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"Research Note"

NUMERICAL SOLUTION AND HYDRODYNAMICS OF GROUT PROPAGATION IN POROUS MEDIA*

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Abstract– A new approach to *modeling injecting domain* has been developed based on changing permeability coefficient in a certain form in theory of consolidation. The proposed model utilizes the variation of the coefficient of permeability with respect to the pore fluid pressure justifying flow characteristic below certain hydraulic gradients and void volume changes. A proposed numerical solution for heterogeneity of media with regards to the permeability coefficient also leads the results to a better grout extension solution. The potential of the proposed model is evaluated in predicting the propagation of grouting material due to a single injection bore hole. The general comparison indicates that this approach is capable of solving the injection boundary value problem.

Keywords– Grout propagation, non-Darcian flow, three level scheme, hydrodynamic permeability

1. INTRODUCTION

The basic phenomenon of grout propagation is described as a continuous fluid-displacement in deformable saturated porous media. The solution of mathematical formulation results in a highly coupled and non-linear system requiring specific numerical techniques. The pressure–displacement formulation is discretized in porous media by application of the weighted residual method. The whole system is then integrated in time by means of a simple three level scheme for non-linear variation of parameters. The mechanism of grout propagation in porous media as a coupled problem is deplored by the lack of design technology. Among the various basic hypotheses necessary to derive a macroscopic model of flow in porous media, the most relevant one may be adopted as a diffusion of grout in the fluid phase.

Modeling coupled solid deformation and fluid flow involving two or more fluid phases has been addressed by many researchers in petroleum or environmental engineering [1-4]. The Finite Element Method is widely applied to obtain the solution of multi-phase flow in a deformable porous medium. To obtain the fully coupled partial differential equations, the fluid should obey Darcy's law at each time increment.

2. HYDRODYNAMIC VARIATION OF PERMEABILITY

Under transient conditions, the effective stress condition may be different in the fluid flow path and the surrounding porous media. If the matrix permeability is different than the flow path permeability, fluid pressure will be different under transient flow conditions. Therefore, if the matrix permeability is less than the flow path permeability, there will be a larger effective stress at the sides of the flow path than the matrix when fluid is withdrawn. A difference in effective stresses will also arise when the pore

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compressibility of the domain is greater than the void compressibility of the flow path. To solve such a complex condition, a certain form of hydraulic conductivity change may be justified in a grout conduction solution procedure.

The parameters that characterize only the conductive property of the medium are known as the intrinsic permeability. The hydraulic conductivities of a geological formation usually show variation through media. At a given point, the hydraulic conductivity depends on the direction of measurement.

A geological formation is inherently heterogeneous if the hydraulic conductivity depends on the position in medium. The hydraulic conductivity depends on the coordinates *x, y, z* or $K(x, y, z) = f(x, y, z)$. However, any anisotropy in deformation can make the homogeneous formation to a heterogeneous medium. Homogeneous geological formations are rather exceptional, although the concept of homogeneous formation is frequently used in theoretical considerations because of simplicity.

For the case of anisotropy not coinciding the principal direction with the *x, y,* and *z* coordinate axes, the generalized form of Darcy's law is employed as follows:

$$V = K\left[\frac{\partial h}{\partial X_i}\right] \tag{1}$$

Clearly, in the most general case the hydraulic conductivity has nine components. These components placed in a matrix form give the hydraulic conductivity tensor. Disregarding the rotation effects in the imposed large deformations leads to a general form of a second order symmetric tensor that is assumed to have the property $K_{xy} = K_{yx}$, $K_{yz} = K_{zy}$, and $K_{zx} = K_{xz}$.

Since the hydraulic head h is a continuous function of x, *y,* and *z,* the hydraulic gradient in an inclined direction of $l = \cos\theta_1$, $m = \cos\theta_2$, and $n = \cos\theta_3$ is written as follows:

$$\frac{\partial h}{\partial S} = \frac{\partial h}{\partial x}\cdot\frac{\partial x}{\partial S} + \frac{\partial h}{\partial y}\cdot\frac{\partial y}{\partial S} + \frac{\partial h}{\partial z}\cdot\frac{\partial z}{\partial S} \tag{2}$$

Geometrically, $\partial x/\partial S = \cos\theta_1$, $\partial y/\partial S = \cos\theta_2$ and $\partial z/\partial S = \cos\theta_3$. These equations yield the following relation to find conductivity on an inclined orientation θ_i.

$$\frac{1}{(K_{\theta i})^2} = \frac{\cos^2\theta_1}{(K_x)^2} + \frac{\cos^2\theta_2}{(K_y)^2} + \frac{\cos^2\theta_3}{(K_z)^2} \tag{3}$$

This means that knowing K_x, K_y, and K_z through three simple permeability tests along major axes, the hydraulic conductivity ellipsoid is known on coordinate axes K_x, K_y, and K_z. To assess the potential of the current hypotheses, the variation of conductivity coefficient along any radius is calculated as follows:

$$\frac{K_x}{K_{x0}} = (e)^{[aLn(u/u_o - \varepsilon_x)+b]},\quad \frac{K_y}{K_{y0}} = (e)^{[aLn(u/u_o - \varepsilon_y)+b]},\quad \frac{K_z}{K_{z0}} = (e)^{[aLn(u/u_o - \varepsilon_z)+b]} \tag{4}$$

K_{x0}, K_{y0} and K_{y0} are the measured conductivity coefficients through water pressure tests along *x, y* and *z* directions, respectively, u_o and u are initial and current pore pressure, *a* and *b* are material constants, *b* is always positive and dilative strains have a negative sign.

To determine the range of values *a* and *b* with the provision of the known initial conditions, all three permeability values are equal to their initial values at the start of the operation, while axial strains are equal to zero. Therefore, the right hand side of Eq. (5) is equal to one, and this leads us to conclude that *a* and *b* are inter-dependent. It is found that *b*, for many soils, is equal to 0.5, however, it may be found through the time-displacement curve of a normal odometer test, comparing experimental and Terzaghi solutions [5].

3. GOVERNING EQUATIONS

The mass balance equation for the solid phase is written as follows:

$$\frac{\partial(1-n)\rho_s}{\partial t} + \nabla \cdot \left[(1-n)\rho_s V_s\right] = 0 \tag{5}$$

For the fluid phase, the equation of conservation of mass is:

$$\frac{\partial n\rho_f}{\partial t} + \nabla \cdot \left[n\rho_f V_f\right] = 0 \tag{6}$$

n is porosity, ρ_s , ρ_f , V_s, and V_f are grain density, fluid phase density, and grain and fluid velocities, respectively. ∇ is the divergence operator. The general momentum equilibrium equation is:

$$\nabla \cdot \sigma_{ij} + \rho g = 0 \tag{7}$$

Substituting Terzaghi's principle of effective stress in the momentum equilibrium, it results in:

$$\nabla \cdot (\sigma'_{ij} - u\delta_{ij}) + \rho g = 0 \tag{8}$$

A mixture density ρ is defined as follows:

$$\rho=(1-n)\,\rho_s+n\,\rho_f \tag{9}$$

In the solid phase continuity equation, matrix porosity variations are expressed in terms of skeleton volumetric deformation and solid density variation to yield:

$$\rho_s \frac{d_s n}{dt} = (1-n)\rho_s \nabla \cdot V_s + (1-n)\frac{d_s \rho_s}{dt} \tag{10}$$

$\frac{d_s(\ )}{dt} = \frac{\partial}{\partial t}(\ ) + \nabla(\ ) \cdot V_s$ is the derivative with respect to the solid phase. Assuming that the solid grains are incompressible yields ($\rho_s = const.$):

$$\frac{d_s n}{dt} = (1-n)\rho_s \nabla \cdot V_s \tag{11}$$

After some manipulations, the continuity of fluid leads to the following equation:

$$\rho_f \nabla \cdot V_s + n\frac{d_s n}{dt} + \nabla \cdot \left[n\rho_f (V_f - V_s)\right] = 0 \tag{12}$$

In this research, the solid is assumed to be linear elastic, and the relative fluid-solid velocity is governed by a Darcy's law for a fluid structured porous medium.

$$n(V_f - V_s) = -\frac{K}{\mu_f}(\nabla u - \rho_f g) \tag{13}$$

In the general immiscible phase, $K=K(n,u)$, is the geometric intrinsic permeability assumed to be function of pore pressure and porosity. $\mu_f = \mu_f(n,u,t)$ is the dynamic viscosity and is a function of porosity, pore pressure, and time. Note that in the following simulation (i.e., miscible phase), μ_f is assumed to be constant. In general, $\rho_f = \rho_f (u)$ is assumed as the effect of fluid compressibility, however the fluid is assumed to be incompressible.

The generalized continuity conditions lead to the following general equation of three dimensional consolidations:

$$(1+e)\left[\frac{\partial}{\partial x}(Kx\frac{\partial u}{\gamma_w \partial x})+\frac{\partial}{\partial y}(Ky\frac{\partial u}{\gamma_w \partial y})+\frac{\partial}{\partial z}(Kz\frac{\partial u}{\gamma_w \partial z})\right]=\frac{\partial e}{\partial t}+\beta\frac{\partial u}{\partial t} \tag{14}$$

For saturated soil the coefficient β is zero. Neglecting the effects of *Kxy, Kyz* and *Kzx*, and also in isotropic conditions *Kx*, K_y, and *Kz* are equal, the use of Eq. (20) leads to the following equation:

$$\frac{\partial \sigma'_{av}}{\partial t}-\frac{[K](1+e)}{\gamma_w \dfrac{\partial e}{\partial \sigma'_{av}}}\left[(\frac{\partial^2 u}{\partial x^2})+(\frac{\partial^2 u}{\partial y^2})+(\frac{\partial^2 u}{\partial z^2})\right]=(1-\frac{\beta}{\dfrac{\partial e}{\partial \sigma'_{av}}})\frac{\partial e}{\partial t}$$
$$\sigma'_{av}=(\sigma'_x+\sigma'_y+\sigma'_z)/3 \tag{15}$$

A three level finite difference time marching scheme employed to solve non-linear consolidation as a coupled solution of both equilibrium and continuity equations can be stated in matrix form as:

$$\begin{bmatrix} KS & -L \\ -L^T & -\dfrac{2\Delta t}{3}H^T \end{bmatrix}\begin{Bmatrix} \delta^{t+\Delta t} \\ u^{t+\Delta t} \end{Bmatrix}=\begin{bmatrix} KS & -L \\ -L^T & +\dfrac{2\Delta t}{3}H^T \end{bmatrix}\begin{Bmatrix} \delta^{t-\Delta t} \\ u^{t-\Delta t} \end{Bmatrix}+\begin{bmatrix} 0 & 0 \\ 0 & +\dfrac{2\Delta t}{3}H^T \end{bmatrix}\begin{Bmatrix} \delta^{t} \\ u^{t} \end{Bmatrix}+\begin{Bmatrix} 2\Delta tf \\ -2\Delta tg \end{Bmatrix} \tag{16}$$

The solutions of these equations represent the values of nodal pore water pressures and deformations. This three level system is a typical central difference form referred to [5].

$$[H]=\int_V\left[\frac{K_x \partial N_r \partial N_i}{\gamma_w \partial x \partial x}+\frac{K_y \partial N_r \partial N_i}{\gamma_w \partial y \partial y}+\frac{K_z \partial N_r \partial N_i}{\gamma_w \partial z \partial z}\right]dv\ ,\ [L]=\int_V N_r\left[\frac{\partial N_i}{\partial x}+\frac{\partial N_i}{\partial y}+\frac{\partial N_i}{\partial z}\right]dv \tag{17}$$

$$[KS]=\int_v [B]^T\left[D^e\right][B]dv\ ,\ \{f\}=\int_v [N]^T\{f_b\}dv+\int_A [N]^T\{f_s\}dA+\int_v [B]^T\left[D^e\right]\{\varepsilon_\circ\}dv\ ,$$
$$\left\{\overset{\circ}{f}\right\}=\int_v [N]^T\left\{\overset{\circ}{f_b}\right\}dv+\int_A [N]^T\left\{\overset{\circ}{f_s}\right\}dA+\int_v [B]^T\left[D^e\right]\{\frac{\varepsilon_\circ}{\partial t}\}dv \tag{18}$$

The complete formulation including displacement is implemented in a fully coupled way, because each of the equations appears to be strongly coupled to the others and a three level scheme resolution procedure leads to divergence of the algorithm solution procedure.

$$[R]^{(t)}\{\phi\}^{(t+\Delta t)}=[R]^{(t)}\{\phi\}^{(t-\Delta t)}+[S]^{(t)}\{\phi\}^{(t)}+[T]^{(t)} \tag{19}$$

4. GROUT PRESSURE DEVELOPMENT

The pressure development profile reveals that the dilution process (growing transition zone) on grouting during the progress of the experiment is typical of a miscible constituent. The above model is applied to a two dimension, axe-symmetric case study of injection of a bore-hole in order to test the program under a simple geometrical configuration and to compare it with numerical results. In the obtained results, only information about the pressure distribution profile is presented.

The result of a bore hole BH-1 carried out at the site of Abshineh Dam in Hamedan city is employed for the solution of the boundary value problem. The geotechnical properties of the considered layers are given in Table 1. Figure 1 shows general boundaries and pressure growth. A plane finite element mesh regarding the adopted axe-symmetric condition of the bore hole and the surrounding affected space is shown in Fig. 2. Three different pressures (6, 4 and 2 bar) applied into the bore-hole and the pressure distribution versus distance to the center of the bore hole at different time levels are shown in Figure 3-a to 3-c. Three soil layers with different Lugeon test results as 3, 6 and 30 Lugeon are injected upon the same pressure of 6 bars. The different grout extension lengths obtained are plotted versus Lugeon ratios in Fig. 4-a, b. Accordingly, the higher Lugeon test ratio leads to the more extended grout through medium.

Three layers of soil strata having the same Lugeon ratios as above, respectively, were injected upon the same conditions and 6 bar grout pressure. Later, the effective grouted area was detected through a set of bore holes. The approximate extended distances of grout in three layers are shown in Fig. 5.

Table 1. Geotechnical parameters

Young's Modulus in saturated condition	9 GPa.
Poisson ratio	0.3
Porosity	30%
Internal Friction Angle	35^{o}
Cohesion	150 kPa.
Dry density	2.59 gr/cm^3
Fluid density	1.0 gr/cm^3
Fluid viscosity	$1.1*10^{-3}$ Pa.sec
Permeability coefficients	4.0E-5, 8.0E-5, and 40.0E-5 cm/sec.

5. CONCLUSION

A simulation of an injecting experiment by a coupled flow and deformation model has been presented. The model utilized an elastic/elastic-plastic constitutive relationship for the behavior of soil. The transfer processes represented included the flow of grout simulated as pressure development associated with deformations of medium.

A linear elastic law is employed for the deformation of skeleton and incompressible fluid. Upon exponential descending/ascending function of a permeability coefficient that is led to a diffusion of flow through media, the flow motion potential is not enough to force the grout to progress more. Subsequent to the above achievement, the validity of the proposed model was brought under scrutiny in a collection of experimental results with the predictions of the model. The capabilities of the model have also been demonstrated.

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