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DAM-RESERVOIR SEISMIC ANALYSIS BY THE EULERIAN METHOD, KOWSAR DAM, A CASE STUDY*

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Abstract– The dynamic interaction of fluid-solid systems has long been a subject of interest in many engineering fields. One of the most important problems of this nature, encountered in civil engineering, is the dynamic interaction of a dam and the reservoir under seismic loading. Recent research indicates the importance of this interaction and the resulting hydrodynamic forces, including the compressibility effects of water on the seismic response of concrete dams.

In this thesis a combination of 2D finite-element and a closed form method based on a Eulerian approach is developed to study the dam-reservoir interaction effect. The mentioned procedure is used to investigate the interaction effect on stresses and displacements of the Kowsar Dam-reservoir system.

To establish the accuracy of the method, the seismic response of the Pine Flat Dam is studied and the results are compared with other exact solutions.

Keywords – Concrete dam, interaction, Kowsar, gravity Dam

1. INTRODUCTION

a) General

The problems of 'fluid-solid' dynamic interaction exist whenever the relative motion of the two phases occurs. The dynamic response of fluid-solid interaction systems has gained a lot of interest in different fields of engineering such as civil, mechanical and aerospace engineering. The hydrodynamic effects caused by earthquakes in dam-reservoirs and water-intakes, the dynamic forces in water tanks and offshore structures, the interaction effects in floating and submerged structures such as ships and submarines, the interaction occurring in the flutter of aircraft wings or the oscillation of suspension bridges are categorized as fluid-solid interaction problems.

There are several methods to investigate the dynamic response of the mentioned systems, and they can be grouped into three main categories; the added mass approximation, the Lagrangian approach and the Eulerian approach.

The first presented solution was based on the added mass method. In this approach, the only effect of fluid was the portion of fluid mass which was added to the solid. The stiffness and damping effects of the fluid was ignored. In this state, the solid was solved without considering the fluid, and the solid mass matrix was modified by a portion of fluid mass. This method was used to analyze stiff and flexible structures such as dams and water tanks. In general, this method gives overestimated results, but is still useful for pre-analysis procedures.

In the Lagrangian approach, the fluid motion equation is the same as the solid. The governing

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equation for the fluid is the Lagrangian equation which acts the same as a solid with or without a small shear stiffness. However the solid and fluid are represented with the same terms, which relax the compatibility problem at the interface nodes. The main advantage of this method is that the fluid elements could be added into structural analysis software.

In the Eulerian approach, the fluid and solid are modeled with different terms. In general, the governing equations for fluid motion are the Navier-Stokes equations. For inviscid fluid and small amplitudes motion, these equations reduce to wave equations. The governing equation for the solid is the Lagrangian equation. To solve a solid-fluid system using this approach, each system should be solved individually and the interaction between them should be considered using an iteration method. The systems also could be solved together. Doing so results in large bandwidth, non-symmetric and non-positive mass and stiffness matrices.

In general this method needs custom software, which demands large computer resources.

b) Objectives

In the present work, the response of an infinitely long gravity dam-reservoir system for a ground acceleration normal to the dam axis is investigated. The hydrodynamic pressure acting on the wet surface of the dam is first evaluated in closed form as a function of the unknown deflections of the dam-reservoir interface and the ground accelerations. Employing finite element techniques, the motion of the dam is investigated. The hydrodynamic pressures enter the equation of motion of the dam as loading in excess of the inertia load. In this approach, the general flexibility of the dam cross-section and the compressibility of water are taken into account. A computer code based upon the mentioned procedure has been developed. The code has been tested with the popular concrete gravity dam, 'Pine Flat Dam'. This dam has been analyzed with all mentioned methods noted earlier

The dam-reservoir effect has been studied for the Kowsar Dam located near Behbahan city, Kohkeelooyeh and Boyrahmad province. The displacements and principal stresses due to earthquake forces have been studied.

2. LITERATURE REVIEW

The fluid coupling problem of a dam was originally considered by Westergaard [1]; his theoretical work showed that the rigid body acceleration of a vertical plane wall will produce a hydrodynamic pressure in the direction of the acceleration. It was pointed out that the hydrodynamic pressure can be expressed in terms of a body of water that is forced to move back and forth with the dam, whilst the rest of the reservoir remains inactive. Westergaard's study was based on the rigid body-movement, i.e., all points of the foundation and dam have the same instantaneous displacement, velocity and acceleration, the reservoir is infinitely long and the water is incompressible and inviscid. The above assumption may be valid when the period of harmonic excitation is greater than the fundamental period of the reservoir-dam system [2], otherwise considerable negative pressure will be calculated, and hence the added mass concept will lose its meaning [3]. On the basis of a simple linear momentum-balance principle, Von Karman [4] obtained distributions of the hydrodynamic pressure force and the total load on a rigid dam with a vertical upstream face, which were very close to the Westergaard results.

Saini *et al.* [5] used infinite elements coupled with finite elements to analyze the two-dimensional behavior of reservoir-dam systems. In this approach, the hydrodynamic effects on a dam are accurately simulated and the method can be adapted to arbitrary geometries.

Chopra and Gupta [6] investigated the linear response of an idealized dam cross-section to harmonic horizontal or vertical ground motion, subject to the combined effects of interaction between the dam and water and interaction between the dam and foundation. They conclude that the dam response at the resonant frequencies of water in the reservoir is strongly influenced by dam water interaction and less by the properties of the foundation rock.

Ahmad and Banerjee [7] used a fictitious vector function to approximate the inertial forces in the free vibration analysis by using the boundary element method with particular integrals. In their approaches, the static fundamental solutions were for solving dynamic problems. However, their study was limited to two-dimensional cases and required sophisticated adjustment for the suitably chosen constant.

Maheri and Taylor [8] used a quadratic fluid element and by placing it in SAP-IV program, they introduced a computer program for analyzing the dynamic response of two-dimensional dam-reservoir systems.

Ahmadi and Ozaka [9] introduced a three-dimensional dynamic analysis program for dam-reservoir systems by the Lagrangian approach. They assumed the fluid was inviscid and included the wave refraction in the dam base and wall for the reservoir element. The difficulty in using this program is that it is a time consuming process.

Bahar [10] used a three-dimensional, eight nodded Lagrangian fluid element implanted into a general-purpose finite element program. He applied his element in the dynamic analysis of the Pine Flat dam.

Tan and Chopra [11] have developed a three-dimensional method to analyze the dynamic response of arch dams including the dam-reservoir-foundation interaction effect and reservoir boundary absorption due to alluvium and sediments at the bottom and possibly at the sides of actual reservoirs.

Nassaj [12] introduced a new 8-27 variable node, and a 3D Lagrangian curvilinear fluid element and placed it into a general-purpose finite element program. He also included the surface wave effect in his formulation.

3. GOVERNING EQUATIONS

a) General

At small vibration amplitudes a concrete gravity dam will behave as a solid even though the joints between the monoliths may slip [13]. However, during large-amplitude motions, the behavior of a dam depends on the extent to which the inertia forces can be transmitted across the joints. For dams with straight contraction joints, either grouted or ungrouted, the inertia forces that develop during large-amplitude motion are much greater than the shear forces that the joints can transmit. Consequently, the joints would slip and the monoliths vibrate independently, as evidenced by the spalled concrete and water leakage at the joints of the Koyna Dam during the Koyna earthquake of 1 December, 1967 [14]. For such dams, a two-dimensional plane stress model of the individual monoliths appears to be appropriate for predicting the earthquake response. On the other hand, for dams with keyed contraction joints, it may be inappropriate to assume that the monoliths vibrate independently. A two- dimensional plane strain model would be better for such dams. The hydrodynamic effects of the water impounded in the reservoir are assumed to be adequately modeled by the two-dimensional wave equation. Water compressibility is included in the analysis, since it can significantly affect the earthquake response of concrete gravity dams [15]. The system is analyzed under the assumption of linear behavior for the concrete dam impounded water. Thus, the possibilities of concrete cracking or water cavitation are not considered.

b) System and ground motion

The system considered consists of a concrete gravity dam supported on the horizontal surface of underlying rigid foundation rock and impounding a reservoir of water. The selected monolith or dam cross-section is idealized as a two-dimensional finite element system in order to model arbitrary geometry and elastic material properties of the dam. Hence, non-overflow sections, overflow sections and appurtenant structures can be modeled. However, certain restrictions are imposed on the geometry of the dam to permit a continuum solution for hydrodynamic pressure in the impounded water. For the purpose of determining hydrodynamic effects, and only for this purpose, the upstream face of the dam is assumed to be vertical. This assumption is reasonable for actual concrete gravity dams because their upstream face is vertical or almost vertical for most of the height, and the hydrodynamic pressure acting on the dam face is insensitive to small departures of the face slope from the vertical, especially if these departures are near the base of the dam, which is usually the case. The water impounded in the reservoir is idealized as a fluid domain of constant depth and infinite length in the upstream direction.

The dam-water interaction effect is dominated by the overlying reservoir bottom materials which may consist of variable layers of alluvium, silt and other sediments, possibly deposited to a significant depth, which are highly saturated and have a small shear modulus. A hydrodynamic pressure wave impinging on such materials will partially reflect back into the water and partially refract, primarily as a dilatational wave, into the layers of reservoir bottom materials. Because of the considerable energy dissipation that results from hysteretic behavior and particle turbulence in the layer of saturated materials, the refracted wave is essentially dissipated before reaching the underlying foundation rock. The dissipation of hydrodynamic pressure waves in the reservoir bottom materials is modeled approximately by a boundary condition at the reservoir bottom that partially absorbs incident hydrodynamic pressure waves.

The earthquake excitation for the dam-water system is defined by the horizontal component of free-field ground acceleration in a cross-sectional plane of the dam. The free-field ground acceleration is assumed to be identical at all points on the base of the dam.

c) Finite element method

General: The finite element method has been fully described in many publications and not described herein [16]. Only the essential features of the method will be outlined, with particular reference to the dynamic response analysis.

The basic concept of the finite element procedure is the idealization of an actual elastic continuum as an assemblage of discrete elements interconnected at their nodal points. For the analysis of two-dimensional stress fields it has been found convenient to use quadrilateral plate elements in the idealization. To maintain compatibility between the edges of adjacent elements, it is assumed that the deformations within each element vary linearly in the x- and y- directions. On the basis of this assumption, it is possible to calculate the stiffness properties of the elements, i.e., the nodal force-deflection relationships. Finally, the stiffness of the complete structural assemblage is obtained by merely superposing the appropriate stiffness coefficients of the individual elements connecting to each nodal point.

d) Equation of motion for the reservoir

I. Assumptions and simplifications: The following conventional assumptions are made regarding the water in the reservoir;

1. The water is homogeneous, inviscid, and linearly compressible.
2. The motion of water is irrotational.
3. The water displacement and their spatial derivatives are small.
4. The effects of waves at free surface are ignored.

The geometry of the reservoir is simplified by the following reasonable approximations:

1. The upstream face of the dam is vertical.
2. The reservoir sides are vertical, and extend to infinity in a direction normal to the dam.

3. The reservoir is of constant depth.

II. Governing fluid equations: Assuming that water is inviscid and compressible and the motion of the water is limited to small amplitudes, the motion of water is governed by

$$\frac{\partial^2 u}{\partial t^2} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{\partial^2 v}{\partial t^2} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{1}$$

Here, u and v are the x and y components of displacement of the water particle, ρ is the density of the water and the hydrodynamic pressure is denoted by p

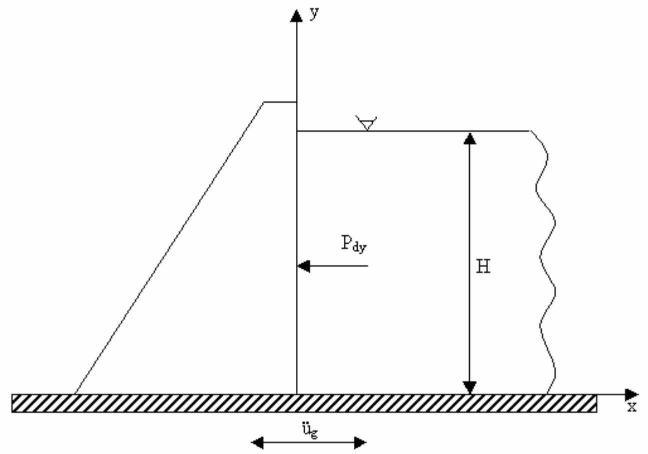


Fig. 1. Geometry of dam-reservoir system

$$p = -k\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{2}$$

Where k is the bulk modulus of water
Employing a velocity potential $\phi(x, y, t)$ defined as:

$$\frac{\partial \phi}{\partial x} = -\frac{\partial u}{\partial t} \qquad \frac{\partial \phi}{\partial y} = -\frac{\partial v}{\partial t} \tag{3}$$

The Eq. (3) yield

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} \tag{4}$$

and

$$p = \rho\frac{\partial \phi}{\partial t} \tag{5}$$

$$c = \sqrt{\frac{k}{\rho}} \tag{6}$$

Here, c denotes the velocity of sound in water given by [17]
The boundary conditions are formulated as follows noting that:

(i) On the free surface of the reservoir the hydrodynamic water pressure vanishes assuming that the effect of surface waves is neglected.
(ii) The velocity component of water normal to the reservoir base vanishes at the reservoir base.
(iii) At the reservoir dam interface, the velocity components of the water and dam normal to the

interface are the same.

(iv) At infinity the velocity potential vanishes.

These conditions provide the following four equations:

$$\frac{\partial \phi}{\partial t}(x, H, t) = 0 \tag{7}$$

$$\frac{\partial \phi}{\partial t}(x, 0, t) = 0 \tag{8}$$

$$\frac{\partial \phi}{\partial t}(x, y, t) = -\frac{\partial}{\partial t}\left[u_g(t) + w(y,t)\right] \tag{9}$$

$$\phi(\infty, y, t) = 0 \tag{10}$$

Here $u_g(t)$ denotes ground displacement parallel to x-axis. The deflections of the dam-reservoir interface is denoted by $w(y,t)$. Initially, we shall take u_g=0.

Assuming that reservoir and dam are initially at rest, initial conditions become:

$$\phi(x, y, 0) = 0 \qquad \frac{\partial \phi}{\partial t}(x, y, 0) = 0 \qquad w(y, 0) = 0 \tag{11}$$

The coupling between the motion of the water and the motion of the dam appears in the boundary condition (9).

By Solving Eq. (4) with boundary conditions (7 ~ 10) and initial conditions (11), we shall have the pressure on the dam interface.

Employing finite element discretization, the equation of motion for the dam can be written as:

$$[M]\ddot{q} + [C]\dot{q} + [K]q = F - \ddot{u}_g [M]a \tag{12}$$

in which **q** denotes the displacements of various nodes, [K] and [M] are the assembled stiffness and mass matrices and [C] is the damping matrix. The dots denote differentiation with respect to time. Vector **a** contains one and zero elements so that $\ddot{\mathbf{u}}_{\mathbf{g}}$[M]**a** represents the effective loading resulting from horizontal ground acceleration $\ddot{\mathbf{u}}_{\mathbf{g}}$(t); vector **F** denotes the external forces applied to nodes. Since the only loaded nodes are the interface nodes, the elements of **F** corresponding to the forces applied at all nodes except interface nodes are zeros.

4. NUMERICAL RESULTS

a) Kowsar Dam configuration

A concrete gravity dam (Fig. 2) with vertical upstream and inclined downstream (1:0.8) faces has the following parameters:

- height 144.0m
- crest length 190.0 m
- crest width 7.0m

In the central part of the dam, a 75 m wide chute spillway is provided. The down-stream face of the dam (both non-spillway and spillway parts) is stepped to simplify the process of dam construction and to improve hydraulic conditions of flood discharge. Step height is a multiple of placed concrete layer thicknesses. At the dam crest (only within its non-spillway parts) from the upstream face, a 1.6 m high concrete parapet is erected to prevent water overflow during PMF discharge.

In the dam body, at elevations of 607.0, 580.0 and 547.0, longitudinal inspection galleries are located for use in the inspection and drainage of the dams. Beneath, for this purpose, transverse galleries at elevations of 527.5 and 510.5 m are used.

The dam body is cut by contraction intersection joints with 40 m spacing, except right-bank and left-bank sections, the length of which is 18 and 12 m, respectively. Each of the 5 dam sections (except the left-bank section) has joint-cuts 2.5 m deep, confined by round stress deconcentrators. Spacing between joint-cuts is 10m. Intersection joints and joint-cuts are over-lapped from the upstream face by two rows of antiseepage waterstops and from the downstream side by one row.

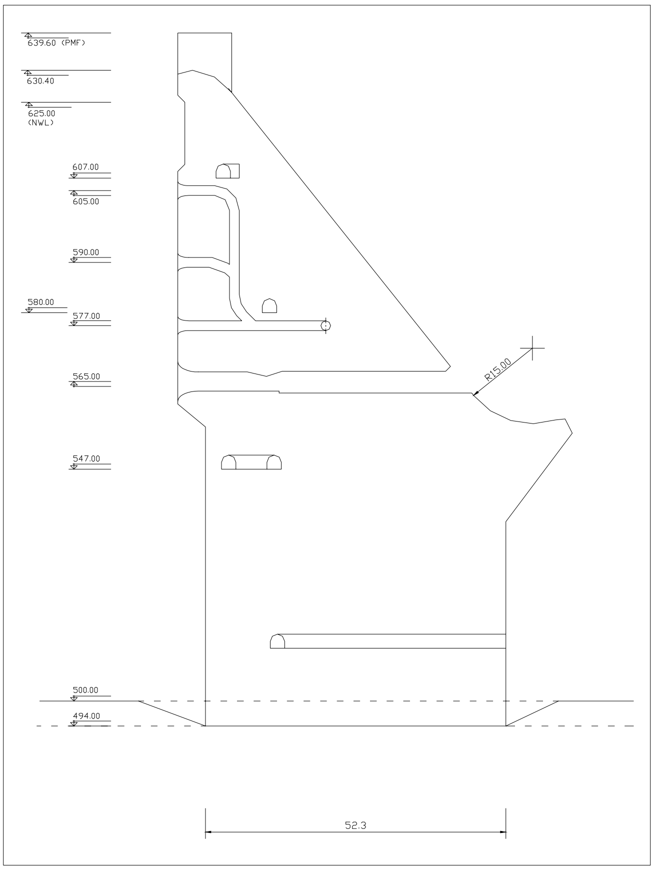


Fig. 2. Kowsar Dam cross section

b) Calibration and verification

I. General: The computer code "SAGAD" in FORTRAN has been written to solve the Eq. (13). In order to verify the mentioned method, we first analyze the Pine Flat Dam, which has been investigated by different methods before [11]. Later we compare the results with the available data. After verification we shall analyze the Kowsar Dam and investigate the stresses and deflections in the dam body.

II. Pine flat dam parameters and configuration: The pine flat concrete gravity dam is constructed of thirty-six monoliths and has a total crest length of 552 m. The tallest, non-overflow monolith is 120 m (400 ft) high, and is selected for analysis. The two-dimensional finite element idealization for this monolith, shown in Fig. 3, consists of 136 quadrilateral elements with 162 nodal points. With rigid foundation considered, the finite element idealization has 306 degrees of freedom. The mass concrete in the dam is assumed to be a homogeneous, isotropic, linear elastic solid with the following properties based, in part, on forced vibration tests of the dam: Young's modulus of elasticity E_s = 27600 MPa, unit weight = 24.4kN/m^2 , and Poisson's ratio = 0.2. Energy dissipation in the dam is represented by a constant hysteretic damping factor of η_s= 0.10. This value corresponds to a viscous damping ratio of 5% in all natural vibration modes of the dam (without impounded water) on rigid foundation rock, which is higher than the 2 to 3.5% determined from forced vibration tests because of the much larger motions and stress levels expected during strong earthquake tremors.

The water in the reservoir impounded by the dam is idealized by a fluid domain that extends to infinity in the upstream direction and has a constant depth of 114.3 m, with the water level at an elevation of 285. This water level is considered as full reservoir condition. The water is assumed to be compressible and have the following properties: velocity of pressure waves C = 1416 m/sec, and unit weight = 9.086 kN/m^3.

The bottom of a reservoir upstream of a dam may consist of highly variable layers of exposed bedrock, alluvium, silt and other sedimentary materials. The value of the wave reflection coefficient α that characterizes the reservoir bottom materials should be selected based on their actual properties, not on properties of the foundation rock. Because there are no available data on the properties of' the reservoir bottom materials upstream of Pine Flat Dam, a wave reflection coefficient α= 0.5 is arbitrarily selected for this example analysis.

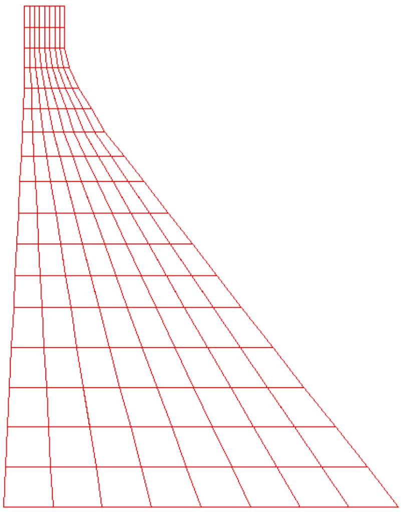


Fig. 3. Pine Flat Dam finite element mesh layout

III. Numerical results for Pine Flat Dam: An analysis of the dam monolith due to the weight of the dam, the static pressure of the impounded water and the simultaneous action of the horizontal components of the ground motion of the Taft Lincoln School Tunnel during the 1952 Kern County earthquake was carried out using the computer program SAGAD. The Frequency response of the first three modal shapes are presented in the Table 1. As it can be seen, the results are of satisfactory accuracy.

Table 1. Frequency response of the first three mode shapes

	Empty reservoir		**Full reservoir**	
	SAGAD	Others(11)	SAGAD	Others(11)
Mode shape 1	3.145	3.154	2.563	2.591
Mode shape 2	6.525	-	6.274	-
Mode shape 3	8.729	-	8.740	-

c) Earthquake response analysis of Kowsar dam

The Kowsar concrete gravity dam has a total crest length of 190 m. The tallest, non-overflow monolith is 144 m high, and is selected for analysis. The two-dimensional finite element idealization for this monolith, shown in Fig. 4, consists of 78 quadrilateral elements with 98 nodal points. With the foundation considered rigid, the finite element idealization has 184 degrees of freedom. The mass concrete in the dam is assumed to be a homogeneous, isotropic, linear elastic solid with the following properties based, in part, on forced vibration tests of the dam: Young's modulus of elasticity E_s = 28000MPa, unit weight = 25 kN/m^3, and Poisson's ratio =0.2. Energy dissipation in the dam is represented by a constant hysteretic damping factor of η_s=0.10. This value corresponds to a viscous damping ratio of 5% in all natural vibration modes of the dam (without impounded water) on rigid foundation rock, which is higher than the 2 to 3.5% determined from forced vibration tests because of the much larger motions and stress levels expected during strong earthquake tremors.

The water in the reservoir impounded by the dam is idealized by a fluid domain that extends to infinity in the upstream direction and has a constant depth of 131 m with the water level at an elevation of 625m. This water level is considered a full reservoir condition. The water is assumed to be compressible and have the following properties: velocity of pressure waves C = 1416 m/sec, and unit weight = 9.086 kN/m^3.

The bottom of a reservoir upstream of a dam may consist of highly variable layers of exposed bedrock, alluvium, silt and other sedimentary material. The value of the wave reflection coefficient α that characterizes the reservoir bottom materials should be selected based on their actual properties, not on properties of the foundation rock. Because there are no available data on the properties of' the reservoir bottom materials upstream of the Kowsar Dam, a wave reflection coefficient α= 0.5 is arbitrarily selected for this analysis.

The dam and foundation rock are assumed to be in a state of generalized plane stress. This assumption, though not strictly appropriate for the foundation rock, is dictated by the expected behavior of the non-keyed joints between the dam monoliths [15].

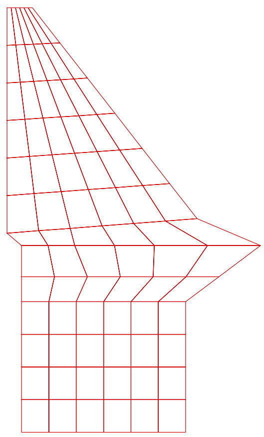
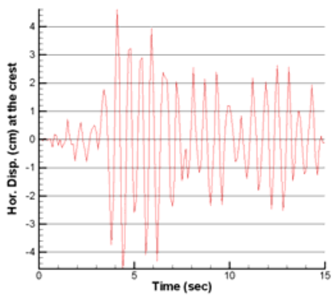
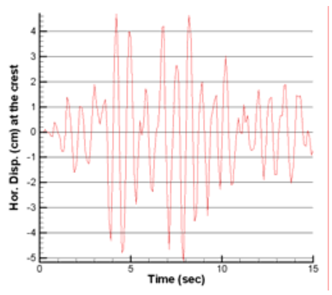


Fig. 4. Kowsar Dam finite element mesh layout

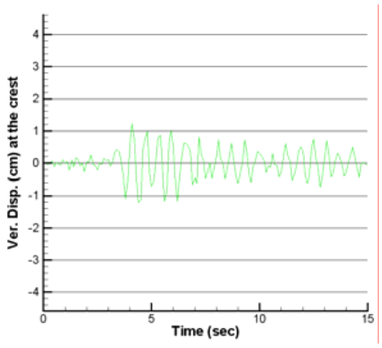


a) Full Reservoir

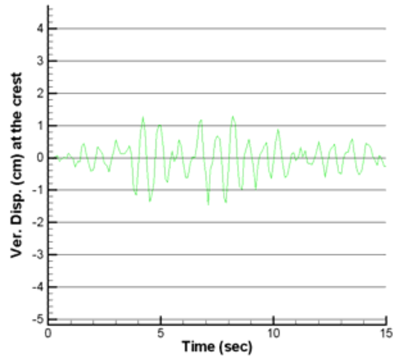


b) Empty Reservoir

Fig. 5. Horizontal displacement

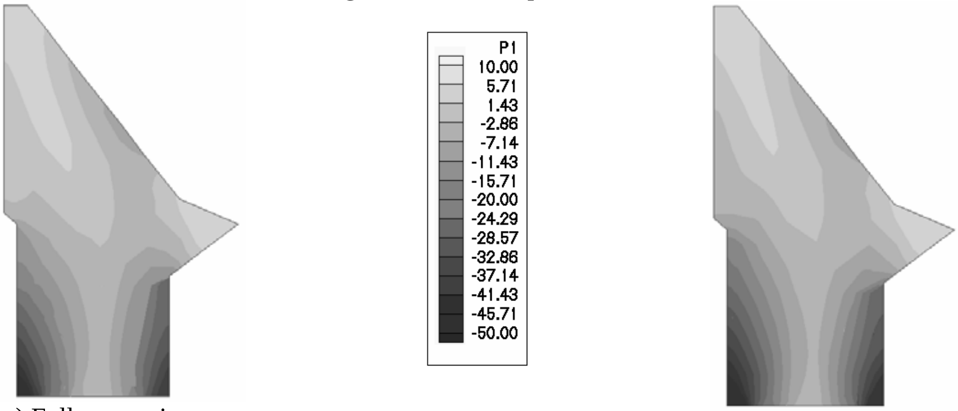


a) Full Reservoir



b) Empty Reservoir

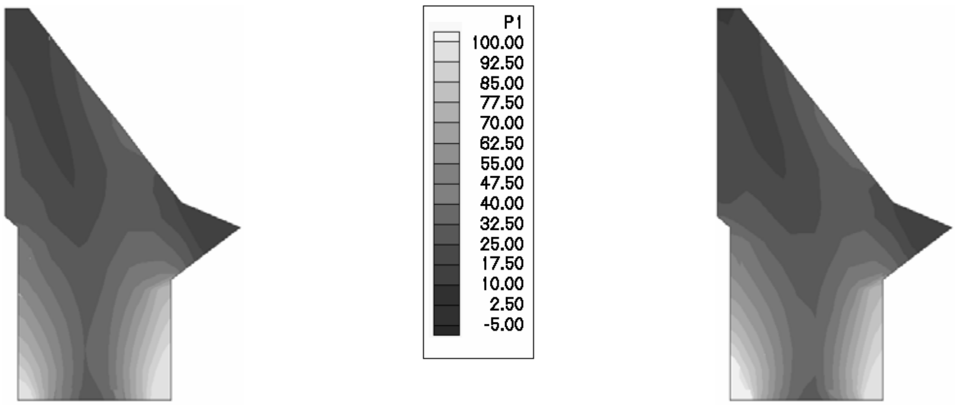
Fig. 6 . Vertical displacement



a) Full reservoir

b) Empty reservoir

Fig. 7. Minimum principal stresses (kg/cm^2)



a) Full reservoir

b) Empty reservoir

Fig. 8. Maximum principal stresses (kg/cm^2)

Table 2. Principal stresses in different states

Principal stresses (kg/cm^2)			
Maximum		Minimum	
Empty	Full	Empty	Full
90.7	105.5	40.3	-48.7

5. SUMMARY OF KEY RESULTS OF ANALYSIS OF KOWSAR DAM FROM HYPOTHEORETICAL EARTHQUAKE

Figures 5 and 6 show the interaction effect influence on displacements. In both figures the interaction effect results in larger displacements in the entire earthquake period, however the maximum displacement remains unchanged. The same phenomenon is seen for the vertical displacement (Fig. 6). Interaction effect influence on principal stresses is shown in Figs. 7 through 8. As it can be seen, the interaction effect leads to larger stress distribution in almost the entire dam body, especially around the base. To get a better view of the stress difference in different states, a table has been provided (Table 2). Examining this table we conclude that in the Kowsar Dam case there is a significant stress differences (about 15%) between two states of empty and full reservoirs, so it's not wise to ignore the interaction effect which is common in simplified analysis procedures. Again we should conclude that for an accurate dam analysis, it is necessary to include the interaction effect in the dynamic analysis, especially for dams which are build on possible earthquake sites.

Although it is predictable to observe the same phenomenon in other similar dams, the results obtained for the Kowsar Dam is not expandable for other such dams. A similar procedure should be applied to other desired dams.

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