"Research Note"

DERIVATION OF RESERVOIR'S AREA-CAPACITY EQUATIONS BASED ON THE SHAPE FACTOR[*]

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Abstract– Area-capacity curves are of the most important physical characteristics of dam reservoirs. These curves are used for reservoir flood routing, reservoir operation, prediction of sediment distribution in reservoirs, etc. In this study, the linear characteristic of a reservoir's depth-capacity data on log-log paper is used and the mathematical equation of the reservoir's capacity-depth curve is obtained based on the shape factor, M. The reservoir's area-depth equation and equation of shape factor are obtained by differentiating the reservoir's capacity-depth equation. The obtained equations are evaluated with the area-capacity data of 8 reservoirs in the United States. The results of this study showed that the obtained equations agree well with the actual data.

1. INTRODUCTION

Dams are important hydraulic structures which are built across rivers for flood control, irrigation, municipal and industrial water supply, hydropower production, etc [1, 2]. Area-capacity curves are of the most important physical characteristics of dams' reservoirs. These curves are used for reservoir flood routing, dam operation, determination of water surface area and capacity corresponding to each elevation, reservoir classification [3] and prediction of sediment distribution in reservoirs [3-7]. As a result, obtaining the area-capacity equations of reservoirs has great significance from a practical aspect.

Mohammadzadeh-Habili et al. [1] derived new equations for reservoir's dimensionless depth-capacity and area-depth curves as

$$p = \ln(v^{N} + 1)/\ln 2 \tag{1}$$

$$a = 0.5\exp.[p\ln(2)]\left(\exp.[p\ln(2)] - 1\right)^{\frac{1-N}{N}} \tag{2}$$

where p = relative depth; v = relative volume; a = relative area; and N = reservoir coefficient. p, v, and a are defined as

$$p = y/y_m \tag{3}$$

$$v = V_y/V_m \tag{4}$$

$$a = A_y/A_m \tag{5}$$

where V_m = reservoir capacity at maximum water level; y_m = reservoir depth from streambed to maximum water level; A_m = reservoir surface area at maximum water level; y = reservoir depth from streambed; V_y = reservoir capacity at depth y; and A_y = reservoir surface area at depth y.

In this study, the linear characteristic of the reservoir's depth-capacity data on log-log paper is used and the mathematical equation of capacity-depth curve is obtained based on the shape factor, M. The reservoir's area-depth equation and equation of shape factor are obtained by differentiating the capacity-depth equation. The obtained equations are evaluated with the area-capacity data of 8 reservoirs in the United States.

2. GOVERNING EQUATIONS

If the reservoir depth is plotted as ordinate against the reservoir capacity as abscissa on a log-log paper, it will be a nearly straight line and the reciprocal of the line slope is equal to the reservoir's shape factor, M [3]. Because of the logarithmic scale of coordinate axes, the dimensionless reservoir's depth-capacity data will be a straight line, too. As an example, the dimensionless depth-capacity data of Pilot Butte Reservoir is shown on log-log coordinate in Fig. 1.
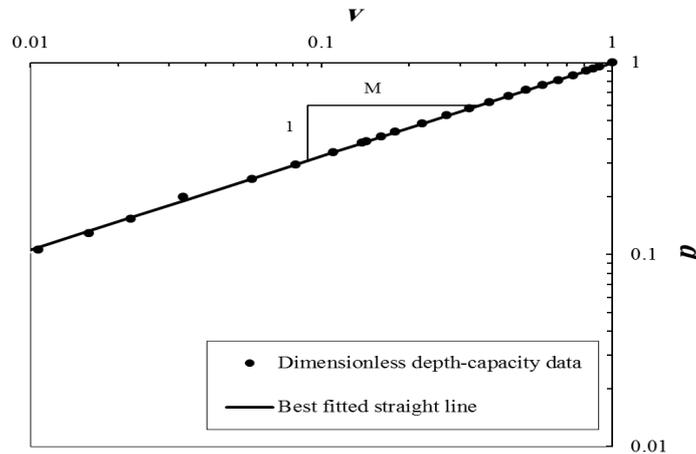

Fig. 1. The dimensionless depth-capacity data of Pilot Butte Reservoir on log-log coordinate

Since the dimensionless depth-capacity data pass through $p=1$ and $v=1$ on log-log coordinate, the relationship between v and p can be expressed as

$$\log(p) = \frac{1}{M}\log(v) \text{ or } p = v^{1/M} \tag{6}$$

Substituting p and v from Eqs. (3) and (4) into Eq. (6) and simplifying, the reservoir's capacity-depth equation is obtained as

$$V_y = V_m \left(y / y_m \right)^{M} \tag{7}$$

By differentiating V_y with respect to y, the reservoir surface area equation can be obtained as

$$A_y = \frac{dV_y}{dy} = \frac{MV_m}{y_m^{M}} y^{M-1} \tag{8}$$

By substituting $y = y_m$ in Eq. (8), the maximum water surface area A_m is derived as

$$A_m = MV_m / y_m \tag{9}$$

It follows from Eq. (9) that the reservoir shape factor M is equal to

$$M = y_m A_m / V_m \qquad (10)$$

Dividing both sides of Eq. (8) by Eq. (9), and using Eqs. (3) and (5) gives

$$a = p^{M-1} \qquad (11)$$

To evaluate the obtained equations, the original (at operation time) area-capacity data of 8 reservoirs [8] in the western and central parts of the United States are used. The percent of average error between the observed and estimated values is calculated as

$$\text{Percent of average error} = \frac{\sum_{i=1}^{n}(100 \times \frac{O_i - E_i}{O_i})}{n} \qquad (12)$$

where O_i = observed value; E_i = estimated value; and n = number of data.

3. RESULTS AND DISCUSSIONS

For validation of the obtained equations, the shape factor of each reservoir should be estimated at the first step. The value of M can be estimated either by minimization of the sum of squared errors (SSE) or using Eq. (10). Estimated value of M from minimization of SSE is more precise, because in this method the value of M is obtained through a trial and error procedure to minimize the value of SSE between the curve of Eq. (6) and the reservoir's dimensionless depth-capacity data so as to ensure the curve of Eq. (6) has the best possible fit to the reservoir's dimensionless depth-capacity data. Estimated value from Eq. (10) can be taken as an initial value to get the exact value of M using minimization of SSE. This initial value can minimize the time of trial and error procedure, and therefore a quicker estimation of the exact value of M can be acheived.

Fig. 2 shows the dimensionless depth-capacity data of the studied reservoirs. For each reservoir, the curve of Eq. (6) is also plotted for comparison (for each reservoir, the value of M in Eq. (6) is obtained by minimization of SSE between the dimensionless depth-capacity data and the curve of Eq. (6)). In these figures, some of the reservoirs have similar value of M.

It follows from Fig. 2 that the dimensionless depth-capacity data of all the reservoirs agree very well with Eq. (6).
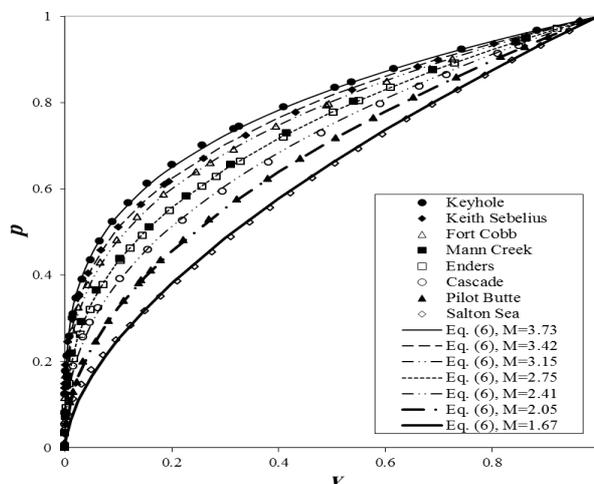

Fig. 2. Dimensionless depth-capacity data of the studied reservoirs and the curves of Eq. (6)

Fig. 3 shows the dimensionless area-depth data of the studied reservoirs. For each reservoir, the curve of Eq. (11) is also plotted for comparison (for each reservoir, the value of M in Eq. (11) is obtained by minimization of SSE between the dimensionless depth-capacity data and the curve of Eq. (6)).

It follows from Fig. 3 that the dimensionless area-depth data of all the reservoirs agree well with Eq. (11).
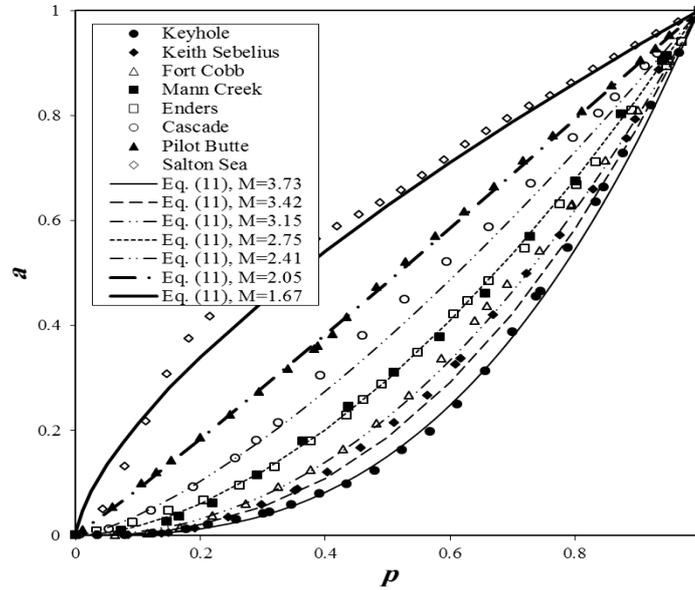

Fig. 3. Dimensionless area-depth data of the studied reservoirs and the curves of Eq. (11)

The percent of average error between the area-capacity data and the curves of the obtained equations is calculated from Eq. (12). Results indicated that, for all the reservoirs, the percent of average error is less than 1 percent. These reveal that the obtained equations fit very well to the area-capacity data.

Sedimentation causes the reservoir geometry to vary with time. Variation of reservoir geometry can be determined by analysis of variation of M. Fig. 4 shows the effect of sedimentation on the dimensionless depth-capacity data of Nambe Falls Reservoir[8].
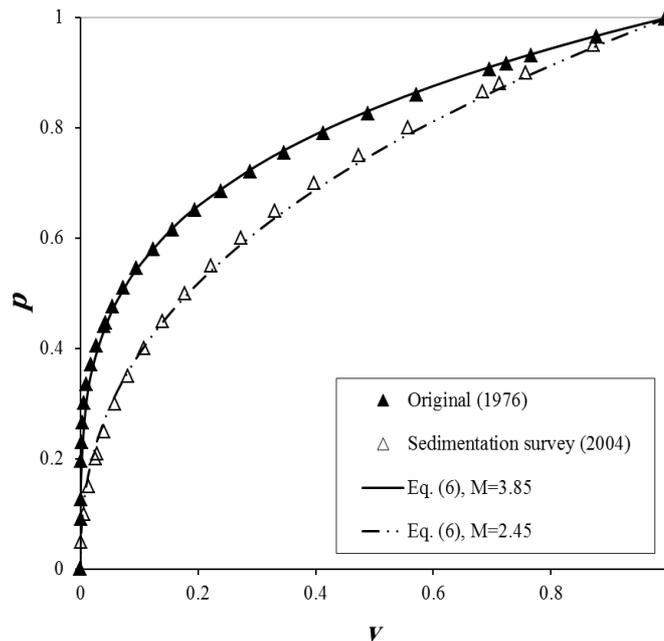

Fig. 4. The effect of sedimentation on the reservoir geometry of Nambe Falls Dam

Obtained equations in this study are based on linear assumption of depth-capacity data on log-log paper. Therefore, for reservoirs whose depth-capacity data on log-log paper are nearly fitted along a straight line, the obtained equations have the best possible fit to the reservoir's data.

4. CONCLUSION

In this study, the linear characteristic of a reservoir's depth-capacity data on log-log paper is used and the mathematical equation of reservoir's capacity-depth curve is obtained based on the shape factor, M. The reservoir's area-depth equation and equation of shape factor are obtained by differentiating the reservoir's capacity-depth equation. The obtained equations are simpler in comparison with the previous equations. They are evaluated with the area-capacity data of 8 reservoirs. Results of this study showed that the obtained equations agree well with the actual data. Variation of reservoir geometry due to sedimentation can also be determined using the obtained equations.

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