STIFFENED MULTI-BAY COUPLED SHEAR WALLS ON ELASTIC FOUNDATION[*]

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Abstract – In this study, the static analysis of a multi-bay coupled shear wall on an elastic foundation, having any number of stiffening beams is studied using the continuous connection method (CCM). The shear wall is considered to be made up of a finite number of sections in the vertical direction, with or without stiffening beams between each pair of consecutive sections. This method of analysis, which accounts for thickness variation and elastic beam-wall connections, treats the discrete connecting beams as a continuous layered medium. To implement the foregoing analysis a computer program has been developed in the MATHEMATICA computer algebra system. To support the validity of the present method comparisons have been carried out with the results of SAP2000 structural analysis program and those in some references.

Keywords – Continuous connection method, multi-bay coupled shear wall, stiffening beam, flexible connection

1. INTRODUCTION

In tall buildings, lateral loads due to winds and earthquakes are generally resisted by shear walls. A solid shear wall is easily designed, since it can be treated roughly as a cantilever beam and more precisely employing a fine finite element mesh. However, the design of shear walls weakened by doors, windows and corridors (Fig. 1), necessitates particular attention since they are highly indeterminate.
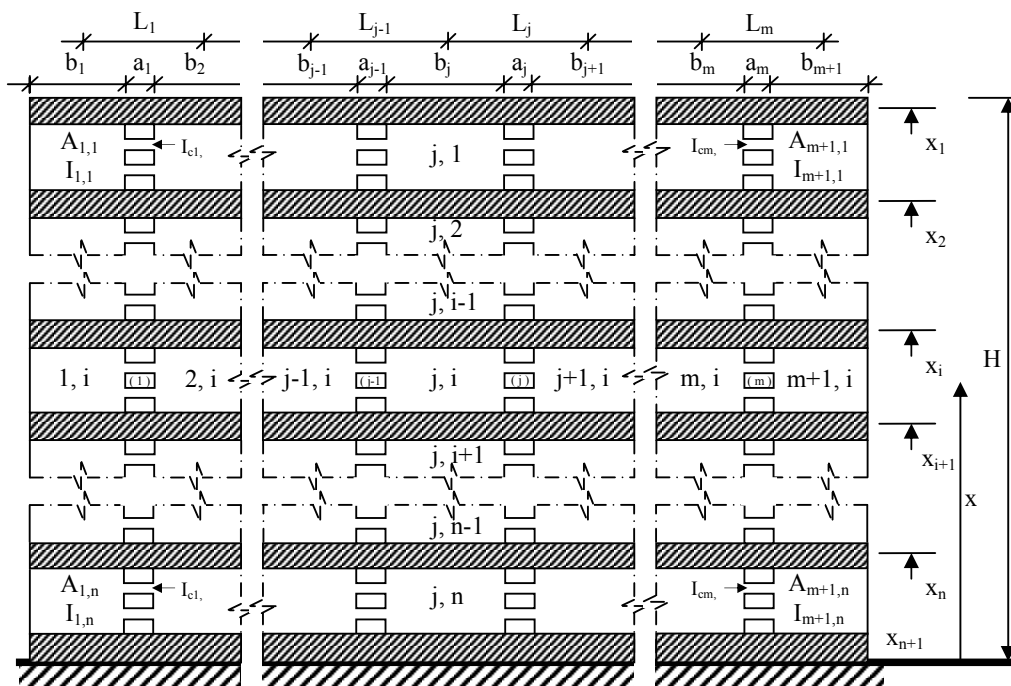

Fig. 1. A multi-bay shear wall with stiffening beams

Continuous Connection Method (CCM) is one of the most widely used methods for the analysis of perforated shear walls. In the analysis of perforated shear walls, an increase in the number of rows of openings causes an increase in the number of unknowns and the consequent tedious computation renders a closed form solution impossible. Hence, the number of rows of openings has been restricted to one, two and three in the pertinent literature, the latter case being applicable to symmetrical problems [1-3], only. There are few papers in the literature on the analysis of multi-bay shear walls. Koo and Cheung [4] have expressed the axial forces in the piers of a multi-bay shear wall in terms of some coordinate functions. However, they have found a high percentage of errors in their results, especially in low stress values compared to those of the finite strip method. Coull *et al*. [5] have used a matrix progression method for multi-bay high-rise buildings having many discontinuities in the vertical direction and resting on a rigid foundation. Soane [6] has analyzed high multi-bay shear walls in complex buildings using analog computers. Although his assertion about the approximate solution of three dimensional buildings may be correct, he has no examples for planar multi-bay coupled shear walls. Hence, there is no proof of the validity of his results for such cases. The results of Elkholy and Robinson [7] which used the finite difference method to solve multi-bay unstiffened coupled shear wall problems, has been used for checking purposes in the present study. If a coupled shear wall is very high, the thickness of the wall can be varied along the height [8] and this necessitates no new feature compared to a similar analysis with different properties in different sections along the height, such as the present one. Coull [9] obtained closed form solutions for the problem of a nonsymmetric coupled shear wall with one row of openings, which has a stiffening beam at the top and a flexible foundation at the bottom. Chan and Kuang [10, 11] used the CCM to study a coupled shear wall with one row of openings, a stiffening beam at any height and a rigid or flexible foundation. Coull and Bensmail [12] studied the coupled shear wall on both a rigid or flexible foundation with two stiffening beams at arbitrary heights using the CCM. Then, research works on the static [13-15] and dynamic [16-19] analyses of coupled shear walls with one row of openings came one after another, not to mention a host of others. However, since most buildings have more than one row of openings, it is worthwhile to work on the case of multi-bay shear walls, which were not studied sufficiently before.

In the present study, the static analysis of a multi-bay coupled shear wall on an elastic foundation with a finite number of stiffening beams along the height is carried out using the CCM. The widths of piers and openings are kept constant while the thickness of the wall, the heights of the storeys and the geometric and physical properties of the piers can be varied from one section to another along the height. The employment of the CCM makes the replacement of the discrete connecting beams by a laminated medium with equivalent stiffness possible. This procedure, in turn, renders possible the formulation of the problem as a one dimensional one instead of two. The governing equations of this approach are the compatibility equations for the relative vertical displacements at the mid-points of the connecting and the stiffening beams. The system of linear second order coupled differential equations, thus obtained, is transformed into an eigenvalue problem. Then, the latter system of equations is solved by a matrix diagonalization procedure which renders the system uncoupled [20]. To verify the present method, a computer program has been prepared in the MATHEMATICA computer algebra system to find results for comparison with the results given in the previous literature. Although some results in the previous literature did not match with those of the present work, the latter were in good agreement with those of the SAP2000 structural analysis program [21] in each and every case considered.

2. ANALYSIS

The primary assumption in the CCM is that the lengths of the connecting beams do not change, i.e. their axial stiffness is infinitely large. This assumption is equivalent to the widely used rigid diaphragm model for storey floors, which is known to yield rather good results. This assumption renders the lateral displacements at the same floor level equal for all piers. Consequently, the slope and curvature at the same level can be assumed to be equal, as well. Furthermore, in this method, it being assumed that every span between two neighboring piers is constant throughout the total height of the wall, and the real connecting beams with

bending stiffness $EI_{cj,i}$ are replaced by a laminated medium with $EI_{cj,i}/h_i$ per unit length in the vertical direction (Fig. 2). In the foregoing expression, E, $I_{cj,i}$ and h_i are, respectively, the elasticity modulus, the moment of inertia of the connecting beams in bay j of section i and the storey height in section i. Likewise, the discrete shear forces in the connecting beams are replaced by a continuous shear force function $q_{j,i}$, per unit length of height along the mid-points of the connecting laminae (i.e. the points of zero moment).
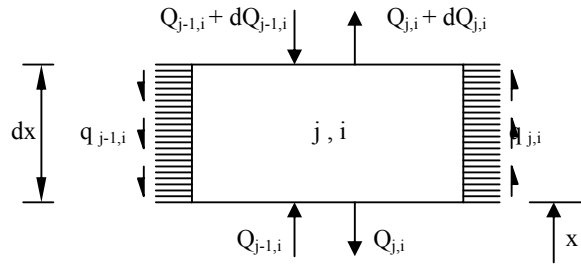

Fig. 2. The vertical forces on an isolated region of the shear wall

By applying the vertical force equilibrium equation to a dx length of pier j of section i, the following equations are found:

$$\frac{dQ_{j-1,i}}{dx} - \frac{dQ_{j,i}}{dx} = q_{j,i} - q_{j-1,i}, \quad j = 1,2,...,m+1, \quad i = 1,2,...,n \tag{1}$$

Here, it should be noted that

$$Q_{0,i} = Q_{m+1,i} = q_{0,i} = q_{m+1,i} = 0, \quad j = 1,2,...,m, \quad i = 1,2,...,n \tag{2}$$

Moreover, each function $Q_{j,i}$ is the integral of the negative of the shear force function $q_{j,i}$ starting with zero value at the top of the shear wall, i.e.

$$Q_{j,i} = -\int_x^H q_{j,i}\, dx, \quad j = 1,2,...,m+1, \quad i = 1,2,...,n \tag{3}$$

The moment-curvature relation for a cross-section of the shear wall at height x can be written in the following form:

$$EI_i \frac{d^2 y_i}{dx^2} = M_e - \sum_{k=1}^{m} Q_{k,i} L_{k,i}, \quad i = 1,2,...,n \tag{4}$$

Here, M_e, $L_{j,i}$ and I_i are, respectively, the moment of external forces above the cross-section of concern with respect to any point in the cross-section, the distance between the axes of the j'th and (j+1)'st piers and

$$I_i = \sum_{k=1}^{m+1} I_{k,i}, \quad i = 1,2,...,n \tag{5}$$

It will be assumed that all rows of connecting laminae will be cut through the mid-points, which are the points of zero moment, thus exposing the shear forces in them. The compatibility of the relative vertical displacements at the ends, on the two sides of the cut sections necessitates their sums to be equal to zero for each lamina, i.e.

$$L_{j,i} \frac{dy_i}{dx} - \frac{h_i a_j^2}{2 C_{cbi}} q_{j,i} - \frac{h_i a_j^3}{12 EI_{cj,i}} q_{j,i} - \frac{1}{E} \sum_{k=i+1}^{n} \left[\left\{ \frac{1}{A_{j,k}} \right\} \int_{x_{k+1}}^{x_k} (Q_{j,k} - Q_{j-1,k})dx + \left\{ \frac{1}{A_{j+1,k}} \right\} \right.$$

$$\left. \int_{x_{k+1}}^{x_k} (Q_{j,k} - Q_{j+1,k})dx \right] - \frac{1}{E} \left[\left\{ \frac{1}{A_{j,i}} \right\} \int_{x_{i+1}}^{x} (Q_{j,i} - Q_{j-1,i})dx + \left\{ \frac{1}{A_{j+1,i}} \right\} \int_{x_{i+1}}^{x} (Q_{j,i} - Q_{j+1,i})dx \right] - \delta_{0j} = 0 \tag{6}$$

$$j = 1,2,...,m, \quad i = 1,2,...,n$$

This compatibility equation can be written for all spans of the shear wall, having in mind the previously defined values in Eq. (2). In Eq. (6), a_j, C_{cbi}, $I_{cj,i}$, $A_{j,i}$ and δ_{0j} are, respectively, open length of span j,

beam-wall connection stiffness in section i, the moment of inertia of the connecting beams in span j of section i, the cross-sectional area of the j'th pier in section i and relative vertical displacement of the bottom of pier j with respect to that of pier j+1. The terms of the compatibility Eq. (6) are the relative vertical displacements of the two ends on the two sides of the cut due, respectively, to the bending of the piers, relative rotation of beams with respect to piers, the bending of the connecting beams due to shear forces, the axial deformations of the parts of the piers between section i and the foundation, the axial deformations of the piers in section i, and the relative vertical displacements in the foundation. Differentiating (6) with respect to x, substituting expressions (3) and (4) and simplifying the resulting equation, the following non-homogeneous second order linear differential equation with constant coefficients is obtained in matrix form

$$\left[\gamma_{j,i}^2 \, Q_{j,i}''\right]_{mx1} - \left[\alpha_{jk}^2\right]_{mxm} \left[Q_{j,i} \, L_{j,i}\right]_{mx1} = -M_e, \quad j=1,2,...,m, \quad i=1,2,...,n \tag{7}$$

where the following definitions apply:

$$\alpha_{jk}^2 = \left(1 + \frac{I_i}{L_{j,i}^2}\left(\frac{1}{A_{j,i}} + \frac{1}{A_{j+1,i}}\right)\right), \qquad (j=k)$$

$$\alpha_{jk}^2 = \left(1 - \frac{I_i}{L_{j,i} \, L_{j-1,i} \, A_{j,i}}\right), \qquad (j=k+1)$$

$$\alpha_{jk}^2 = \left(1 - \frac{I_i}{L_{j,i} \, L_{j+1,i} \, A_{j+1,i}}\right), \qquad (j=k-1)$$

$$\alpha_{jk}^2 = 1, \qquad (j < k-1)$$

$$\alpha_{jk}^2 = 1, \qquad (j > k+1)$$

$$\gamma_{j,i}^2 = \frac{E \, I_i}{L_{j,i}}\left(\frac{h_i \, a_j^2}{2 \, C_{cbi}} + \frac{h_i \, a_j^3}{12 \, E \, I_{cj,i}}\right), \quad k=1,2,...,m, \quad j=1,2,...,m, \quad i=1,2,...,n$$

(8)

The set of differential Eqs. (7) is coupled and moreover, as the number of rows of openings increases, a closed form solution is not feasible, if at all possible. In this work, the matrix orthogonalization method will be used for solving the set of differential Eqs. (7). For this purpose, using the variable transformation

$$Q_{j,i} L_{j,i} = Z_{j,i} \implies Q_{j,i} = \frac{Z_{j,i}}{L_{j,i}} \implies Q_{j,i}'' = \frac{Z_{j,i}''}{L_{j,i}}, \quad j=1,2,...,m, \quad i=1,2,...,n \tag{9}$$

the equation set (7) can be written in the following form for the new variables $Z_{j,i}$:

$$\underbrace{\begin{bmatrix} \frac{\gamma_{1,i}^2}{L_{1,i}} & 0 & \cdot & \cdot & 0 \\ 0 & \frac{\gamma_{2,i}^2}{L_{2,i}} & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{\gamma_{m,i}^2}{L_{m,i}} & \\ 0 & 0 & \cdot & \cdot & \frac{\gamma_{m,i}^2}{L_{m,i}} \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} Z_{1,i}'' \\ Z_{2,i}'' \\ \cdot \\ \cdot \\ \cdot \\ Z_{m,i}'' \end{bmatrix}}_{\underline{Z}''} + \underbrace{\begin{bmatrix} -\alpha_{11}^2 & -\alpha_{12}^2 & \cdot & \cdot & -\alpha_{1m}^2 \\ -\alpha_{21}^2 & -\alpha_{22}^2 & \cdot & \cdot & -\alpha_{2m}^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ -\alpha_{m1}^2 & -\alpha_{m2}^2 & \cdot & \cdot & -\alpha_{mm}^2 \end{bmatrix}}_{\underline{B}} \underbrace{\begin{bmatrix} Z_{1,i} \\ Z_{2,i} \\ \cdot \\ \cdot \\ \cdot \\ Z_{m,i} \end{bmatrix}}_{\underline{Z}} = \underbrace{\begin{bmatrix} -M_e \\ -M_e \\ \cdot \\ \cdot \\ \cdot \\ -M_e \end{bmatrix}}_{\underline{M_e}} \quad i=1,2,...,n \tag{10}$$

where $\underline{A}$ and $\underline{B}$ are m×m and $\underline{Z}$ and $\underline{M_e}$ are m×1 dimensional matrices. The homogeneous part of this matrix equation, which is an eigenvalue problem in the following form

$$\underline{A}\,\underline{Z}'' + \underline{B}\,\underline{Z} = \underline{0} \tag{11}$$

is solved and the eigenvectors corresponding to the eigenvalues all together yield the transformation matrix T. Since the coefficient matrices $\underline{A}$ and $\underline{B}$ are constant, Eq. (10) can be diagonalized. For this purpose, the following transformation can be used

$$\underline{Z} = \underline{T}\,\underline{Y}, \quad \underline{Z}'' = \underline{T}\,\underline{Y}'' \tag{12}$$

which, when substituted in Eq. (10), yield

$$\underline{A}\,\underline{T}\,\underline{Y}'' + \underline{B}\,\underline{T}\,\underline{Y} = \underline{M}_e \tag{13}$$

Here, $\underline{T}$ and $\underline{Y}$ are, respectively, the transformation matrix of dimension m×m and the vector of independent functions of variable x of dimension m×1. Multiplying both sides of Eq. (13) by the transpose of $\underline{T}$, the two coefficient matrices $\underline{A}$ and $\underline{B}$ are diagonalized to yield

$$\widetilde{\underline{A}}\,\underline{Y}'' + \widetilde{\underline{B}}\,\underline{Y} = \widetilde{\underline{M}}_e \tag{14}$$

which is an uncoupled system of differential equations. The set of expressions yielding the solution of Eq. (14) for all sections is of the following form:

$$Y_{j,i} = C_{j,i}\cosh\left(\sqrt{\frac{\widetilde{B}_{jj,i}}{\widetilde{A}_{jj,i}}}\,x\right) + D_{j,i}\sinh\left(\sqrt{\frac{\widetilde{B}_{jj,i}}{\widetilde{A}_{jj,i}}}\,x\right) + \frac{1}{\widetilde{B}_{jj,i}}\left[\sum_{k=0}^{\infty}\left(\frac{\widetilde{B}_{jj,i}}{\widetilde{A}_{jj,i}}\right)^{-k}\frac{d^{2k}}{dx^{2k}}\left(\widetilde{M}_e\right)\right] \tag{15}$$

$$j = 1,2,...,m, \quad i = 1,2,...,n$$

This solution is only valid for the polynomial forms of the external moment function M_e. Hence, it applies for the special cases of uniformly distributed forces, linearly distributed forces and a point force at the top in a straightforward manner. When M_e is given by other functions, the corresponding particular solutions must be found. There are 2×m×n integration constants $C_{j,i}$ and $D_{j,i}$ in Eq. (15). To determine those constants, the boundary conditions at the top, bottom and between each pair of consecutive sections are used.

Before writing down the boundary conditions, the shear forces in the stiffening beams must be determined. For this purpose, compatibility Eq. (6) must be written both for section i at level x_i and the stiffening beam i and solved simultaneously. Thus, employing the definitions

$$\eta_{j,i} = \frac{1}{H}\frac{\left(\dfrac{h_i}{C_{cbi}} + \dfrac{h_i\,a_j}{6\,E\,I_{cj,i}}\right)}{\left(\dfrac{1}{C_{sbi}} + \dfrac{a_j}{6\,E\,I_{si}}\right)}, \quad j = 1,2,...,m, \quad i = 1,2,...,n \tag{16}$$

the shear forces in the stiffening beams are found as follows:

$$V_{j,i} = -\eta_{j,i}\,H\,\frac{dQ_{j,i}}{dx}\bigg|_{x=x_i} \quad j = 1,2,...,m, \quad i = 1,2,...,n \tag{17}$$

The integration constants in the equation set (15) are found from four different types of boundary conditions:

a) The first type of equation is written for the equilibrium of vertical forces at the top of each row of openings

$$Q_{j,1}\big|_{x=H} - V_{j,1} = 0, \quad j = 1,2,...,m \tag{18}$$

As a special case, if there is no stiffening beam at the top:

$$Q_{j,1}\big|_{x=H} = 0, \quad j = 1,2,...,m \tag{19}$$

Applying the compatibility equation for the lowermost section at its lower end, taking the rotation of the foundation to be

$$\left.\frac{dy_n}{dx}\right|_{x=0} = \frac{M_e - \sum\limits_{k=1}^{m} Q_{k,n} L_{k,n}}{\sum\limits_{k=1}^{m+1} K_{rk}} \tag{20}$$

and the relative vertical displacements to be

$$\delta_{0j} = \frac{Q_{j,n} - Q_{j-1,n}}{K_{vj}} + \frac{Q_{j,n} - Q_{j+1,n}}{K_{v(j+1)}}, \quad j = 1,2,...,m \tag{21}$$

the second type of equation is written as

$$\frac{L_{j,n}}{\sum\limits_{k=1}^{m+1} K_{rk}}\left(M_e - \sum\limits_{k=1}^{m}\left(Q_{k,n}\big|_{x=0} L_{k,n}\right)\right) + \left(\frac{h_n a_j^2}{2C_{cbn}} + \frac{h_n a_j^3}{12 E I_{cj,n}}\right)\frac{Q_{j,n}}{dx}\bigg|_{x=0} - \left(\frac{1}{K_{vj}} + \frac{1}{K_{v(j+1)}}\right)$$

$$Q_{j,n}\big|_{x=0} + \frac{1}{K_{vj}} Q_{j-1,n}\big|_{x=0} + \frac{1}{K_{v(j+1)}} Q_{j+1,n}\big|_{x=0} = 0, \qquad j = 1,2,....,m \tag{22}$$

As a special case, for rigid foundation

$$\left.\frac{Q_{j,n}}{dx}\right|_{x=0} = 0, \quad j = 1,2,...,m \tag{23}$$

b) From the equilibrium of vertical forces for the stiffening beam i, comes the third type of boundary conditions as

$$Q_{j,i-1}\big|_{x=x_i} - V_{j,i} - Q_{j,i}\big|_{x=x_i} = 0, \quad j = 1,2,...,m, \ i = 2,3,...,n \tag{24}$$

As a special case, if there is no stiffening beam at $x = x_i$ height, $V_{j,i}$ has to be taken as zero.

c) The fourth type of boundary conditions are obtained by equating the slopes of two consecutive sections, i-1 and i, using the compatibility equations applied at the boundary between them as

$$\left(\frac{h_i a_j^2}{2C_{cbi}} + \frac{h_i a_j^3}{12 E I_{cj,i}}\right)\frac{dQ_{j,i}}{dx}\bigg|_{x=x_i} - \left(\frac{h_{i-1} a_j^2}{2C_{cbi-1}} + \frac{h_{i-1} a_j^3}{12 E I_{cj,i-1}}\right)\frac{dQ_{j,i-1}}{dx}\bigg|_{x=x_i} = 0 \tag{25}$$

$$j = 1,2,...,m, \ i = 2,3,...,n$$

As a special case, for two consecutive sections with equal storey heights and moments of inertia of connecting beams and beam-wall connection stiffnesses, the corresponding equation of the fourth type simplifies to

$$\left.\frac{dQ_{j,i}}{dx}\right|_{x=x_i} = \left.\frac{dQ_{j,i-1}}{dx}\right|_{x=x_i} \tag{26}$$

Substituting $Q_{j,i}$, which was obtained using the foregoing boundary conditions and Eqs. (9), (12) and (15), in the system of Eqs. (4) and integrating twice with respect to x,

$$y_i = \frac{1}{E I_i} \int\left[\int\left(M_e - \sum\limits_{k=1}^{m} Q_{k,i} L_{k,i}\right)dx\right]dx + H_i x + G_i \quad i = 1,2,...,n \tag{27}$$

To find the 2×n integration constants above, the following conditions are used:

a) The displacements and slopes between two neighboring sections being equal

$$y_i\big|_{x=x_i} = y_{i-1}\big|_{x=x_i}, \quad i = 2,3,...,n, \qquad \left.\frac{dy_i}{dx}\right|_{x=x_i} = \left.\frac{dy_{i-1}}{dx}\right|_{x=x_i}, \quad i = 2,3,...,n \tag{28}$$

b) The horizontal displacement at the bottom being

$$y_n\big|_{x=0} = \frac{W}{\sum\limits_{j=1}^{m+1} K_{hj}} \tag{29}$$

where W is the total horizontal force on the shear wall and the sum in the denominator is the total equivalent horizontal stiffness of the foundation.

c) The rotation at the bottom being

$$\frac{dy_n}{dx}\bigg|_{x=0} = \frac{\sum\limits_{k=1}^{m+1} M_{k,n}}{\sum\limits_{k=1}^{m+1} K_{rk}} \tag{30}$$

If the foundation is rigid, Eqs. (29) and (30) simplify to

$$y_n\big|_{x=0} = 0, \qquad \frac{dy_n}{dx}\bigg|_{x=0} = 0 \tag{31}$$

3. NUMERICAL RESULTS

To verify the foregoing analysis and its implementation in the MATHEMATICA computer algebra system, two examples have been solved for comparison purposes.

Example 1 is taken from Elkholy and Robinson [7] and can be seen in Fig. 3. This example has five rows of openings and is solved for various kinds of soils, first without stiffening beams as in the literature, and then with stiffening beams at one fourth and three fourths of the height. The horizontal displacements at the top of the wall are presented in Table 1 and Table 2, respectively, for the unstiffened and the stiffened walls. The results have been compared with those of the SAP2000 structural analysis computer program [21]. The height of the wall is 200 ft, the storey height is 10 ft, the wall thickness is 1 ft, $K_h = \infty$, the height of the connecting beams is 2 ft, the moment of inertia of the stiffening beams is 144 ft^4 and the elasticity modulus is 2×10^5 kip/ft^2.
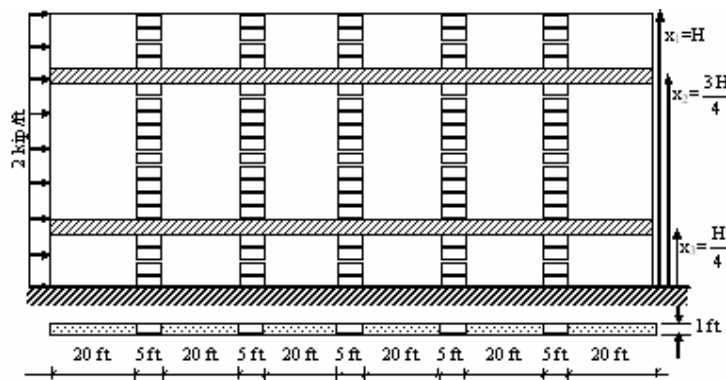

Fig. 3. Shear wall with five rows of openings

Table 1. Comparisons of the horizontal displacement at the top of an unstiffened
shear wall with five rows of openings

Soil properties		y_H (ft)			Difference* (%)	
K_v kip/ft	K_r kip-ft/rad	Ref. [7]	Present study	Ref. [21]	Ref. [7]	Present study
∞	∞	0.00945	0.008856	0.009007	4.92	1.67
9×10^6	3×10^8	0.00972	0.009110	0.009319	4.3	2.15
4×10^6	∞	0.00959	0.008880	0.009031	6.19	1.67
∞	1.5×10^8	0.00985	0.009250	0.009505	3.63	2.68
4×10^6	1.5×10^8	0.01000	0.009277	0.009531	4.92	2.66

* Difference with respect to SAP2000 results [21]

Example 2 is a coupled shear wall on elastic foundation with three stiffening beams and four rows of openings with various spans (Fig. 4). Foundation properties are:

Type 1 (Rigid) : $K_r=\infty$, $K_v=\infty$, $K_h=\infty$

Type 2 (Stiff)   : $K_r=2.72\times10^7$ kN-m/rad, $K_v=8.78\times10^5$ kN/m, $K_h=7.52\times10^6$ kN/m

Type 3 (Soft)   : $K_r=1.36\times10^7$ kN-m/rad, $K_v=4.39\times10^5$ kN/m, $K_h=3.76\times10^6$ kN/m

Table 2. Comparisons for the horizontal displacement of the top of a stiffened shear wall with five rows of openings

Soil properties		y_H (ft)		Difference * (%)
K_v kip/ft	K_r kip-ft/rad	Present study	Ref. [21]	Present study
∞	∞	0.00745414	0.007539	1.13
9×10^6	3×10^8	0.00769314	0.007821	1.63
4×10^6	∞	0.00747819	0.007562	1.11
∞	1.5×10^8	0.00781189	0.007987	2.19
4×10^6	1.5×10^8	0.00783869	0.008013	2.18
3×10^4	∞	0.01048323	0.010580	0.91
3×10^3	1×10^5	0.04891905	0.049458	1.09
6×10^2	2×10^4	0.21137822	0.211918	0.25

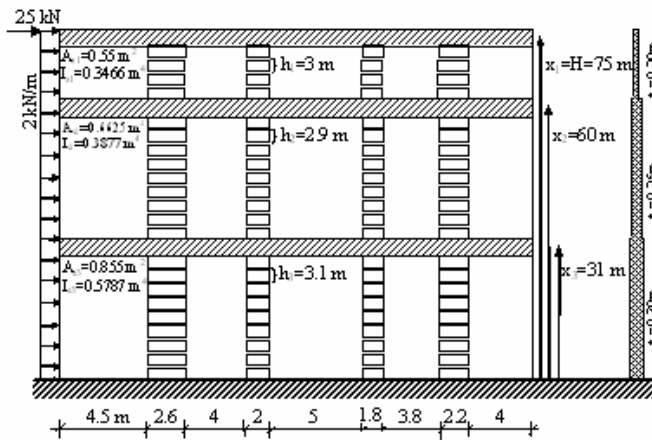* Difference with respect to SAP2000 results [21]

Fig. 4. Shear wall with four rows of openings

The results found by the present method and SAP2000 structural analysis program [21] are presented together in Table 3.

Table 3. Some results found by the present method and SAP2000 in Example 2

	Soil properties					
	Type 1		Type 2		Type 3	
	Present study	Ref. [21]	Present study	Ref. [21]	Present study	Ref. [21]
T_1 (kN)	172.4	174.1	192.7	193.7	202.7	203.5
T_2 (kN)	70.9	71.8	81.3	82.1	87.6	88.2
T_3 (kN)	37.1	35.8	47.6	45.7	43.7	42.1
T_4 (kN)	-109.7	-108.7	-129.1	-125.7	-131.9	-128.7
T_5 (kN)	-172.2	-172.9	-196.6	-195.8	-206.3	-205.2
$\sum M$ (kNm)	1938	1907	1178	1190	868	886
y_H (m) $\times10^{-5}$	222	221	301	299	351	351

4. CONCLUSIONS

The method proposed in the present work is advantageous in two ways. The first advantage stems from the fact that the data preparation is easier than the other methods which deal with one dimensional members.

Moreover, in trying different possibilities, minor changes in the data suffices for the purpose. The second advantage lies in the fact that the computer time needed to solve a problem with the present method is roughly one fifth of that needed to solve it by the finite difference approach of Elkholy and Robinson [7]. Hence, the present method proves to be an effective procedure for predesign purposes. In deciding on the properties of the shear wall and the positions of the stiffening beams, to be able to try a host of possibilities in a short amount of time and selecting the most suitable system, the present method seems to work just fine. However, the final solution should preferably be carried out using a more precise method.

NOMENCLATURE

a_j	open span of j'th bay
$A_{j,i}$	cross-sectional area of the j'th pier in section i
b_j	width of the j'th pier
C_{cbi}	connecting beam-pier connection stiffness in section i
C_{sbi}	stiffening beam-pier connection stiffness in section i
E	elasticity modulus
h_i	storey height in section i
H	total height of the wall
I_{si}	moment of inertia of the stiffener at height x_i
$I_{cj,i}$	moment of inertia of the connecting beams in the j'th bay of section i
$I_{j,i}$	moment of inertia of the j'th pier in section i
I_i	sum of the moments of inertia of the piers in section i
i	numbers of sections and borderlines between consecutive sections
j	numbers of bays and piers
K_{vj}, K_{hj}, K_{rj}	equivalent vertical, horizontal and rotational stiffness coefficients of the foundation of the j'th pier
$L_{j,i}$	distance between the axes of the j'th and (j+1)'st piers in section i,
$M_{j,i}$	bending moment in the j'th pier in section i
$M_e(x)$	moment of the load above w.r.t. height x
m	total number of bays
n	total number of sections in vertical direction
$q_{j,i}$	shear flow function in the j'th bay in section i
$Q_{j,i}$	integral of the shear flow function in the j'th bay, starting from the top of the shear wall in section i
T	transformation matrix
$V_{j,i}$	shear force in the j'th bay of the stiffener at height x_i
x_i	height of the uppermost point of section i
y_i	horizontal displacement function in section i
$w(x)$	horizontal load function
δ_{0j}	relative vertical displacement of the bases of the j'th and (j+1)'st piers

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