

OPTIMAL PLACEMENT OF SEMI ACTIVE DAMPERS BY POLE ASSIGNMENT METHOD*

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Abstract– The determination of the optimal locations of controllers in active or semi active structural control has received very little attention. The optimal placement, compared with the non optimal case, provides control performance with a fewer number of controllers and smaller control force. In this research, the effect of the locations of the controllers on the control force and control performance was studied. This project uses the ‘Pole-Assignment’ method for the purpose of determining control forces, and for considering the semi active state. In the semi active control state system, the control force is determined by specifying a value to produce this state. During this study, two optimization methods have been used for active control. In the first method, all cases are studied to find the optimal case. Then several evaluation criteria have been used to find an optimal case. The second method is based on defining a performance index related to each story. In this method, with a series of repeatable operations, stories with a maximum index have been selected as optimum. The numerical examples show that the results of the first and second method are somewhat different. In addition, the findings of this study indicate that the number of controllers can be reduced in most cases. General results have been obtained for the optimal placement of controllers in common structures.

Keywords– Optimal placement, semi active control, pole assignment method

1. INTRODUCTION

Low expense of the semi active control systems, compared with active control systems, is one of the reasons that this technology has been rapidly expanding. An adequate understanding of the optimal locations of the controllers in the active or semi active structural control is currently lacking. In the case of optimal placement, in contrast with the non optimal case, the control is achieved using a lesser number of controllers and a smaller control force. Limited studies of optimal control have been done by several investigators.

Cheng *et al.* studied the optimal placement of controllers by using a controllability index [1]. Zhang and Soong introduced a sequential method for optimal viscoelastic damper placement. They used a transfer matrix for calculating the controllability index [2]. Takewaki performed optimal placement with transfer function minimization [3]. Abdullah used gradient-based optimization techniques to study the optimal placement of controllers in shear-type and slender structures [4, 5]. Xing *et al.* studied optimal placement of controllers subjected to excitation acting on an intermediate storey [6]. Abdullah *et al.* used a genetic algorithm in conjunction with gradient-based optimization techniques for the purpose of the optimal placement of controllers [7].

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2. OPTIMIZATION WITH POLE ASSIGNMENT METHOD

In this study, two methods for optimization in active control are presented and used. The first method investigates all possible cases. The optimal case is then selected by some evaluation criteria. The criteria uses the maximum and RMS of responses and control force. The second method uses the performance index related to each storey. The details of this method are described later in the paper. The results of these two methods are then compared. The determination of the control force for both methods has been done by the pole assignment method. In this case, after finding the poles, the gain matrix is determined from the Brogan algorithm [9], and the control force is obtained from the following relation:

$$u(t) = F \cdot q(t) \quad (1)$$

Where F is the gain matrix, $q(t)$ is the state vector and $u(t)$ is the control force.

To investigate the performance of each case, a set of evaluation criteria is used. The criteria are related to the peak value of the response, the RMS of the response and the maximum control force. The criteria related to the peak value of the response are represented by the following four relations:

$$EC1 = \frac{\max|y_{con}(t)|}{\max|y_{unc}(t)|} \quad (2)$$

$$EC2 = \frac{\max|x_{con}(t)|}{\max|x_{unc}(t)|} \quad (3)$$

$$EC3 = \frac{\max|\ddot{y}_{con}(t)|}{\max|\ddot{y}_{unc}(t)|} \quad (4)$$

$$EC4 = \frac{\max|V_{0con}(t)|}{\max|V_{0unc}(t)|} \quad (5)$$

In the above relations, indices 'con' and 'unc' are related to controlled and uncontrolled cases, respectively. y is storey displacement; x is storey relative displacement, \ddot{y} storey acceleration; and V_0 is the force of the base shear. For the second criteria, the RMS of the response (instead of peak value) is used.

$$z = T / \Delta t \quad (6)$$

$$RMS(.) = \sqrt{\left[\sum_{i=1}^n \sum_{j=1}^z (.^2_{ij}) \right] / (n \cdot z)} \quad (7)$$

In the above relations, n is the number of system degrees of freedom; z is the number of the time interval, T is the total time; and Δt is the time interval used in calculations. The criteria related to the RMS of responses are presented by the following relations. In this case, $EC5$ is the ratio of RMS of storey displacement, $EC6$ is the ratio of RMS of storey relative displacement, $EC7$ is the ratio of RMS of storey acceleration, and $EC8$ is the ratio of RMS of storey shear force:

$$EC5 = \frac{RMS|y_{con}(t)|}{RMS|y_{unc}(t)|} \quad (8)$$

$$EC6 = \frac{RMS|x_{con}(t)|}{RMS|x_{unc}(t)|} \quad (9)$$

$$EC7 = \frac{RMS|\ddot{y}_{con}(t)|}{RMS|\ddot{y}_{unc}(t)|} \quad (10)$$

$$EC8 = \frac{RMS|V_{0con}(t)|}{RMS|V_{0unc}(t)|} \quad (11)$$

The last criterion is $EC9$, and is related to the necessary control force for each case:

$$EC9 = \frac{\max|U(t)|}{W} \quad (12)$$

where W is the seismic weight of the building model and is obtained from the following relations:

$$M_i = \{\phi\}_i^T \cdot [m] \cdot \{\phi\}_i \quad (13)$$

$$L_i = \{\phi\}_i^T \cdot [m] \cdot \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix} \quad (14)$$

$$W_i = g \frac{L_i^2}{M_i} \quad (15)$$

$$W = \sqrt{\sum_{i=1}^n W_i^2} \quad (16)$$

Where M_i , L_i and W_i are modal mass, partnering coefficient and modal weight, respectively. $[m]$ is the matrix of the structural mass and g is the coefficient of ground gravitation. The last criteria are based on the performance index for the comparison of the necessary control force and displacement of the controlled structure. The performance index (J), along with the Q and R matrices are shown below

$$J = \int_0^{t_f} (y_{con}(t)^T \cdot Q \cdot y_{con}(t) + U(t)^T \cdot R \cdot U(t)) dt \quad (17)$$

$$R = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & & \\ \vdots & & \ddots & \vdots \\ 1 & \dots & 1 & \end{bmatrix} \quad ; \quad Q = 10^{10} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & & \\ \vdots & & \ddots & \vdots \\ 1 & \dots & 1 & \end{bmatrix}$$

Lower values for the above indices imply that the control system is more efficient. To begin the analysis, given the number of time intervals, the structural response is determined from the known external force, and if necessary, it is calculated from the control force. The above method is explained in the form of the first algorithm which is summarized bellow

- *First Algorithm*

1. The total given interval $[0 \ T]$ is divided into a z subinterval with a length Δt . The analysis is done in the subintervals.
2. The external force $[P_{i-1} \ P_i]$ is used in the time interval $[t_{i-1} \ t_i]$.
3. The initial conditions of the analysis is $q_0 = q_{i-1}$.
4. The time interval is divided into z' subinterval with length $\Delta t'$ to increase calculation accuracy.
5. External force in the time interval is obtained by linear interpolation.
6. Using initial conditions and external force, the uncontrolled response of the structure is obtained.
7. The maximum value of the displacement of an uncontrolled structure is called XM in interval $[t_{i-1} \ t_i]$.

8. If $X_M \leq (X_P + X_{ERR})$, (in which X_P is the peak value and X_{ERR} is the permissible tolerance value of displacement) it is not necessary to use control force in the interval. But if $X_M > (X_P + X_{ERR})$, the new control forces are determined by the pole assignment method to satisfy the necessary requirements.
9. Perform steps 2 to 8 until $t_i = T$.
10. Since the device is semi active, every control force cannot be produced. The control force is considered as a step force, and thus it will have special values. In this case, the semi active control device (stiffener or damper or both) act as a parallel system to produce the necessary control force. The maximum value of producible control force is limited to 10 % of the total weight of the structure. Based on these explanations, it is necessary to change the calculations of the gained control force to the nearest producible value.
11. For response control, external force and semi active control force act together to the structure. Then, the response in consecutive time intervals is calculated. Finally, the response is checked to determine if the maximum response of the controlled structure in the total time interval is less than the allowable value.
12. After checking the response, the values of performance criteria $EC1$ to $EC9$ are obtained.

To find the optimal case, all possible cases are first investigated. Let us assume that the goal is to place r controllers in a structure with n stories ($r \leq n$). Thus, the number of possible ways to perform this calculation is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (18)$$

Since the gain matrix is not a unit matrix, different gain matrices must be investigated to find the optimal gain matrix. Here the location of the controllers is changed by the hypothesis that the columns of the determinant $|I_n - F\psi(\lambda)|$ are zero (in which I is identity matrix, F is gain matrix, λ is eigenvalues of a close-loop system and ψ is a matrix with columns of λ). In other words, the locations of the controllers are changed by changing the *ones* in the E matrix which is the location matrix.

Therefore, for a given E matrix as described above, the total number of gain matrices for the special case is

$$r^{(n-r)} \quad (19)$$

As a result, to investigate the optimal location of r controllers in a structure with n stories, it should be analyzed in the following number of times:

$$\binom{n}{r} \times r^{(n-r)} \quad (20)$$

To calculate optimal gain matrix, it must be analyzed $r^{(n-r)}$ times. The algorithm for this calculation is:

- Second Algorithm

1. $r^{(n-r)}$ E matrices are produced for given r and n .
2. $r^{(n-r)}$ times analysis is done on the basis of the first algorithm. If the control force needs to be calculated, it can be done by the related E matrix.
3. If in finding the poles any instability or divergence occurs, the calculation of this case is stopped and other cases with new E are investigated.
4. Steps 2 and 3 are repeated $r^{(n-r)}$ times.
5. Finally, the case which has the minimum response and also the control force is determined as the optimal gain matrix from all the cases, which obtains a result without instability or divergence.

The second method of optimization involves the use of the performance index of a storey. Using this method, a storey with less displacement needs less control force. On this basis, the performance index of a storey is defined to include a displacement response and the control force of that storey. Integration is done on a total time interval. In this case, the performance index of a storey is defined as follows:

$$J_s = \int_0^{t_f} \left(10^5 \times y_{s \text{ con}}^2(t) + U_s^2(t) \right) \cdot dt \quad (21)$$

Where $y_{s \text{ con}}$ and U_s are the controlled displacement and control force of the story. The algorithm used in this method is shown below:

- Third Algorithm
 1. r controllers are placed in r stories from n stories of structure.
 2. Steps 1 to 11 of the first algorithm are continued.
 3. Value J_s for all stories is calculated. For each storey that does not have a controller, the value of the control force related to that story is set to zero.
 4. J_s of stories are put in numerical order and r stories which have maximum J_s are chosen as optimal r stories. If the r stories are the same as the r stories which were chosen in the first step, the results can be taken as optimal case, and if they were not the same, then new r stories are chosen as stories which have controller and restart calculations.
 5. Steps 2 to 4 are repeated until r stories obtained in the two stages become equal. In this case, r stories will represent the given optimal case.

3. ANALYTICAL MODEL

The models used in this study include three structures with the characteristics shown in Tables 1 through 4.

Table 1. Characteristics of structure type A [2]: five-storey frame

| Storey | | 1 st | 2 nd | 3 rd | 4 th | 5 th |
|-------------------|------------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|
| Story mass | (tons) | 593.5 | 593.5 | 593.5 | 593.5 | 593.5 |
| Elastic stiffness | 10^2 (kN/m) | 5641 | 1988 | 1991 | 1912 | 2245 |
| Damping ratio | | Damping ratio for all modes is 2%. | | | | |

Table 2. characteristics of structure type B [4]: eight-storey frame

| Storey | | 1 st | 2 nd | 3 rd | 4 th | 5 th | 6 th | 7 th | 8 th |
|-------------------|------------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Story mass | (tons) | 345.6 | 345.6 | 345.6 | 345.6 | 345.6 | 345.6 | 345.6 | 345.6 |
| Elastic stiffness | 10^2 (kN/m) | 3404 | 3404 | 3404 | 3404 | 3404 | 3404 | 3404 | 3404 |
| Damping ratio | | Damping ratio for all modes is 2%. | | | | | | | |

Table 3. characteristics of structure type C [6]: ten-storey frame

| Storey | | 1 st | 2 nd | 3 rd | 4 th | 5 th | 6 th | 7 th | 8 th | 9 th | 10 th |
|-------------------|------------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|
| Story mass | (tons) | 1010 | 810 | 810 | 610 | 610 | 610 | 600 | 600 | 600 | 600 |
| Elastic stiffness | 10^2 (kN/m) | 1070 | 870 | 870 | 760 | 700 | 680 | 640 | 630 | 615 | 590 |
| Damping ratio | | Damping ratio for all modes is 2%. | | | | | | | | | |

Table 4. Summary of studied examples

| Example | Structural type | Number of controller | Sign |
|-----------------|-----------------|----------------------|--------|
| 1 st | A | 2 | E-5-2 |
| 2 nd | A | 3 | E-5-3 |
| 3 rd | B | 2 | E-8-2 |
| 4 th | B | 3 | E-8-3 |
| 5 th | B | 4 | E-8-4 |
| 6 th | C | 2 | E-10-2 |
| 7 th | C | 3 | E-10-3 |
| 8 th | C | 5 | E-10-5 |

The models were subjected to the loading effect of ground movement. The ground movement is the accelerometer data of the *El Centro 1940 NS* earthquake, shown in Fig. 1. The structural response is investigated in 12.5 seconds.

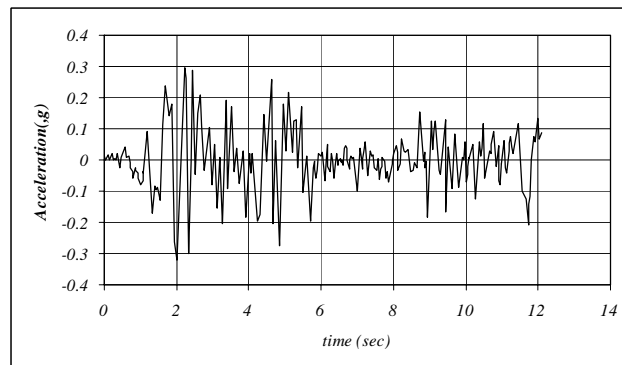


Fig. 1. Accelerometer data of El Centro 1940 NS earthquake

In every example from all cases, the cases with convergence responses were first investigated. Among these cases, the case with the most number of minimum evaluation criteria was selected as the optimal case.

4. EVALUATION

For cases (E-5-2) and (E-5-3), the optimal locations of two and three controllers are found as (2, 4) and (2, 3, 5), respectively. This selection is based on their evaluation criteria as compared with other cases. Comparison of optimal cases (E-5-2) and (E-5-3) are shown in Figs. 2 and 3, and in Table 5.

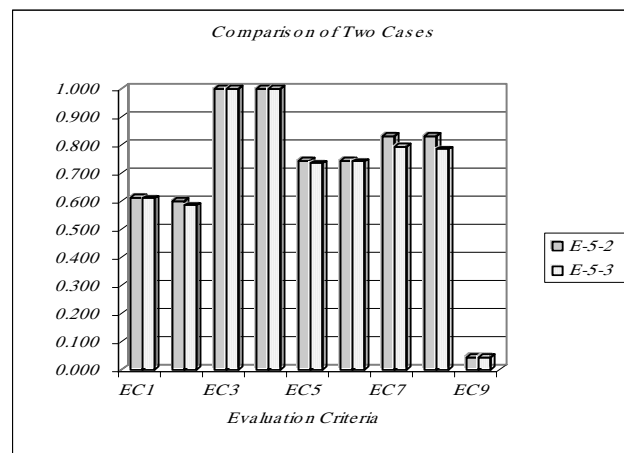


Fig. 2. Comparison of evaluation criteria of (E-5-2) and (E-5-3)

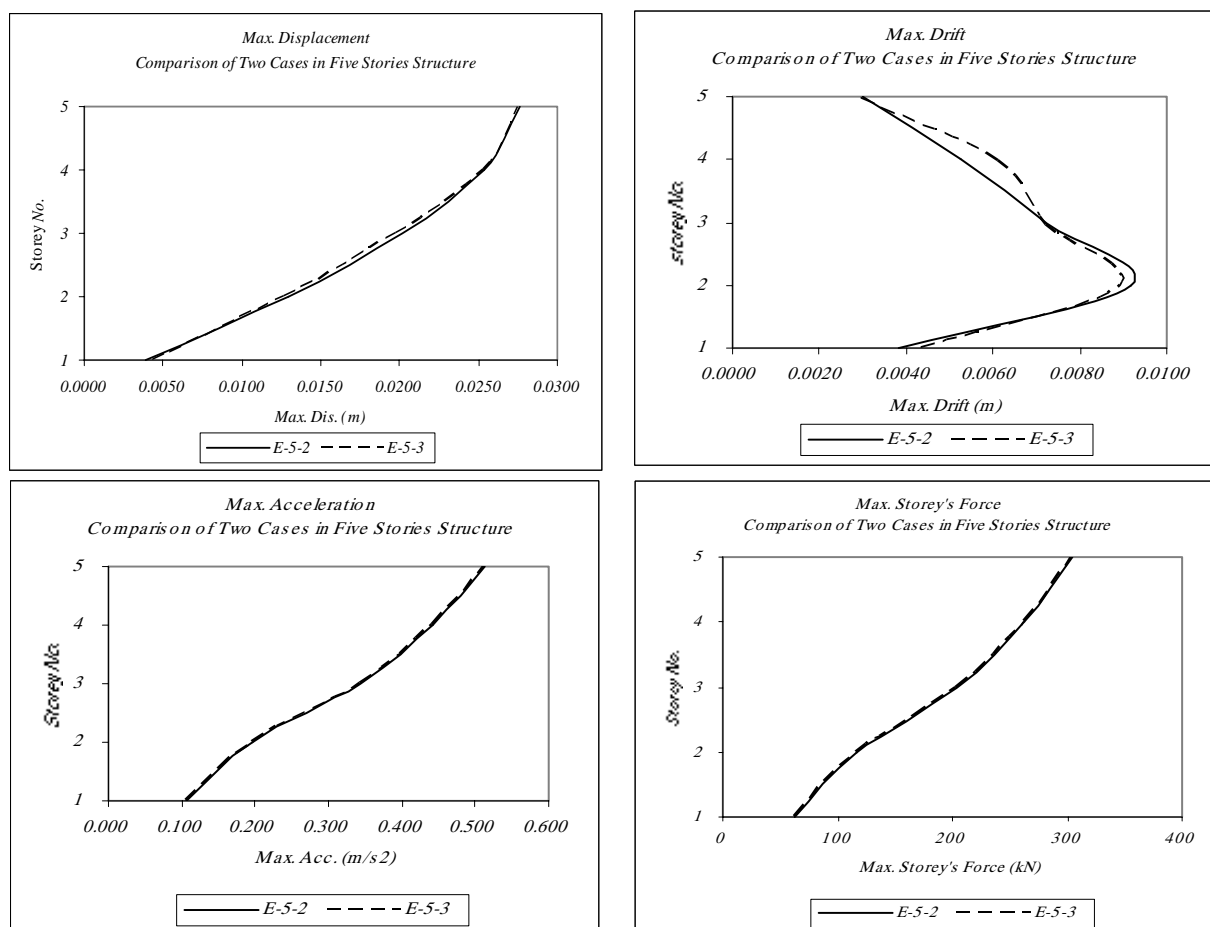


Fig. 3. Comparison of maximum response of optimal cases of (E-5-2) and (E-5-3)

Table 5. comparison of optimal cases in structure type A

| Case | E-5-2 | E-5-3 |
|--|-------|-------|
| Reduction in displacement (%) | 38.8 | 39.0 |
| Reduction in relative displacement (%) | 14.3 | 17.1 |
| Reduction in acceleration (%) | - | - |
| Maximum control force (KN) | 1000 | 1000 |

For cases (E-8-2), (E-8-3) and (E-8-4), optimal locations of two, three and four controllers are found as (5, 6), (5, 6, 7) and (3, 5, 6, 7), respectively. Comparison of optimal cases (E-8-2), (E-8-3) and (E-8-4) are shown in Table 6, and in Figs. 4 and 5.

Table 6- Comparison of optimal cases in structure type B

| Case | E-8-2 | E-8-3 | E-8-4 |
|--|-------|-------|-------|
| Reduction in displacement (%) | 43.7 | 61.5 | 83.3 |
| Reduction in relative displacement (%) | 5 | 10 | 15 |
| Reduction in acceleration (%) | - | 32.9 | 20.8 |
| Maximum control force (KN) | 3000 | 2000 | 1000 |

For cases (E-10-2), (E-10-3) and (E-10-5), optimal cases of two, three and four controllers are found as (7, 8), (5, 9, 10) and (2, 4, 7, 9, 10) respectively. Comparison of optimal cases (E-10-2), (E-10-3) and (E-10-5) are shown in Figs. 6 and 7, and in Table 7.

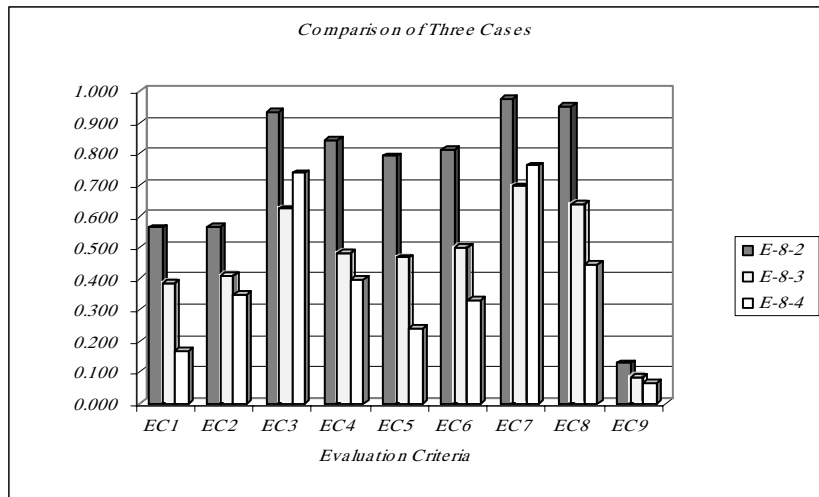


Fig. 4. Comparison of evaluation criteria of (E-8-2), (E-8-3) and (E-8-4)

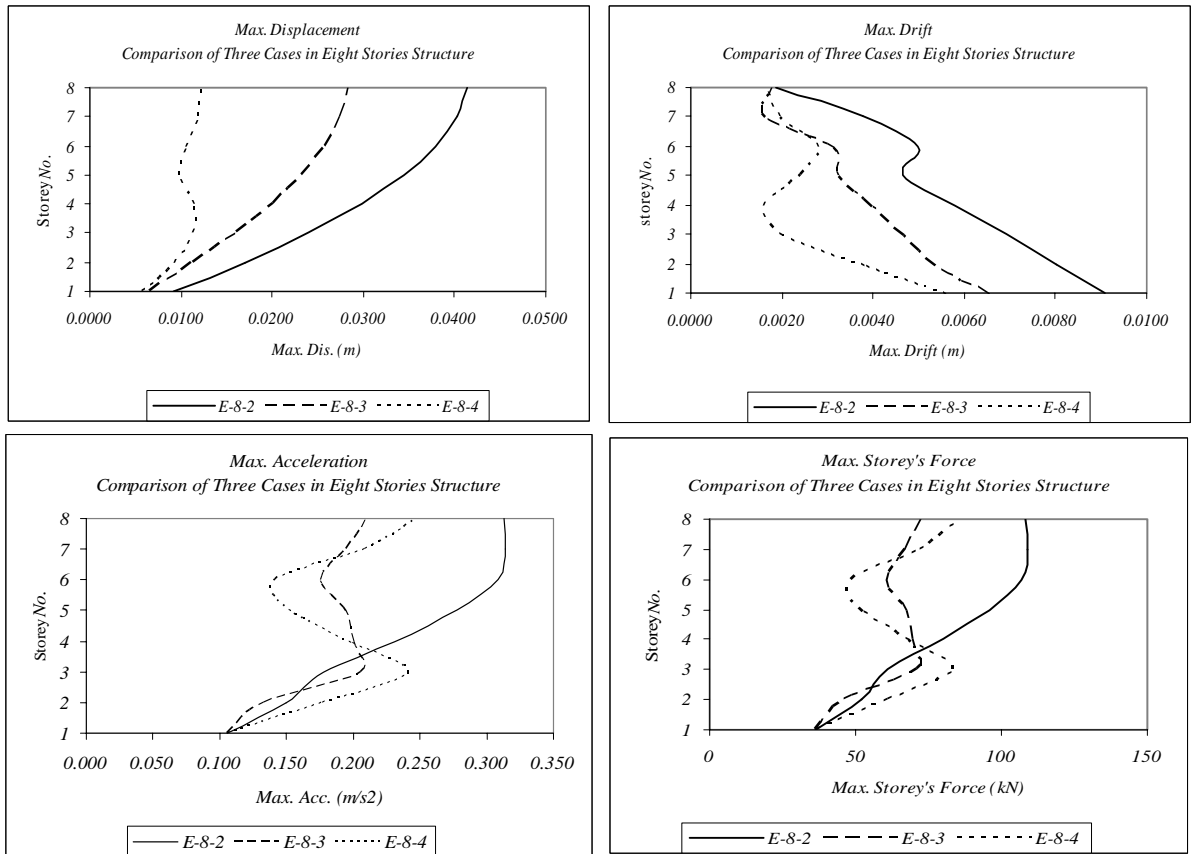


Fig. 5. Comparison of maximum response of optimal cases of (E-8-2), (E-8-3) and (E-8-4)

Table 7. Comparison of optimal cases in structure type C

| Case | E-10-2 | E-10-3 | E-10-5 |
|--|--------|--------|--------|
| Reduction in displacement (%) | 56.9 | 57.5 | 57.6 |
| Reduction in relative displacement (%) | 13.6 | - | 54.5 |
| Reduction in acceleration (%) | 4.4 | 4.4 | 4.4 |
| Maximum control force (KN) | 3500 | 3500 | 4500 |

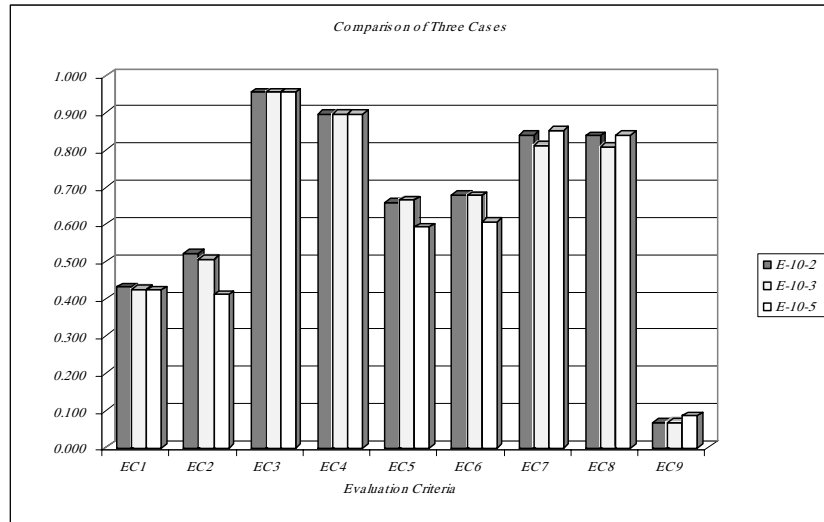


Fig. 6. Comparison of evaluation criteria of (E-10-2), (E-10-3) and (E-10-5)

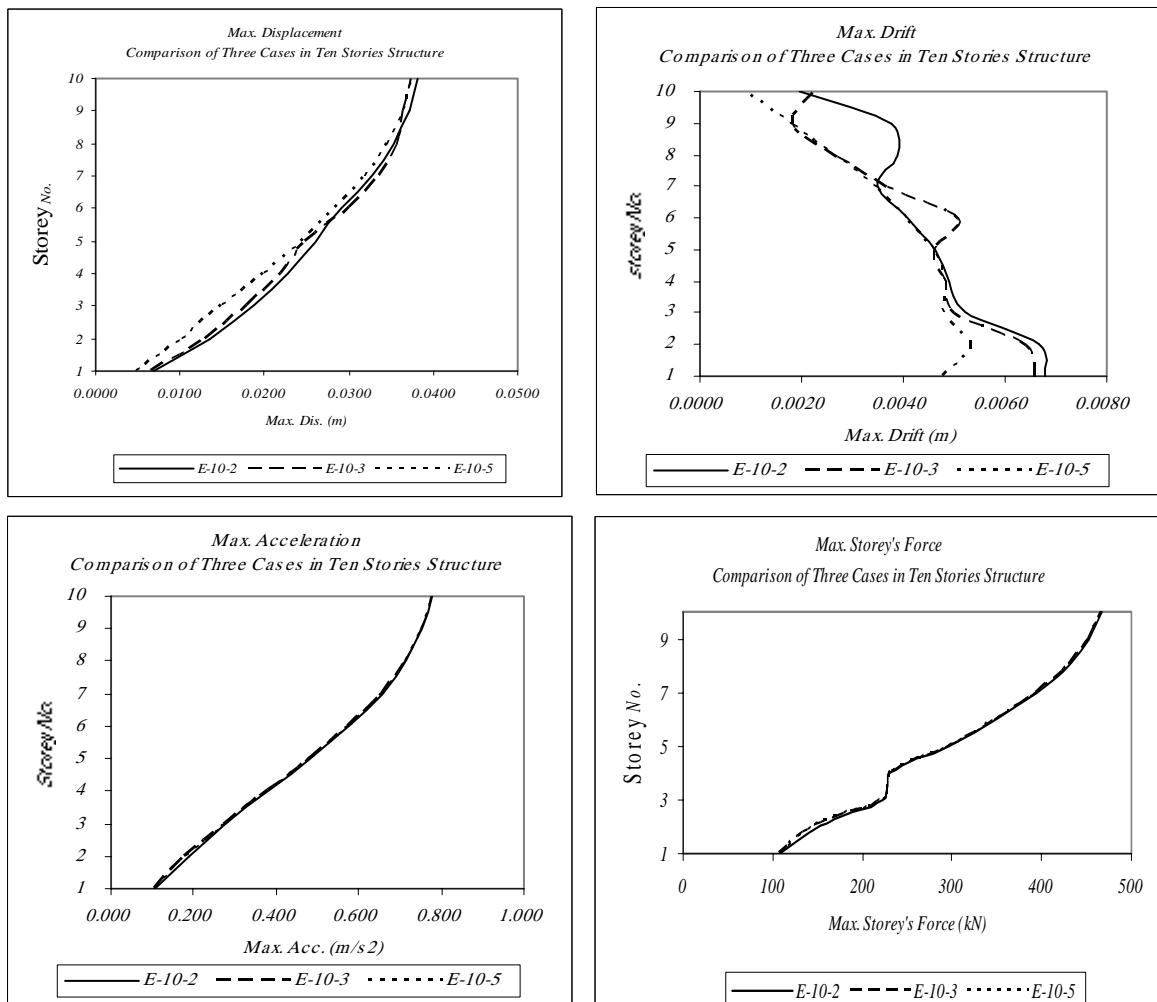


Fig. 7. Comparison of maximum response of optimal cases of (E-10-2), (E-10-3) and (E-10-5)

The next objective is to find the optimal location of the two controllers for case (E-10-2) by the performance index of the storey. To accomplish this objective, suppose that the optimal location of two controllers is stories (6, 9). Thus, according to the designed algorithm for the second method (third

algorithm), the index of stories for all stories are obtained and the two stories with the maximum indices are determined. Figure 8 (left) shows the diagram of the index of stories. It can be noted from this figure that the two stories which have the maximum indices are (9, 10). Since the stories are different from the stories that were first assumed, the analysis is done again. This time, controllers are placed in stories (9, 10).

In the second stage, for the stories with the maximum index, it's gained stories (9, 10) (Figure 8 right). Since the answer is the same as the one assumed at the beginning of the analysis, the optimal case gained from this method is (9, 10). But it should be noted that this optimal case is different from the optimal case gained from the first method.

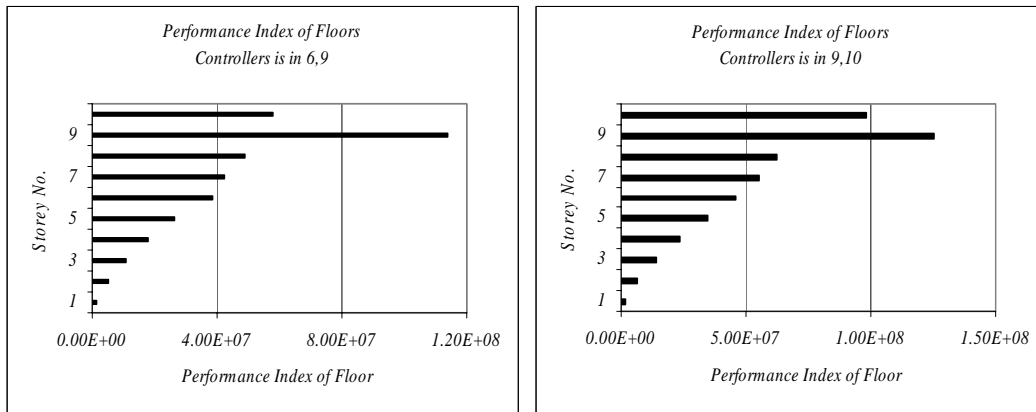


Fig. 8. Performance index of stories, left: case (6, 9) and right: (9, 10)

Then the optimal locations of the three controllers for case (E-10-3) were done by the performance index of the storey. To accomplish this objective, suppose that optimal locations of three controllers are stories (3, 7, 8). Thus, according to the designed algorithm for the second method (third algorithm), the index of stories for all stories are obtained and the three stories with the maximum indices are determined. Figure 9 (left) shows the diagram of the index of stories. It can be seen from this figure that the three stories with the maximum indices are stories (8, 9, 10). Since these stories are different from the stories that were first assumed, the analysis is done again. This time, controllers are placed in stories (8, 9, 10).

In the second stage, for the stories with the maximum index, the gained stories are (8, 9, and 10) (Fig. 9 right). Since the answer is the same as the one assumed at the beginning of the analysis, the optimal case obtained from this method is (8, 9, 10). But this optimal case is different from the optimal case which was gained from the first method.

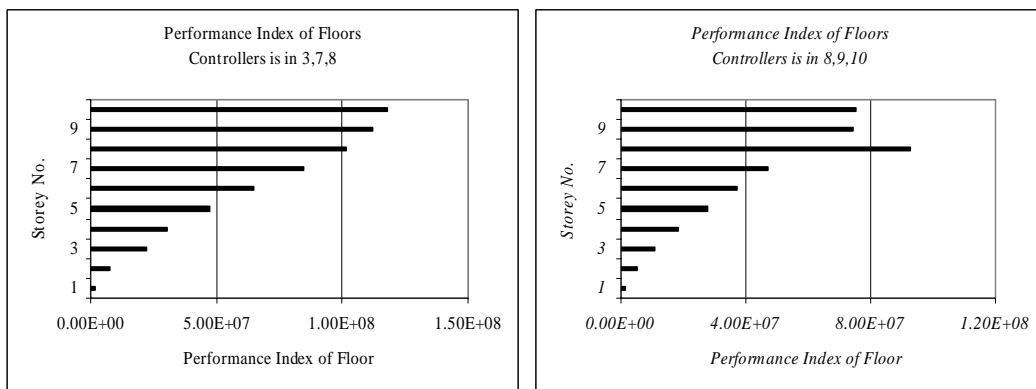


Fig. 9. Performance index of stories, left: case (3, 7, 8) and right: case (8, 9, 10)

5. CONCLUSION

The findings of this study demonstrate that the optimal control analysis results in less control force and a lesser number of controllers in comparison with the non optimal case. In most cases, the number of controllers does not have a greater effect on the desired control performance, and controlling can be done with fewer optimal controllers. In common structures with a moderate number of stories, the optimal location of the controllers are in the upper stories of the structure with an increasing number of controllers in the middle and lower stories. The results of the numerical analysis in this study can be used as initial data to train an artificial neural network. Optimal placement of controllers can then be done by the artificial neural network.

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