A COMBINATION OF FRACTAL ANALYSIS AND ARTIFICIAL NEURAL NETWORK TO FORECAST GROUNDWATER DEPTH[*]

G. R. RAKHSHANDEHROO[1**] AND Z. GHADAMPOUR[2]

[1]Dept. of Civil Engineering, Shiraz University, Shiraz, I. R. of Iran
Email: rakhshan@shirazu.ac.ir
[2]Islamic Azad University, Estahban branch, I. R. of Iran

Abstract– A concern that researchers usually face in different applications of Artificial Neural Network (ANN) is determination of the size of effective domain in time series. In this paper, fractal analysis was used on groundwater depth time series to determine the size of effective domain in the series in an observation well in Union County, New Jersey, U.S. The variation method was applied to the sets considering different domains of 20, 40, 60, 80, 100, and 120 preceding days and the fractal dimension was determined. The fractal dimension remained constant (1.52) when the length of the domain decreased below 80 days. Data sets in different domains were fed to a Feed Forward Back Propagation ANN with one hidden layer and the the groundwater depths were forecasted. Root Mean Square Error (RMSE) and the correlation factor (R^2) of estimated and observed groundwater depths for all domains were determined. In general, groundwater depth forecast improved, as evidenced by lower RMSEs and higher R^2s, when the domain length increased from 20 to 120. However, 80 days was selected as the effective domain because the improvement was less than 1% beyond that. Forecasted groundwater depths utilizing measured daily data (set #1) and data averaged over the effective domain (set #2) were compared. It was postulated that the more accurate nature of the measured daily data was the reason for a better forecast with lower RMSE (*0.1027 m* compared to *0.255 m*) in set #1. However, a major drawback was the size of the input data in this set which was 80 times the size of the input data in set #2; a factor that may increase the computational effort unpredictably. Hence, it was concluded that fractal analysis may be successfully utilized to lower the size of input data sets considerably, while maintaining the effective information in the data set.

Keywords– Neural networks, groundwater depth, forecast, fractal analysis

1. INTRODUCTION

Groundwater is one of the major sources of supply for domestic, industrial, and agricultural purposes. In some areas groundwater is the only dependable source of supply, while in other regions it is chosen because of its availability [1]. Groundwater models provide a scientific and predictive tool for determining appropriate solutions to water allocation, surface water–groundwater interaction [2], landscape management or the impact of new development scenarios. For many practical problems of groundwater hydrology, such as aquifer development, contaminated aquifer remediation, or performance assessment of planned water supply projects, it is necessary to predict the water table and its fluctuation during the year. While depletion of groundwater supplies, conflicts between groundwater and surface water users, and the potential for groundwater contamination are concerns that will become increasingly important in any basin, the consequences of aquifer depletion can lead to local water rationing, excessive reductions in yields, wells going dry or producing erratic groundwater quality changes. Changes in flow patterns of ground water may also result, for example, in the inflow of poorer quality water and sea water intrusion in

coastal areas. Below normal groundwater recharge to creeks and streams during low flow periods could result in reduced supplies for surface water sources. Therefore, continuous groundwater level monitoring is extremely important. Groundwater levels, if forecasted well in advance, may help the administrators plan groundwater utilization more effectively. Also, for the overall development of the basin, a continuous forecast of the groundwater level is required to effectively use any simulation model for water management [1, 3].

Groundwater systems possess features of complexity, non-linearity, multi-scale and randomicity, all influenced by natural/anthropic factors, which make predictions highly complicated. In fact, when sufficient data are unavailable, and obtaining accurate predictions is more important than understanding the actual physics of the situation, empirical models remain a good alternative method and can provide useful results without the requirement of costly calibration time [4]. To date, a wide variety of models have been developed and applied for groundwater forecasting [5]. These models can be categorized into empirical time series models and physical descriptive models. Empirical time series models have been widely used for groundwater level modeling. [5]. Physics based models practically require enormous data, in particular, data pertaining to soil physical properties of the unsaturated zone, that is generally difficult or expensive, to simulate water table fluctuations [6].

In view of these factors, a stochastic model based on certain observation data requires no special experiments and brings enormous convenience to the prediction of regional-scale groundwater levels [7]. In fact, fluctuations of groundwater levels are typically nonlinear and hence, many hydrologists have attempted to use modern statistical models and techniques in water resources forecasting, including ANN, in recent years [4, 8]. Although ANN was first developed in 1943, it was not employed in water science until the 1990s. ANN has been proven to be very effective in modeling virtually any nonlinear function to an arbitrary degree of accuracy. It covers a wide range of applications involving hydrologic analyses and predictions, and the evaluation of water quality [7, 9, 10]. The main advantage of ANN over traditional methods is that it does not require the complex nature of the underlying processes under consideration to be explicitly described in a mathematical form; a feature that makes ANN an attractive tool for modeling water table fluctuations [4, 11-14].

Most of the published ANN models have been restricted in their use owing to their complex structure and algorithm [4, 10, 15]. Among different structures, Back Propagation ANN (BPANN) has a simpler structure and algorithm and has been applied widely in surface water, groundwater, and other fields with encouraging results, though it has some defects [7, 8, 9, 15, 16]. In particular, BPANN has been applied to arid and semi-arid areas of western Jilin province in China. The simulation results indicated that BPANN was accurate in reproducing (fitting) the groundwater levels [17].

A concern that researchers usually face in different applications of ANNs is determination of the size of effective domain in time series [14]. The main approach to address this concern has been trial and error [14]. It is well-known that the scale invariance is the intrinsic property of some irregular structures and patterns possessing self-similarity on certain space and time scales [18, 19]. This fact is utilized in fractal analysis to detect the scale invariance in a particular data set. When analyzing empirical data for scale invariance with fractal theory, some approaches have been proposed to estimate the dimension of the data set, a dimension that may be interpreted as the degree of irregularity by which the set is distributed [20].

In this paper, a combination of fractal analysis and artificial neural network was applied to forecast groundwater depth. Fractal analysis was used on groundwater depth time series to determine the size of effective domain in the series. The effective domain was then used in a BPANN to optimize its performance in forecasting groundwater depth in an observation well in Union County, New Jersey, U.S. In order to determine the effectiveness of this combination, different domains of the series were fed as inputs to the ANN and the results were compared.

2. ARTIFICIAL NEURAL NETWORKS

Neural networks have gone through two major development periods; the early 60's, and the mid 80's. They were a key development in the field of machine learning. Artificial Neural Networks were inspired by biological findings relating to the behavior of the brain as a network of units called neurons [21].

The fundamental building block in an Artificial Neural Network is the mathematical model of a neuron as shown in Fig. 1. The three basic components of an artificial neuron are:

1. The synapses or connecting links that provide weights, w_j, to the input values, x_j for all nodes (j =1,..., m);

2. An adder that sums the weighted input values to compute the input to the activation function

$$v = w_0 + \sum_{j=1}^{m} x_j w_j$$

where w_0 is called the bias (not to be confused with statistical bias in prediction or estimation); a numerical value associated with each neuron. It is convenient to think of the bias as the weight for an input x_0 whose value is always equal to one.

3. An activation function g (also called squashing function) maps v to g(v); a monotone function that represents the output value of the neuron.
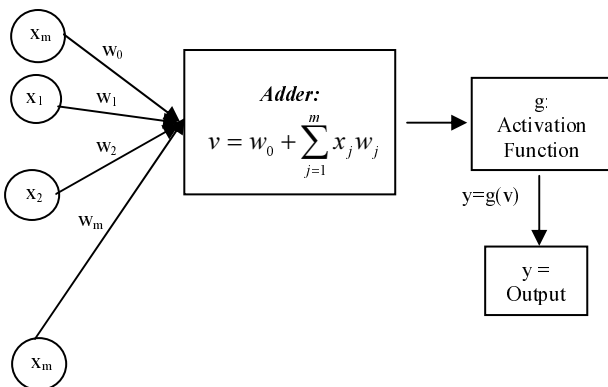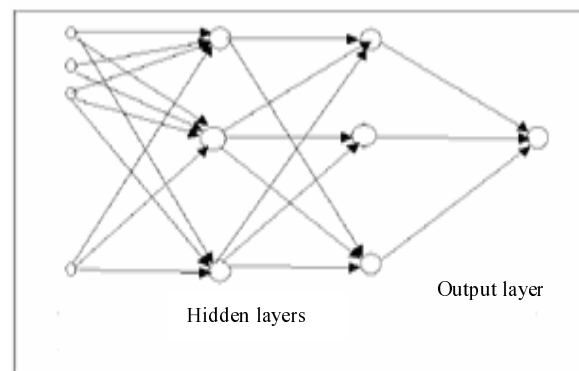

Fig. 1. Schematics of a mathematical model of a neuron

Fig. 2. Diagram of a feed forward back propagation network

a) Network architecture

While there are numerous different ANN architectures that have been studied by researchers, the most successful applications in data mining have been multilayer feedforward networks. Figure 2 is a diagram for this architecture [21].

1. Feed forward back propagation (FFBP) neural network: Training an ANN is a mathematical exercise that optimizes all of the ANN's weights and threshold values, using some fraction of the available data. Optimization routines can be used to determine the ideal number of units in the hidden layer and the nature of their transfer functions [21]. The present study employed a standard back propagation algorithm for training, and the number of hidden neurons is optimized by a trial and error procedure. In these networks, there is an input layer consisting of nodes that simply accept the input values and successive layers of nodes that are neurons as depicted in Fig. 1. The outputs of neurons in a layer are inputs to neurons in the next layer. The last layer is called the output layer. Layers between the input and output layers are known as hidden layers.

2. Evaluation criteria: Root Mean Square Error (RMSE) criterion is used by researchers in order to evaluate the effectiveness of each network in its ability to make precise predictions [10]. It is calculated by

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{N}}$$

Where y_i is the observed data, $\hat{y}_i$ is the calculated data, and N is the number of observations. Qualitatively speaking, RMSE reflects the discrepancy between the observed and calculated values. The lower the RMSE, the more accurate the prediction.

3. FRACTAL ANALYSIS

The fractal concept, first developed by Mandelbrot (1982) who coined the term from the Latin adjective "fractus", provides a useful tool to quantify the inherent irregularity of phenomena**.** Fractals are often studied with the aid of fractal dimension**.** There are many methods available for estimating the fractal dimension of a data set. The most popular ones for assigning fractal dimensions to time series are the box-counting method, rescaled range analysis, and variation method. From these three, the variation method is very good for time series analyses [18]. Dubuc *et al.* described the variation method as a method that gives more accurate results than the standard box-counting method, as well as being more robust and efficient [19]. The method uses coverings built out of intervals, ε, rather than boxes. A very simple example of covering a curve constructed by the variation method is shown in Fig. 3. The covering is constructed by determining oscillations at interval points along the curve. If the curve is labeled *x (t)*, the oscillation at a point *x (t₀)* is simply:

$$V(x(t_0),\varepsilon) = \max\ x(\tau) - \min\ x(\tau) \qquad \text{for} \qquad \tau \in (t_0 - \varepsilon, t_0 + \varepsilon) \qquad (1)$$

This corresponds to the height of the cover shown in Fig. 3. The interval span, ε, gives the scale at which one measures the oscillations, much similar to the span of boxes in the box counting method. For a certain ε the coverage surface area, found by integrating V over the curve, is known as the variation of *x*, and is denoted $V(\varepsilon)$. Obviously, the dependence of $V(\varepsilon)$ to ε is such that as ε decreases, so does $V(\varepsilon)$. In order to find the fractal dimension, the rate at which the area, $V(\varepsilon)$, decreases as ε tends to *0* is calculated. The approach to derive fractal dimension is a mathematical procedure explained in the references. It turns out that a log-log plot of $V(\varepsilon)/\varepsilon^2 vs. 1/\varepsilon$ gives the fractal dimension as its slope [18, 19].
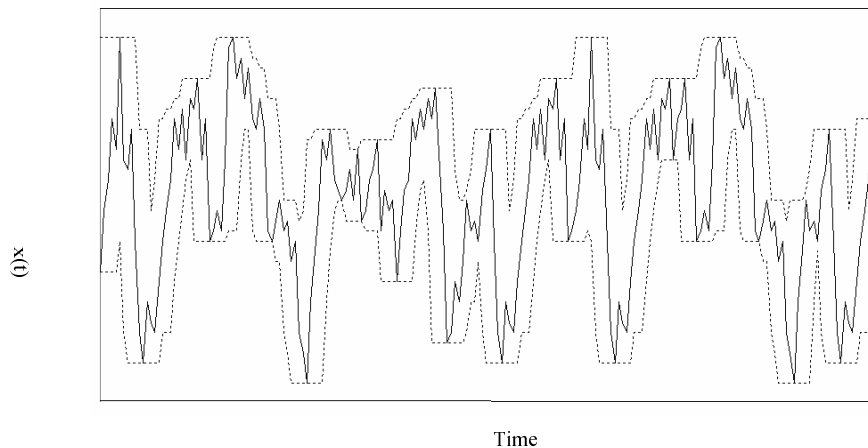

Fig. 3. A typical covering for an arbitrary record by variation method [18]

4. STUDY AREA

Union County Well (well # 39-119) is located in New Jersey, US (Site # 404106074171901) with 40˚41´ 06´´north latitude and 74˚14´19 ´´east longitude. The well depth is 290 ft and ground surface elevation is 69.00 ft above mean sea level. The well was completed in "Early Mesozoic basin aquifers"

(N300ERLMZC) in the US national aquifer (Fig. 4). The daily data are recorded by USGS for a period of 65 years (from 1943 to 2008) except for an 8 year gap (1975 to 1983) [22]. In this paper a combination of fractal dimension (variation method) with ANN (FeedForward Backpropagation) was used to forecast groundwater depths recorded in the well.

Fig. 4. Location of Union County Well in New Jersey, US

5. MODEL STRUCTURE

a) Input vector selection

One of the most important steps in the model development process is the determination of significant input variables and the effective domain of each parameter. Usually, not all of the potential input variables will be equally informative since some may be correlated, noisy, or have no significant relationship with the output variable being modeled [23]. Generally, some degree of *a priori* knowledge is used to specify the initial set of candidate inputs [24, 25]. Although *a priori* identification is widely used in many applications and is necessary to define a candidate set of inputs, it is dependent on an expert's knowledge, and hence, is very subjective and case dependent. When the relationship to be modeled is not well understood, then an analytical technique, such as Principal Axes Component, is often employed [26-28].

In this paper, daily depth to groundwater was used as the input. The fractal dimension of measured data was used to evaluate the effective domain. Two sets of data, measured daily data (set #1) and data averaged over the effective domain (set #2) were used to forecast groundwater depths.

b) Hidden neurons optimization

In order to ensure good generalization ability by an ANN model, a number of empirical relationships between the number of training samples and the number of connection weights have been suggested in the literature [18]. However, network geometry is generally highly problem dependent and these guidelines do not ensure optimal network geometry, where optimality is defined as the smallest network that adequately captures the relationships in the training data (principle of Parsimony). In addition, there is quite a high variability in the number of hidden nodes suggested by various rules. While research is being conducted in this direction by the scientists working in ANNs, it may be noted that traditionally, optimal network geometries have been found by trial and error [18].

In this paper, the number of hidden layers in the network, which is responsible for capturing the dynamic and complex relationship between input and output variables, was identified by various trials. The trial and error procedure started with one hidden layer initially, and the number of hidden layers was

increased up to 3 with a step size of 1 in each trial. For each set of hidden layers, the network was trained in *batch mode* to minimize the mean square error at the output layer. The training was stopped when there was no significant improvement in the efficiency, and the model was then tested for its general properties.

c) Internal parameters of the model

A sigmoid function was used as the activation function in both hidden and output layers. As the sigmoid transfer function has been used in the model, the input-output data have been scaled appropriately to fall within the function limits. A standard back propagation algorithm has been employed to estimate the network parameters [18]. The learning rate was held constant throughout training in the standard steepest descent (back propagation) process.

6. RESULTS

Monitored daily groundwater depths in the Union County Well from Mar 1985 to Mar 2007 are shown in Fig. 5 as a time series. The fluctuating nature of the data reflects a fractal character which has an average, minimum, maximum, and standard deviation of 6.79 *m*, 3.45 *m*, 10.75 *m*, and 0.956 *m*, respectively. Variation method was applied to the data set considering different domains of 20, 40, 60, 80, 100, and 120 preceding days and the fractal dimensions were determined (Table 1). Figure 6 shows a typical fractal dimension determination curve for a domain of 80 days. As shown on Table 1, the fractal dimension remained constant (*1.52*) when the length of the domain decreased below 80 days.

Data sets in different domains were fed to ANN and groundwater depth was forecasted. Figures 7, 8, and 9 depict estimated versus observed groundwater depths for a 22-year period considering the domains of 20, 80, and 120 preceding days, respectively. Root Mean Square Error (RMSE) and the correlation factor (R^2) of the estimated and observed groundwater depths for all domains are also presented in Table 2. In general, when the domain length increased from 20 to 120, groundwater depth forecast was improved as evidenced by the lower RMSEs (*0.563509* compared to *0.098972 m*) and higher R^2s (*0.69023* compared to *0.99273*). However, the improvement was less than 1% beyond the domain length of 80 days; an achievement which may not offset the drawbacks.

Comparing Tables 1 and 2, the data in the 80-day domain was selected as the effective data which required a reasonable computational effort and yielded an acceptable R^2 (*0.99218*) and RMSE (*0.102719 m*). It was concluded that fractal dimension may be successfully utilized to lower the size of input data sets considerably, while maintaining the effective information in the data set. The calculated effective domain (80 days) agrees well with the effective domain of a few months reported by other researchers on groundwater depth time series.

Statistical parameters of training and testing the network with the two data sets considering different numbers of hidden layers are shown in Table 3. As shown, errors are bound to acceptable values (less than *0.037*) and typically smaller in lower numbers of hidden layers for both data sets. No over flowing was observed in either sets of data and it was concluded that data with one hidden layer was sufficient for the accuracy of the network.

Comparison of the estimated data (based on data averaged over the effective domain; set #2) and the observed data is shown in Fig. 10. Similar comparison for the estimated data (based on daily data in the effective domain; set #1) and the observed data was shown in Fig. 8. Comparing Figs. 8 and 10, it was postulated that the more accurate nature of data set #1 (shown in Fig. 8) was the reason for a better prediction with a lower RMSE (*0.1027 m* compared to *0.255 m*). However, one major drawback was the size of the input data in set #1 which was 80 times the size of the input data in set #2; a factor that may increase the computational effort unpredictably. Hence, it was concluded that fractal analysis may be successfully utilized to lower the size of input data sets considerably, while yielding acceptable RMSE (*0.255 m* in our case) as well. In other words, the proposed method may be utilized to forecast

groundwater depth effectively. However, further research seems necessary before the application of the method to other data sets.
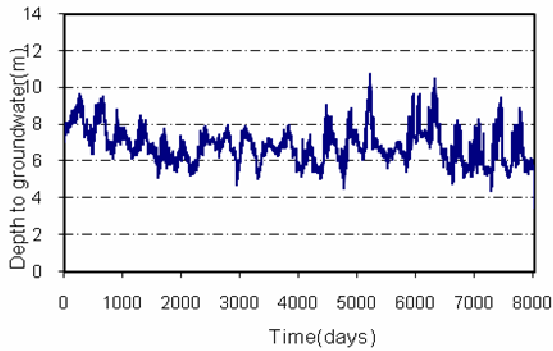

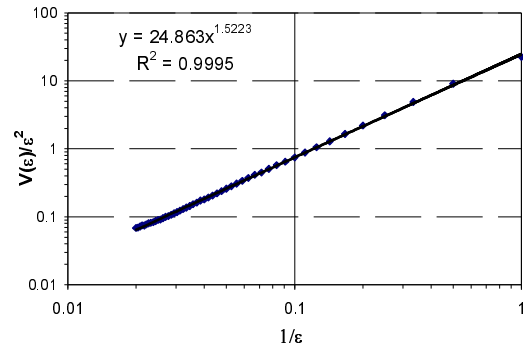Fig. 5. Observed groundwater depth in Union County Well

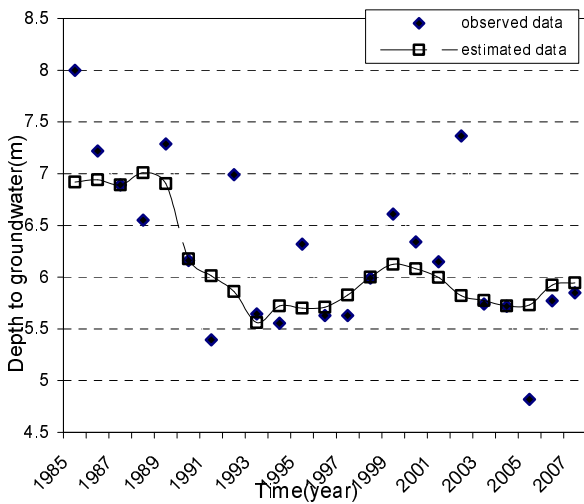Fig. 6. Fractal dimension for data over a domain of 80 days

Fig. 7. Comparison of the estimated data (based on daily data in the 20-day domain) and observed data
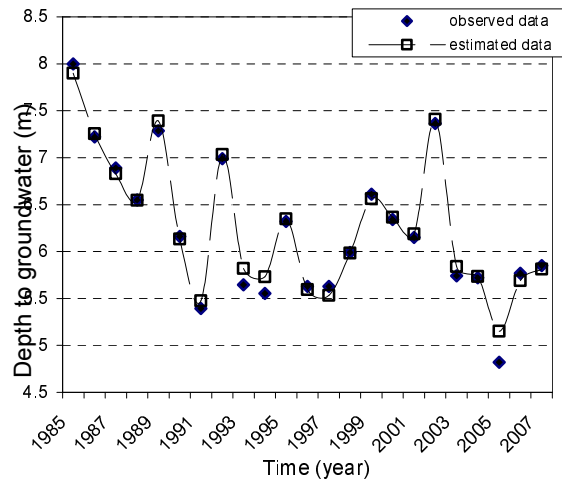

Fig. 8. Comparison of the estimated data (based on daily data in the effective domain; set #1) and observed data
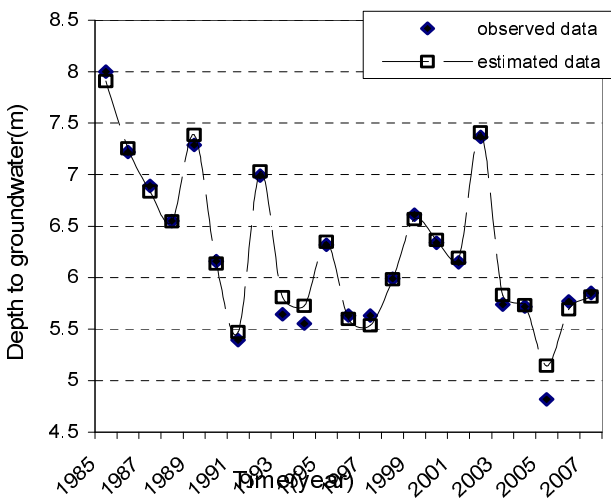

Fig. 9. Comparison of the estimated data (based on daily data in the 120-day domain) and observed data
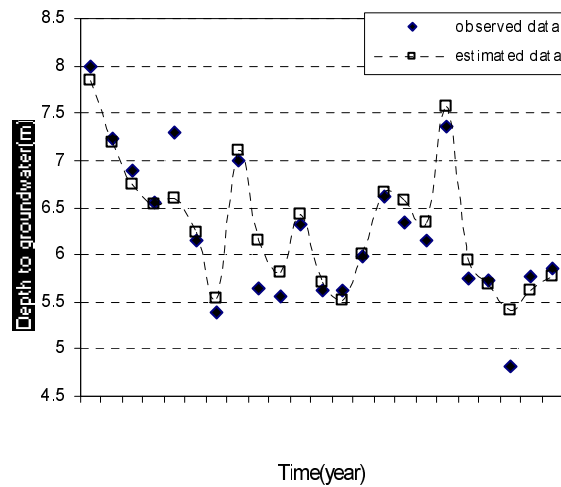

Fig. 10. Comparison of the estimated data (based on data averaged over the effective domain; set #2) and observed data

G. R. Rakhshandehroo and Z. Ghadampour

Table 1. Fractal dimension calculated for different domain lengths

Domain length (days)	Fractal dimension
20	1.52
40	1.52
60	1.52
80	1.52
100	1.4943
120	1.4906

Table 2. Root Mean Square Error (RMSE) and correlation factor (R^2) for different domain lengths

Domain length (days)	R^2	*RMSE (m)*
20	0.69023	0.563509
40	0.85301	0.284730
60	0.89763	0.163450
80	0.99218	0.102719
100	0.99225	0.099437
120	0.99273	0.098972

Table 3. Statistical parameters for training and testing data sets

Data Sets	No. of Hidden Layers	Error in Training Data			Error in Testing Data		
		Ave (m)	Max (m)	Pd (%)	Ave (m)	Max (m)	Pd (%)
Set #1	1	0.026	0.13	91.4	0.035	0.16	88.46
	2	0.03	0.14	89.14	0.037	0.15	87.5
	3	0.09	0.33	43.67	0.11	0.32	35.58
Set #2	1	0.037	0.16	85.52	0.046	0.16	79.81
	2	0.037	0.15	85.7	0.046	0.15	77.88
	3	0.095	0.33	42.08	0.11	0.34	35.58
Pd: percent of data with less than *0.05 m* error							

7. CONCLUSION

A combination of fractal analysis and ANN was utilized to forecast groundwater depths in two different sets of data. Variation method was applied to the sets considering different domains of 20, 40, 60, 80, 100, and 120 preceding days and the fractal dimension was determined. Fractal dimension remained constant (1.52) when the length of the domain decreased below 80 days. Hence, it was selected as the effective domain and utilized in the ANN to enhance its performance. This combination of fractal analysis and ANN decreased the computational effort and, at the same time, yielded acceptable R^2 and RMSE. The

number of hidden layers was optimized by trial and error to one hidden layer for both data sets. It was postulated that the more accurate nature of the measured daily data was the reason for a better forecast with the lower RMSE (*0.1027 m* compared to *0.255 m*) in set #1. However, one major drawback was the size of input data in this set which was 80 times the size of the input data in set #2; a factor that may increase the computational effort unpredictably. It was concluded that fractal analysis may be successfully utilized to lower the size of the input data sets considerably, while maintaining the effective information in the data set. In other words, the proposed method may be utilized to forecast groundwater depth effectively. However, further research seems necessary before application of the method to other data sets.

REFERENCES

1. Knotters, M. & Van Walsum, P. E. V. (1997). Estimating fluctuation quantities from the time series of water table depths using models with a stochastic component. *Journal of Hydrology*, Vol. 197, pp. 25–46.

2. Safavi, H. R., Afshar, A., Ghaheri, A. & Marino, M. A. (2004). A coupled surface water and groundwater flow model. *Iranian Journal of Science and Technology, Transaction B: Engineering*, Vol. 28, No. B1, pp. 38-48.

3. Van Geer, F. C. & Zuur, A. F. (1997). An extension of Box-Jenkins transfer noise models for spatial interpolation of groundwater head series. *Journal of Hydrology*, Vol. 192, pp. 65–80.

4. Daliakopoulos, I. N., Coulibaly, P. & Tsanis, I. K. (2005). Groundwater level forecasting using artificial neural networks. *J. Hydrol.*, Vol. 309, pp.229–240.

5. Bierkens, M. F. P. (1998). Modeling water table fluctuations by means of a stochastic differential equation. *Water Resources Research*, Vol. 34, pp. 2485–2499.

6. Knotters, M. & Bierkens, M. F. P. (2000). Physical basis of time series models for water table depths. *Water Resources Research*, Vol. 36, No. 1, pp. 181–188.

7. Castellano-Méndez, M., W., González-Manteiga, M., Febrero-Bande, J. M., Prada-Sánchez, & Lózano-Calderon, R. (2004). Modeling of the monthly and daily behavior of the runoff of the Xallas River using Box–Jenkins and neural networks methods. *J. Hydrol.*, Vol. 296, pp. 38-58.

8. Wong, H., Ip, W. C., Zhang, R. Q. & Xia, J. (2007). Non-parametric time series models for hydrological forecasting. *J. Hydrol.*, Vol. 332, pp. 337-347.

9. Kuo, Y. M., Liu, C. W. & Lin, K. H. (2004). Evaluation of the ability of an artificial neural network model to assess the variation of groundwater quality in an area of blackfoot disease in Taiwan. *Water Res.*, Vol. 38, pp. 148-158.

10. Aqil, M., Kita, I., Yano, A. & Nishiyama, S. (2007). Analysis and prediction of flow from local source in a river basin using a Neuro-fuzzy modeling tool. *J. Environ. Manage.*, Vol. 85, pp. 215-223.

11. Rizzo, D. M. & Dougherty, D. E. (1994). Characterization of aquifer properties using artificial neural networks: Neural kriging. *Water Resources Research*, Vol. 30, No. 2, pp. 483–497.

12. Ranjithan, R. S. Eheart, D. E. & Garrett, J. H. (1995). *Application of neural network in groundwater remediation under conditions of uncertainty, in new uncertainty concepts in hydrology and water resources*. Edited by Z. W. Kundzewicz, pp. 133–144, Cambridge University Press, New York.

13. Morshed, J. & Kaluarachchi, J. J. (1998). Parameter estimation using artificial neural network and genetic algorithm for free-product migration and recovery. *Water Resources Research.* Vol. 34, No. 5, pp. 1101–1113

14. Coulibaly, P., Anctil, F., Aravena, R. & Bobee, B. (2001). Artificial neural network modeling of water table depth fluctuations. *Water Resources Research*, Vol. 37, No. 4, pp. 885–896.

15. Coulibaly, P., Anctil, F., Aravena, R. & Bobée, B. (2001). Artificial neural network modeling of water table depth fluctuations. *Water Resour. Res.*, Vol. 37, pp. 885-896.

16. Jiang, S. Y., Ren, Z. Y., Xue, K. M. & Li, C. F. (2008). Application of BPANN for Prediction of backward ball spinning of thin-walled tubular part with longitudinal inner ribs. *J. Mater. Process*, Vol. 196, pp. 190-196.

17. Yang, Z. P., Lu, W. X., Long, Y. Q. & Li, P. (2009). Application and comparison of two prediction models for groundwater levels: A case study in Western Jilin Province, China, *Journal of Arid Environments*, pp. 1–6

18. Dubuc, B., Quiniou, J. F., Roques-Carmes, C., Tricot, C. & Zucker, S. W. (1989). Evaluating the fractal dimension of profiles. *Phys Rev A*, Vol. 39, No. 3, pp. 1500-1512.

19. Dubuc, B. & Dubuc, S. (1996). Error bounds on the estimation of fractal dimension. *SIAM J Numer Anal*, Vol. 33, No. 2, pp. 602-626.

20. Lee, C. K., Ho, D. S., Yu, C. C. & Wang, C. C. (2003). Fractal analysis of temporal variation of air pollutant concentration by box counting. *Environmental Modeling & Software*, Vol. 18, pp. 243–251.

21. Rumelhart, D. E., Hinton, G. E. & Williams, R. J. (1986). Learning representations by backpropagating errors. *Nature*, Vol. 323, pp. 533–536.

22. http://waterdata.usgs.gov/nwis/dv/?site_no=404106074171901&referred_module=gw

23. Maier, H. R. & Dandy, G. C. (2000). Neural networks for the prediction and forecasting of water resources variables: A review of modeling issues and application. *Environmental Modeling and Software*, Vol. 15, pp. 101–124.

24. Campolo, M., Andreussi, P. & Soldati, A. (1999). River flood forecasting with neural network model. *Water Resources Research*, Vol. 35, No. 4, pp. 1191–1197.

25. Thirumalaiah, K. & Deo, M. C. (2000). Hydrological forecasting using neural networks, *Journal of Hydrologic Engineering*, Vol. 5, No. 2, pp. 180–189.

26. Luk, K. C., Ball, J. E. & Sharma, A. (2000). A study of optimal model lag and spatial inputs to artificial neural network for rainfall forecasting. *Journal of Hydrology*, Vol. 227, pp. 56–65.

27. Coulibali, P., Anctil, F. & Bobee, B. (2001). Multivariate reservoir inflowforecasting using temporal neural network. *Journal of Hydrologic Engineering*, Vol. 6, No. 5, pp. 367–376.

28. Sudheer, K. P., Gosain, A. K. & Ramasastri, K. S. (2002). A data-driven algorithm for constructing artificial neural network rainfall-runoff models. *Hydrological Processes*, Vol. 16, pp. 1325–1330.