, designing the HSC structures for ductility is important. A considerable amount of ductility can be ensured by a few relatively simple and inexpensive design details, especially when the structures are made of NSC [1]. In other words, a limited research report is available when considering the HSC structures.

2. DUCTILITY OF REINFORCED HSC BEAMS

Here, the easiest and most common method of defining the ductility of a cross section called curvature ductility ratio, $\mu = \phi_u / \phi_y$ is taken [2]. Using Fig. 1a; the yield curvature ϕ_y of a section can be computed with adequate accuracy from the straight line theory of Eq. (1).

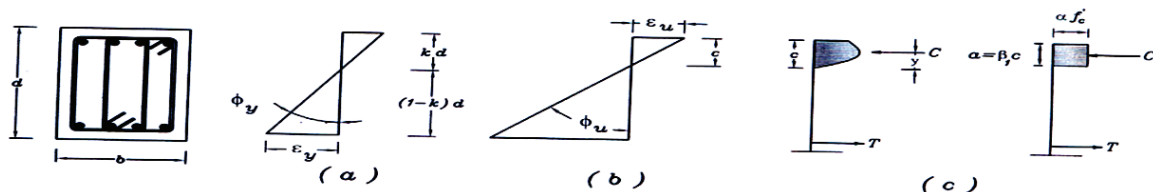


Fig. 1. (a) Yielding curvature ϕ_y , (b) Ultimate curvature and (c) Stress block's parameters

a) Singly reinforced beams

$$\phi_y = \epsilon_y / d(1-K) \tag{1}$$

where, kd is depth of the compression zone computed using straight line theory;

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$K = -\rho n + [2\rho n + \rho^2 n^2]^{1/2}$, $\rho = A_s / bd$, $n = E_s / E_c$ and the ultimate curvature, ϕ_u can be calculated as

$$\phi_u = \varepsilon_u / c \quad (2)$$

where $c = \rho(f_y / f'_c)(d / \alpha \beta_1) = \omega(d / \alpha \beta_1)$ and ε_u is concrete compressive strain at crushing of concrete with a uniform stress block of $\alpha f'_c$. However, for NSC the α is respectively assumed equal or smaller than $0.85 f'_c$ for HSC (see the following). Using Eq. (1) and (2) gives;

$$\mu = \varepsilon_u (\alpha \beta_1 f'_c) E_s (1 + \rho n - (2\rho n + \rho^2 n^2)^{0.5}) / \rho f_y^2 \quad (3)$$

The value of μ in Eq. (3) is directly affected by the value of ultimate strain ε_u assumed in the calculation and is a function of a number of variables including the concrete strength. When stirrups are not available in the beam, it seems this value can be replaced by ε_{cu} . Although the ultimate compressive strain varies with concrete strength a value equal to 0.003 represents the test results satisfactorily [3,4]. This value is specified in standards [5-7]. Thus for design use it is recommended that $\varepsilon_u = \varepsilon_{cu} = 0.003$ be used, unless special binding reinforcement is provided to increase ε_u [1,7]. Now, substituting in Eq. (3), the Eq. (4) can be used to compute design values of the ductility ratio of both (NSC and HSC).

$$\mu = 600(\alpha \beta_1 f'_c)(1 + \rho n - (2\rho n + \rho^2 n^2)^{0.5}) / \rho f_y^2 \quad (4)$$

b) Doubly reinforced beams

Here, $\phi_y = f_y / E_s d(1-k)$ and $\phi_u = \varepsilon_u / C_u$ and $K = [n^2 (\rho + \rho')^2 + 2n(\rho + \rho' d' / d)]^{1/2} - n(\rho + \rho')$

However, when the A'_s yields or does not yield we have respectively;

$$\phi_u = \varepsilon_u \alpha \beta_1 f'_c b / (A_s - A'_s) f_y \text{ or } \mu = \phi_u / \phi_y = E_s \varepsilon_u (1-k) d / f_y C_u .$$

In a beam without A'_s and limiting the value of $\rho_{\max} = 0.75\rho_b$, the Eq. (5) can be obtained as;

$$\rho_{\max} = 0.75(\alpha \beta_1 f'_c / f_y)(\varepsilon_{cu} / \varepsilon_{cu} + \varepsilon_y) \quad (5)$$

So, where the confinement mechanism is available the ε_u is calculated with the actual value.

3. HIGH STRENGTH COMPRESSIVE CONCRETE STRESS BLOCK'S PARAMETERS

Although the ultimate compressive strain varies with concrete strength for HSC a value equal to 0.003 represents, satisfactorily, the test results. This value is specified in the ACI, Australian and New Zealand standards [5-7]. However, in the Canadian standard [8] it is taken as 0.0035.

$\alpha = 0.85 - 0.004 (f'_c - 55)$ It is generally accepted that the uniform stress should be smaller than $0.85 f'_c$ for HSC [3]. In [7], the depth of equivalent rectangular stress block is taken as γ times the depth of neutral axis and the uniform stress is taken is $\alpha f'_c$, where, $\gamma = 0.85 - 0.008(f'_c - 30)$; within the limits $0.65 \leq \gamma \leq 0.85$ (it is assumed that $\gamma = \beta_1$), and within the limits $0.75 \leq \alpha \leq 0.85$, note that $\alpha = 0.85$ when $f'_c \leq 55 \text{ MPa}$ and $\alpha = 0.75$ when $f'_c \geq 80 \text{ MPa}$).

However, when $f'_c \geq 80 \text{ MPa}$, $\alpha \beta_1 (1 - 0.5\beta_1) b c^2 f'_c = C y$ and $\alpha \beta_1 b c f'_c = C$, and C is the resultant compressive force which is placed in y distance from the neutral axis. For variables α and β_1 in HSC, comprehensive information is available in [9].

4. CONFINEMENT EFFECT OF TRANSVERSE REINFORCEMENT

Enhancement of strength and ductility has been observed for all grades of concrete with lateral confinement. This leads to the speculation that the beneficial effect of lateral confinement would reduce with the increase in compressive strength. Data reported for Poisson's ratio of HSC are evident in that there is a large scatter of results. However, the value proposed by [9] which is 0.2, appears to be the median of the experimental results.

Analytical model for confinement mechanism in rectangular beams including closed stirrups is shown in Fig. 2. Based on this assumption, the stress-strain relationship for confined core concrete (imprisoned concrete on the central line of stirrups), shown in Fig. 2.

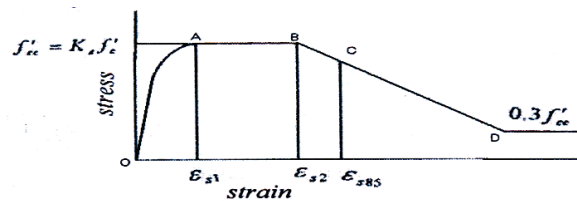


Fig. 2. General model for stress-strain concrete curve proposed by [10]

This model includes: the OA part, which is a parabolic curve with the peak point, A, at f'_{cc} stress and ϵ_{s1} strain, f'_{cc} is confined concrete's compressive strength and is equal to $K_s f'_c$. f'_c is unconfined concrete's compressive strength and K_s is increasing strength's factor. Strains ϵ_{s1} and ϵ_{s2} are the minimum and maximum strain values associated with maximum stress, and ϵ_{s85} is strain associated with 85 percent of maximum stress on the declining part of the stress-strain curve. AB and BC are the straight lines on the curve.

$$K_s = 1 + b^2 / 140 P_{occ} \left[\left(1 - n_1 S_c^2 / 5.5 b^2 \right) \left(1 - S_v / 2b \right)^2 \right] (\rho_s f_{yv})^{0.5} \quad (6)$$

The value of ϵ_{s2} is a concrete ductility index and is obtained by transverse reinforcement. Here, an equation proposed by Sargin's model is proposed for ϵ_{s2} , which includes stirrups shape and spacing, and the magnitude and characteristic of transverse reinforcement. The proposed equation is in the following form;

$$\epsilon_{s2} / \epsilon_{00} = 1 + 248 / S_c \left[1 - 5(S_v / b)^2 \right] (\rho_s f_{yv} / (f'_c)^{0.5}) \quad (7)$$

ϵ_{00} is strain corresponding to the maximum stress in the plain concrete (i.e., 0.0022), S_c is the spacing of longitudinal reinforcement in the compressive concrete region (mm) and stresses are in MPa, and the stirrup reinforcement's strain corresponding to 85 percentage of maximum stress is $\epsilon_{s85} = 0.225 \rho_s (b / S_v)^{0.5} + \epsilon_{s2}$ and therefore, ultimate compressive concrete strain is;

$$\epsilon_u = \epsilon_{s85} \quad (8)$$

So Eq. (8) can be substituted in Eq. (3) to calculate ductility [10].

5. FLEXURAL DUCTILITY DESIGN OF HIGH STRENGTH CONCRETE SECTION

There are different opinions on what should be taught in the way of design and design office work for ductility design of HSC sections. In this paper, the tools such as design tables and graphs are prepared and presented for both HSC unconfined and confined bending sections.

a) Singly reinforced bending sections

Considering (Fig.1) an ultimate compressive concrete strain for unconfined conditions and for different steel ratio, concrete compressive and steel yield strength, by using Eq. (4), the ductility ratios are calculated and their corresponding values are given in Table 1a.

Table 1(a). Steel percentages for various ductility ratios in unconfined state

f'_c MPa	ρ or $(\rho - \rho')$		Ductility ratio μ
	$f_y=280$ MPa	$f_y=420$ MPa	
60	0.0196	0.0099	8
	0.0247	0.0127	6
	0.0340	0.0178	4
80	0.0234	0.0119	8
	0.0295	0.0152	6
	0.0406	0.0213	4
100	0.0287	0.0146	8
	0.0360	0.0186	6
	0.0495	0.0260	4

Table 1(b). Different stirrups percentages ρ_s for $\mu_3 = \epsilon_u / \epsilon_{cu}$ value in confined sections

ρ_s		$\mu_3 = \epsilon_u / \epsilon_{cu}$	b / S_v
$f_{yv} / (S_c \sqrt{f'_c}) = 0.15$	$f_{yv} / (S_c \sqrt{f'_c}) = 1.0$		
0.00423	0.00285	1.33	3
0.00892	0.00601	2	
	0.00917	2.67	
0.00316	0.00192	1.33	5
0.00668	0.00404	2	
0.01020	0.00617	2.67	
	0.00830	3.33	10
0.00228	0.00146	1.33	
0.00481	0.00309	2	
0.00735	0.00472	2.67	
0.00988	0.00634	3.33	

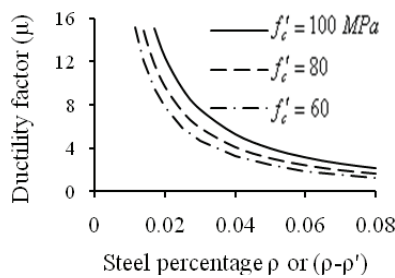


Fig. 3. Relationship between ρ and ductility, unconfined HSC ($f_y = 280 MPa$)

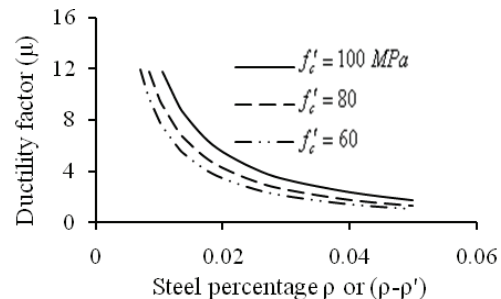


Fig. 4. Relationship between ρ and ductility, unconfined HSC ($f_y = 420 MPa$)

a) Doubly reinforced unconfined bending sections

Considering the affecting parameters on the ductility of bending section having compressive steel reinforcement, the ductility factor μ can be calculated using, $\phi_u = \epsilon_u \alpha \beta_1 f'_c b / (A_s - A'_s) f_y$. By referring the parameter μ in Eq. (4) as μ_1 with the value of $\rho - \rho'$ instead of ρ , $K = -\rho n + [2\rho n + \rho^2 n^2]^{1/2}$ as $k_1(\rho)$ and also the parameter $K = [n^2 (\rho + \rho')^2 + 2n(\rho + \rho' d'/d)]^{1/2} - n(\rho + \rho')$ as $k_2(\rho, \rho', d'/d)$, we can compute the ductility factor μ in the form of $\mu = \mu_1(\rho - \rho') (k_2(\rho, \rho', d'/d) / k_1(\rho - \rho'))$. It is important that the parameters k_2 and k_1 are functions of $\rho, \rho', d'/d, n$, where $n = E_s / E_c$ is a parameter of f'_c , while numerical calculations proved that the affect of n in μ_2 is less than 4%. By referring the $k_2(\rho, \rho', d'/d) / k_1(\rho - \rho')$ as the compressive steel index μ_2 , the ductility factor is obtained as $\mu = \mu_1 \times \mu_2$. The value of parameter μ_2 has been calculated and drawn in Figs. 5 and 6.

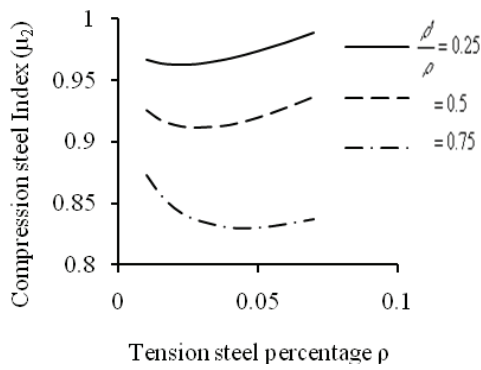


Fig. 5. Relationship between ρ and compression steel index μ_2 ($d'/d = 0.1$)

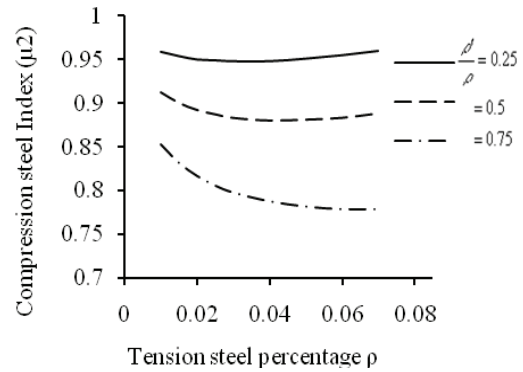


Fig. 6. Relationship between ρ and compression steel index μ_2 ($d'/d = 0.25$)

The significance of a particular value of the ductility ratio μ for HSC sections is difficult to define. Since this term is only an arbitrary measure of the inelastic rotation capacity of a cross section and the value of ϵ_u used in computing μ [9].

Akbarzadeh and Maghsoudi [11] tested HSC beams singly or doubly reinforced with different steel percentage and concluded that the minimum lower value of $\mu = 3$ can be reached when for the singly reinforced beams the value of ρ is greater than 2.0%, whereas for doubly reinforced beams the ratio of ρ can be increased even up to 4.0%. For design purposes, it is recommended that the ductility ratio for HSC from Eq.(4) or Figs. 3, 4 should be at least 4 in structures in seismic regions and at least 3 in structures requiring limited ductility.

c) Confined bending sections

Considering parameters affecting the ductility of sections having transverse steel (confined sections), the value of ϵ_u has been determined using Eq. (8) for values $f_{yv}/S_c(f'_c)^{0.5}$, b/S_v and ρ_s and also the parameter confinement index $\mu_3 = \epsilon_u / \epsilon_{cu}$ has been calculated and given in Table 1b. Then, using previous data for ductility, the value of μ (ductility ratio) in HSC confined sections is obtained in the form of $\mu = \mu_1 \times \mu_2 \times \mu_3$. To design for ductility of sections, the charts 7 and 8 are prepared. As can be seen, for confined sections, the confinement index parameter $\mu_3 = \epsilon_u / \epsilon_{cu}$ is obtained by interpolating charts 5 and 6, which are referred to $f_{yv}/S_c(f'_c)^{0.5} = 0.15, 1.0$ respectively, and then by substituting in $\mu = \mu_1 \times \mu_2 \times \mu_3$ the ductility ratio (μ) in HSC confined sections can be determined.

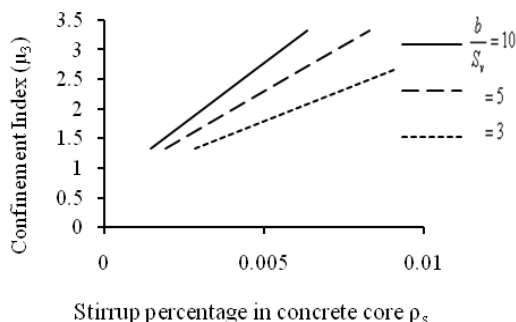


Fig. 7. Relation between ρ_s and ultimate strain in HSC confined sections $f_{yv}/S_c(f'_c)^{0.5} = 1.0$

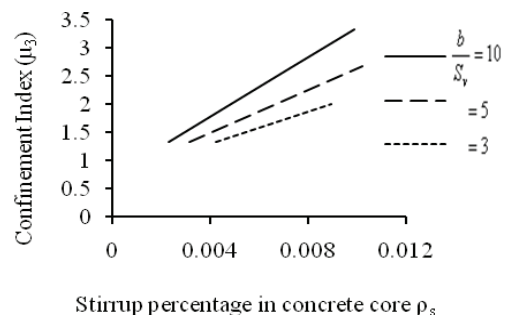


Fig. 8. Relation between ρ_s and ultimate strain in HSC confined sections $f_{yv}/S_c(f'_c)^{0.5} = 0.15$

6. WORKED EXAMPLES FOR UNCONFINED AND CONFINED HSC SECTION OF BEAMS

It is desirable to determine the ductility factor of the unconfined HSC section shown in Fig. 9a for the beam properties of $f'_c = 80 \text{ MPa}$, $f_y = 420 \text{ MPa}$, $A_s = 4 \Phi 25$, $\rho = 0.0201$;

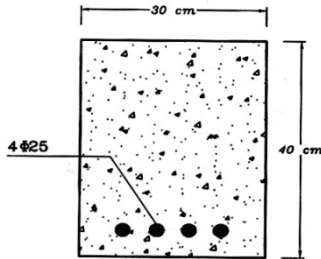


Fig. 9(a). Unconfined HSC beam section

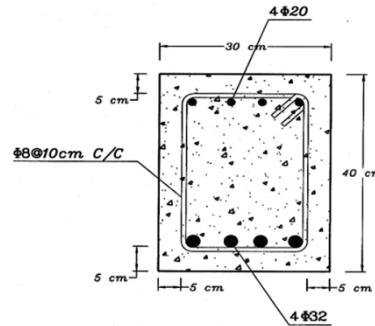


Fig. 9(b). Confined HSC beam section

Using equivalent stress block for HSC; thus, $\gamma = 0.65$ and $\alpha = 0.75$; $E_c = 3.66 \times 10^4 \text{ MPa}$, $n = 5.46$, $\rho = 0.0201$ and therefore the ductility factor is;

$$\mu = 600 (\alpha \beta_1 f'_c) (1 + \rho n - (2 \rho n + \rho^2 n^2)^{0.5}) / \rho f_y^2 = 4.16$$

Also, for the given values, the ductility factor can be calculated using Fig. 4 and it is as $\mu = 4.2$. To determine the ductility factor of the confined HSC section of Fig. 9b with the following properties;

$f'_c = 80 \text{ MPa}$, $f_y = 420 \text{ MPa}$, $A_s = 4 \Phi 32$, $A'_s = 4 \Phi 20$, $\rho - \rho' = 0.0201$, $b = 300 \text{ mm}$, $n_1 = 3$, $S_c = 53 \text{ mm}$, $A_v = 2 \Phi 8$, $S_v = 10 \text{ mm}$, $\rho_s = 0.0087 \text{ mm}$ and $P_{occ} = 3500 \text{ KN}$. Again using equivalent stress block for HSC; $K_s = 1 + b^2 / 140 P_{occ} [(1 - n_1 S_c^2 / 5.5 b^2) (1 - S_v / 2b)^2] (\rho_s f_{y_v})^{0.5} = 1.24$, $f'_{cc} = K_s f'_c = 99.2 \text{ MPa}$, $\epsilon_{s2} / \epsilon_{00} = 1 + 248 / S_c [1 - 5(S_v / b)^2] (\rho_s f_{y_v} / (f'_c)^{0.5}) = 1.84$, $\epsilon_{s2} = 1.84 \times 0.0022 = 0.0041$, $\epsilon_{s85} = 0.225 \rho_s (b / S_v)^{0.5} + \epsilon_{s2}$, $\epsilon_u = \epsilon_{s85} = 0.0074$, $n = 5.46$, $\rho = 0.0330$, $\rho' = 0.0129$, $k = 0.42$.

Therefore, by substituting, the value of $\mu = \alpha \beta_1 f'_c \epsilon_u E_s / f_y^2 (\rho - \rho') (1 - k) = 11.71$.

The ductility factor can also be calculated using charts (i.e., Fig. 4) for $f_y = 420 \text{ MPa}$, the value of $\rho - \rho' = 0.0201$ and the $f'_{cc} = 88.8 \text{ MPa}$, therefore the ductility index is as $\mu_1 = 5.1$.

Using proposed Figs. 5 and 6, from $\rho' / \rho = 0.39$, $\rho = 0.0330$ and $d' / d = 0.21$, the obtained compression steel index value is $\mu_2 = 0.92$.

Using proposed Figs. 7 and 8, from $b / S_v = 3$, $\rho_s = 0.0087$ and $f_{y_v} / S_c (f'_c)^{0.5} = 0.88$ the obtained confinement ductility is $\mu_3 = 2.5$. Therefore the ductility factor is obtained as:

$$\mu = \mu_1 \times \mu_2 \times \mu_3 = 5.1 \times 0.92 \times 2.5 = 11.71$$

So, ductility design of HSC sections can be computed using the proposed equation, tables or (charts), which will cause a reduction in numerical errors.

7. CONCLUSION

By considering the affecting parameters on ductility of confined or unconfined HSC sections, the equations are drawn and also for more convenience, the tables and curves are prepared. Using these tools,

it is possible to determine easily and safely the ductility factor of singly or doubly reinforced flexural confined and unconfined sections.

For singly reinforced unconfined section, the ductility factor is calculated easily by determining the ductility index μ_1 from the prepared curves to calculate the ductility index and replacing it as $\mu = \mu_1$. For doubly reinforced unconfined section the ductility factor is calculated by determining the compression steel index μ_2 from the prepared curves to calculate the compression steel index and replacing it as $\mu = \mu_1 \times \mu_2$. For doubly reinforced confined section the ductility factor is calculated by determining the confinement index μ_3 from the prepared curves to calculate the confinement index and replacing it as $\mu = \mu_1 \times \mu_2 \times \mu_3$.

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